MATHCOUNTS®

2013
■ State Competition ■
Countdown Round
Problems 1–80

This section contains problems to be used in the Countdown Round.

1. (\$) _____ At Rocket Dogs, four hot dogs and three orders of fries cost \$9. Six hot dogs and five orders of fries cost \$14. What is the total cost of one hot dog and one order of fries?

2. _____ If $a + b - 4z = 500$ and $a + b + 6z = 1200$, what is the value of $a + b$?

3. _____ What is the value of $1^0 + 2^0 + 3^0 + \cdots + 2013^0 - (1 + 2 + 3 + \cdots + 2013)^0$?

4. _____ What is the positive difference between the median and the mean of 18, 21, 24, 27, 31, 35 and 40?

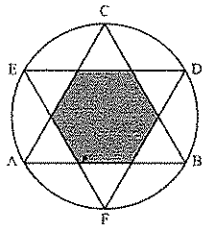
5. _____ (calories) If one bar of chocolate squares is 210 calories, and there are 15 squares in a bar, how many calories are in 1 chocolate square?

6. _____ (minutes) How many minutes will it take to drive 30 miles at an average rate of 40 miles per hour?

7. _____ (days) If three painters can paint three houses in three days, how many days does it take one painter to paint two houses?

8. _____ What is the units digit of the sum $1^2 + 2^2 + 3^2 + \cdots + 10^2$?

9. _____ Equilateral triangles ABC and DEF are inscribed in a circle with their corresponding vertices at diametrically opposite points, as shown. What fraction of the area of ΔABC is gray? Express your answer as a common fraction.



10. _____ What is the value of $(1^{-1} + 2^{-1})^{-1}$? Express your answer as a common fraction.

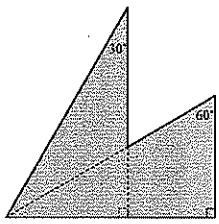
11. _____ The eighth term of an arithmetic sequence is ten times the first term, and the third term of the sequence is 50. What is the first term of the sequence?

12. _____ (factors) How many positive integer factors does 108 have?

13. _____ (dimes) Twelve coins, consisting of dimes and nickels, are worth 85 cents. What is the number of dimes?

14. _____ If $\frac{5}{m} \geq \frac{2}{3}$, what is the largest possible value of m? Express your answer as a decimal to the nearest tenth.

15. _____ (in^2) Two 30-60-90 triangles, each with hypotenuse 12 inches, overlap so that they share a vertex and part of a side, as shown. What is the total area, in square inches, of the resulting figure? Express your answer in simplest radical form.



16. _____ If $x^2 - 45b^2 = 4xb$, what is the largest possible ratio of b to x? Express your answer as a common fraction.

17. _____ (integers) How many positive two-digit integers are multiples of 6?

18. _____ Both p and q are positive integers. Each of the numbers 29, 84 and 128 leaves a remainder of p when divided by q. What is the sum of $p + q$?

19. _____ (marbles) A bag contains only red and black marbles. Originally the ratio of the black marbles to red marbles was 4:5. After 18 black marbles were added, the ratio of black marbles to red marbles became 5:4. How many marbles were in the bag originally?

20. _____ (points) If Consuela has a mean score of 84 out of 100 points for five tests, what is the lowest possible test score she could have received?

21. _____ In the sequence 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, …, each positive integer k appears k times. If the integer 12 first appears in the sequence as the nth term, what is the value of n?

22. _____ (inches) The legs of a right triangle measure $1\frac{2}{3}$ inches and 4 inches. In inches, what is the length of the hypotenuse? Express your answer as a mixed number.

23. _____ What is the greatest single-digit positive integer n for which $n!$ leaves no remainder when divided by 2^{n-1}?

24. _____ What is the value of $1313^2 - 1212^2$?

25. _____ (minutes) Sam glances at the clock and notices that the minute hand is 55° behind the hour hand. In how many minutes will the hands overlap?

26. _____ For what positive integer n is $2^n < 100 < 2^{n+1}$?

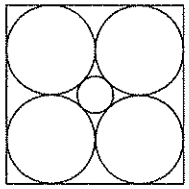
27. _____ What is the least value of n, such that $n > 1$ and the value of $\sqrt{1+2+3+\cdots+n}$ is an integer?

28. _____ (zeros) How many zeros are to the right of the last nonzero digit of 78!?

29. _____ (units^2) A triangle is bounded by the lines $x = 3$, $y = 10$ and $y = -x + 11$. What is its area, in square units?

30. _____ (students) All but two students in the algebra class took a test, and the mean score was 84. When the scores of 100 and 94 for the missing two students were included, the average increased by 1. How many students are in the class?

31. _____ (units) In the figure, each of the four large circles is tangent to two of the other large circles, the small circle, and two sides of the square. If the radius of each of the large circles is 1 unit, what is the radius of the small circle, in units? Express your answer in simplest radical form.



32. _____ For what positive integer n is the mean of the first n positive integers equal to 2013?

33. _____ For positive integers a and b, $\frac{2}{7}$ of the product ab is 60, and $\frac{3}{5}$ of the quotient $\frac{a}{b}$ is $\frac{9}{14}$. What is the value of a?

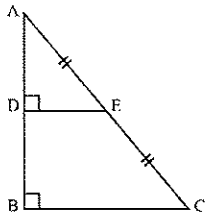
34. _____ An arithmetic progression has 20 as its 12th term and 12 as its 20th term. What is the 2013th term?

35. _____ What number is 10% of one-third of 750?

36. _____ (points) During the basketball season, Cara scored 25 points more than Annika, and Maria scored 37 points less than Annika. What is the positive difference between the average number of points scored by the three players and the number of points that Annika scored?

37. _____ The quadratic equation $x^2 + bx + c = 0$ has real roots 4 and -6. What is the value of $b + c$?

38. _____ If a randomly thrown dart hits the interior of ΔABC, shown here, what is the probability that it also hits the interior trapezoid BDEC? Express your answer as a common fraction.



39. _____ (mm) The diagonals of rhombus MNOP, have lengths of 12 mm and 16 mm. What is the perimeter, in millimeters, of rhombus MNOP?

40. _____ What is the sum of the positive factors of 45?

41. _____ (cm) An isosceles trapezoid with one base twice as long as the other has a height of 12 cm. The shorter base is 18 cm. In centimeters, what is the perimeter of this trapezoid?

42. _____ What is the largest three-digit positive integer with exactly three positive factors?

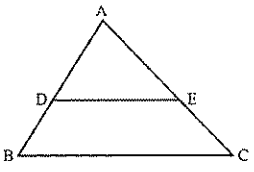
43. _____ (dollars) A fuel tank holds 200 gallons of gasoline when full. The tank is currently $\frac{4}{5}$ full. If fuel costs \$5.50 per gallon, how many dollars worth of fuel are needed to fill the rest of the tank?

44. _____ (odd numbers) Using four of the five digits 1, 5, 6, 8 and 9, how many four-digit odd numbers can be formed, with no repeated digits?

45. _____ (inches) In a right triangle, the longer leg and the hypotenuse have consecutive integer lengths whose sum is 121 inches. In inches, what is the perimeter of the triangle?

46. _____ What is the units digit of the product $1 \times 3 \times 5 \times 7 \times 9 \times \ldots \times 2013$?

47. _____ (cm^2) In the figure, the area of trapezoid DBCE is 80 cm^2. The ratio of the bases DE to BC is 3:5. What is the area of triangle ADE, in square centimeters?

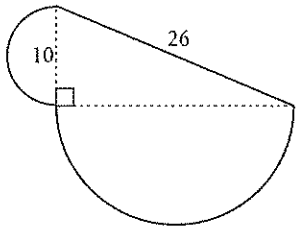


48. _____ What is the units digit of the product 2×4^{130}?

49. _____ If $\frac{2}{x} = w$ and $x \neq 0$, what is the value of $(wx)^{-3}$? Express your answer as a common fraction.

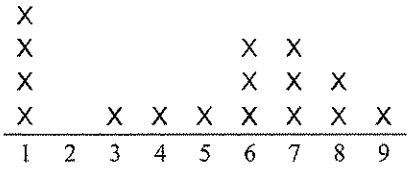
50. _____ What is the value of $133^2 + (2)(67)(133) + 67^2$?

51. _____ (mm) The figure shown is composed of two semi-circles and one right triangle with a leg of 10 mm and a hypotenuse of 26 mm. What is the perimeter of the figure, in millimeters? Express your answer in terms of π.



52. _____ If 25% of 16 is x, what is x% of 200?

53. _____



What is the mean of the data represented in the frequency distribution shown?

54. _____ (dollars) Dianna's weekly paycheck is 15% less than Barbara's paycheck. Dianna's and Barbara's paychecks have a combined total \$740. How much is Dianna's paycheck, in dollars?

55. _____ What is the least positive integer n, for which there are positive integers x and y such that $x^2 + y^3 = n$ and $n > 60$?

56. _____ (dots) On the number line shown, there are dots at every integer and three dots between each pair of consecutive integers. If all of the dots are evenly spaced, how many dots lie strictly between a (not shown) and $\sqrt{a}$?

57. _____ (cm^2) A square has a diagonal of length a cm. In square centimeters, what is twice the area of the square in terms of a?

58. _____ (integers) How many positive integers are a factor of 7! but not a factor of 6!?

59. _____ If $k = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots + \frac{1}{20}$, and $a < k < a + 1$ where a is an integer, what is the value of a?

60. _____ If k is a constant, and $2x + 5 = 3kx + 5$ for all values of x, what is the value of k? Express your answer as a common fraction.

61. _____ If $\frac{1}{c^2} = b^2 + 4b + 4$, where b and c are positive, then what is the value of c in terms of b? Express your answer as a common fraction.

62. _____ What is $\frac{2}{3}$ of 30% of 362,970?

63. _____ If $f(x) = \frac{x+3}{3x-1}$ and $g(x) = \sqrt{2x^3}$, what is the value of $f\left(\frac{1}{2}\right) + g(2)$?

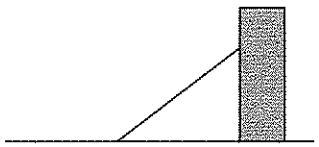
64. _____ What is the maximum integer value of x for which 2^x is less than each of the numbers: 4^{13}, 8^9 and 16^7?

65. _____ (points) Anne was absent for the last algebra test. Her 19 classmates had a class average of exactly 77 points on this test. How many points must Anne score on this test to raise the class average by exactly one point?

66. _____ If $f(x) = x - 11$ and $g(x) = 2x^2 + 3x - 2$, what is the value of $g[f(10)]$?

67. _____ (degrees) Three angles of a convex pentagon measure 88°, 124° and 92°. The remaining two angles are congruent. What is the degree measure of each of the remaining angles?

68. _____ (feet) The top of a pole rests 8 ft above the ground along a wall. If the bottom of the pole is moved another 2 ft from the wall, the entire pole will lie on the ground with the top of the pole touching the wall. In feet, how long is the pole?



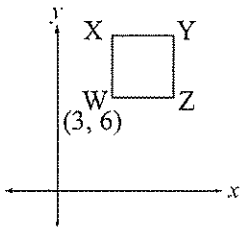
69. _____ (integers) How many integers x satisfy $|13\pi x| < -\pi$?

70. _____ What is the closest integer to 10.01^3?

71. _____ In the equation $(x - 8)(x - k) = x^2 - 5kx + m$, k and m are positive integers. If the equation is true for all values of x, what is the value of m?

72. _____ If $3^x + 3^x + 3^x + 3^x + 3^x + 3^x + 3^x + 3^x + 3^x = 3^n$, what is the value of n, in terms of x?

73. _____ Square WXYZ is located in the first quadrant so that segments WX and YZ are parallel to the y-axis. The coordinates of W are (3, 6), and the square has area 16 units^2. If square WXYZ is translated 2 units up and 6 units to the right, what is the sum of the coordinates of the image of Y?



74. _____ If $4 + \sqrt{b} = 7.2$, what is the value of $4 - \sqrt{b}$? Express your answer as a decimal to the nearest tenth.

75. _____ (days) A chef spends c hours each day at her restaurant, $\frac{c}{3}$ hours of which she spends preparing desserts. At this rate, after how many days will the chef have spent 24 hours preparing desserts? Express your answer as a common fraction in terms of c.

76. _____ The points (6, 1) and (8, 1) lie on the graph of $y = f(x)$, where $f(x) = |x - k|$ and k is a constant. What is the value of $f(0)$?

77. _____ (ways) Four sets of lights surround a baseball field. The field is sufficiently lit when at least two of the sets are on. In how many different ways can lights be turned on so that the field is sufficiently lit?

78. _____ If $(m + n)^{k-3} = m^6 + 6m^5n + 15m^4n^2 + 20m^3n^3 + 15m^2n^4 + 6mn^5 + n^6$, what is the value of k?

79. _____ (turns) Bart played a game in which he could score either 9 or 14 points on each turn. If Bart scored exactly 100 points, on how many turns did he score 9 points?

80. _____ (boys) In January 2013, b boys and g girls belonged to a math club. Three girls and some boys joined the math club in February 2013 and the ratio of boys to girls remained the same. In terms of b and g, how many boys joined the math club in February 2013? Express your answer as a common fraction.

Countdown Round

1. ($) 2.50
2. 780
3. 2012
4. 1
5. 14 (calories)
6. 45 (minutes)
7. 6 (days)
8. 5
9. $\frac{2}{3}$
10. $\frac{2}{3}$
11. 14
12. 12 (factors)
13. 5 (dimes)
14. 7.5
15. $30\sqrt{3}$ (in^2)
16. $\frac{1}{9}$
17. 15 (integers)
18. 18
19. 72 (marbles)
20. 20 (points)
21. 67
22. $4\frac{1}{3}$ (inches)
23. 8
24. 255,025
25. 10 (minutes)
26. 6
27. 8
28. 18 (zeros)
29. 2 (units^2)
30. 26 (students)
31. $-1 + \sqrt{2}$ (units) or $\sqrt{2} - 1$
32. 4025
33. 15
34. −1981
35. 25
36. 4 (points)
37. −22
38. $\frac{3}{4}$
39. 40 (mm)
40. 78

Countdown Round

41. 84 (cm)

42. 961

43. 220 (dollars)

44. 72 (odd numbers)

45. 132 (inches)

46. 5

47. 45 (cm^2)

48. 2

49. $\frac{1}{8}$

50. 40,000

51. $17\pi + 26$ (mm)
or $26 + 17\pi$

52. 8

53. 5

54. 340 (dollars)

55. 63

56. 14 (dots)

57. a^2 (cm^2)

58. 30 (integers)

59. 2

60. $\frac{2}{3}$

61. $\frac{1}{b+2}$

62. 72,594

63. 11

64. 25

65. 97 (points)

66. -3

67. 118 (degrees)

68. 17 (feet)

69. 0 (integers)

70. 1003

71. 16

72. $x + 2$

73. 25

74. .8 or 0.8

75. $\frac{72}{c}$ (days)

76. 7

77. 11 (ways)

78. 9

79. 8 (turns)

80. $\frac{3b}{g}$ (boys)