

The 2016-2017 School Handbook Problems

Start here to choose a math problem to solve! We have reformatted the 2016-2017 MATHCOUNTS School Handbook exclusively for the Math Video Challenge. In this playbook, all 250 of the handbook problems are organized by math topic.

Because problems are organized by math topic, the problem numbers are not in sequential order. We've written the problem number to the left of each problem. You can find the answer key on [pgs. 38-39](#) and a complete Problem Index, including difficulty level and mapping to the Common Core State Standards, on [pgs. 40-42](#).

If your team would prefer to use the version of the handbook formatted for the Competition Series (divided into Warm-Ups, Workouts and Stretches), you can go to www.mathcounts.org/handbook to download a free copy of that version. **Here is where you can find this year's handbook problems in this playbook.**



Algebraic Expressions
+ Equations
[pgs. 16-17](#)



Number Theory
[pgs. 21-23](#)



Proportional Reasoning
[pgs. 34-35](#)



Coordinate Geometry
[pgs. 17-18](#)



Percents + Fractions
[pgs. 24-25](#)



Sequences, Series
+ Patterns
[pgs. 35-36](#)



General Math
[pgs. 18-19](#)



Plane Geometry
[pgs. 25-29](#)



Solid Geometry
[pgs. 36-37](#)



Logic
[pgs. 19-20](#)



Probability, Counting
+ Combinatorics
[pgs. 30-33](#)



Statistics + Data
[pg. 37](#)



Measurement
[pgs. 20-21](#)

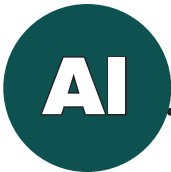


Problem Solving
(Misc.)
[pg. 33](#)



These playbook problems are not in sequential order.

They're based on the 2016-2017 MATHCOUNTS School Handbook, and we've put the problem number to the left of each math problem. You can find the answers on [pg. 38](#).



Algebraic Expressions + Equations

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Get the answers on [pg. 38](#).



Problem #

25 Esme is thinking of two integers. One integer is 4 times the other, and their sum is 18 more than 3 times the smaller integer. What is the smaller of the integers Esme is thinking of?



33 The population of lemmings on an island in Norway varies drastically. In a base year, the population was n lemmings. The next year, it tripled. The third year, the number dropped by 3000 from the second year. The fourth year, the population was $\frac{1}{2}$ that of the third year. The fifth year, it increased by 1300 to a total of 1450 lemmings. What is the value of n ?

49 In exchange for 5 ziggles and 4 zoggles, Jefferson gets 30 zaggles. In exchange for 2 ziggles and 3 zoggles, Monroe gets 19 zaggles. How many zaggles should Carter expect to get in exchange for 1 ziggle and 1 zoggle?

53 The mean of a and b is 8. The mean of b and c is 16. The mean of a and c is 14. What is the value of $a + b + c$?

56 If $(n) = n^2 - n$, what is the value of (5) ?

58 What is the value of $2x^2 + 3y^2 - 4x + 2y - 17$ when $x = 3$ and $y = -2$?

64 The quotient $\frac{x^2(x^2)^3}{x^2}$ can be expressed as x^y . What is the integer value of y ?

79 The absolute difference between two numbers is 6, and the absolute difference between their squares is 24. What is the product of the two numbers?

80 If $a \odot b$ is defined as $a^2 - 2b^2$. What is $5 \odot (4 \odot 3)$?

87 The cost of 1 binder with photos of celebrities on the cover plus the cost of 8 regular binders is a total of \$32.60. The cost of 1 binder with photos of celebrities on the cover plus the cost of 12 regular binders is a total of \$46.00. How much more does it cost to buy a celebrity binder than a regular binder?

88 Mr. Jones makes 3% commission on his sales of widgets. At a different company, Mr. Smith makes 5% commission selling the same widgets at the same price. Mr. Smith sold 500 fewer widgets than Mr. Jones, and they both earned the same commission. How many widgets did Mr. Smith sell?

89 The table of values shows the relationship between x and y , which can be modeled with the equation $y = ax^b$, for integers a and b . What is the value of $a + b$?

x	2	3	5	6
y	8	27	125	216

92 If $a = 12$, $b = 4$, $c = 5$ and $x = \frac{1}{2}$, then what is the value of $\frac{\left(\frac{abc}{x}\right) - (6b^2 - 4)}{0.5}$?

Problem #

- 102** If $f(x) = x^2 - 2$ and $g(x) = 2x + 4$, what is the value of $f(g(-3))$?
- 111** If $\frac{x}{y} = 10$, $x = 3z$ and $z = 20$, what is the value of y ?
- 123** The graphs of $y = x^2 - 3x + 3$ and $4x - 12y = -19$ intersect in two points. What is the sum of the x -coordinates of those points? Express your answer as a common fraction.
- 141** Two positive integers have a sum of 11 and a product of 24. What is the absolute difference between those two numbers?
- 144** Joe has some nickels, dimes and quarters. He has 37 coins in all, with 4 more nickels than dimes and 2 more quarters than nickels. How many quarters does Joe have?
- 158** What is the value of the expression $\frac{2017^2 + 11(2017) - 42}{2014}$?
- 170** If x and y are integers, such that $x > y$, $(x + y)^2 = 9$ and $x^2 + y^2 = 29$, what is the smallest possible value for x ?
- 172** Because of a traffic jam, Alana's 18-mile commute to work took 4 minutes longer than usual, and her average speed was decreased by 9 mi/h. How many minutes did it take her to get to work that day?
- 188** A rectangle has a diagonal of length 8 inches and an area of 26 square inches. What is its perimeter? Express your answer in simplest radical form.
- 196** A private jet made a trip from Denver to Los Angeles in 3 hours, flying against a steady headwind. On the return trip the wind speed doubled and became a tailwind. The return trip took only 2.5 hours. If the plane's speed on the return trip was 450 mi/h, what was the speed of the original headwind?
- 201** Given $a - b = 3$ and $a^2 + b^2 = 65$, what is the value of $a^3 - b^3$?
- 204** $M(a, b)$ is the midpoint of the longest side of the triangle bounded by the lines $x + 2y = 8$, $5x + 2y = 48$ and $x - 2y = 0$. What is the value of $a + b$? Express the answer as a common fraction.



Coordinate Geometry

These playbook problems are **not** in sequential order.

Get the answers on [pg. 38](#).

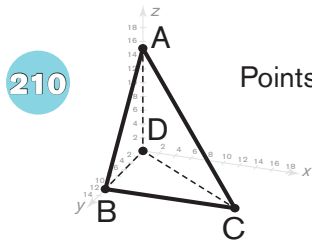


Problem #

- 12** How many units away from the origin is the point $(-3, -4)$?
- 20** On a coordinate grid, point B is located 8 units below and 2 units to the left of $A(0, 6)$. What is the length of segment AB? Express your answer in simplest radical form.
- 113** What is the area of a circle that has diameter AB with endpoints $A(-2, 4)$ and $B(10, 2)$? Express your answer in terms of π .

Problem #

- 179 Jason's line has a slope of $-\frac{1}{3}$ and contains the point $(5, -2)$. Amisha's line is perpendicular to Jason's and passes through the point $(4, 1)$. If the intersection of these lines is (x, y) , what is the value of $x + y$? Express the answer as a common fraction.



Points $A(0, 0, 15)$, $B(0, 12, 0)$, $C(16, 12, 0)$ and $D(0, 0, 0)$ determine the vertices of a tetrahedron as shown. What is the shortest distance from B to the face ADC ? Express your answer as a common fraction.

- 219 Circle A has center $(0, 0)$ with radius 4. Circle B has center $(40, 40)$ with radius 6. The radius of circle B increases at twice the rate as the radius of circle A increases. When the circles are externally tangent at $P(x, y)$, which is located between the centers of circles A and B , what is the value of x ? Express your answer as a fraction in simplest radical form.



General Math

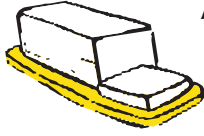
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Get the answers on [pg. 38](#).

Problem #

- 1 A time capsule was sealed in 1940 and will be opened on the same date in 2017. How long will the capsule remain sealed?
- 4 What is the value of $4 \div \frac{2}{3} - 5$?
- 8 In scientific notation, what is the product of 1.2×10^3 and 1.4×10^2 ? Express your answer to two significant figures.
- 10 Minnie paid a one-time registration fee of \$30 for dance lessons. Additionally, she paid \$20 per lesson. If she took seven lessons, how much did she pay altogether?
- 14 In Oregon, which has no sales tax, Gloria bought three notebooks for \$1.57 each. If she paid \$5.00, what is the least number of U.S. coins that she could get in change?
- 24 A room has 23 rows of 27 chairs each. How many chairs are in the room?
- 43 What is the coordinate of the point on a number line that is $\frac{2}{3}$ of the way from -1.3 to $3\frac{1}{8}$? Express your answer as a decimal to the nearest hundredth.



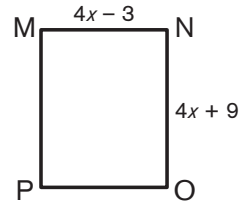
44



A stick of butter is a $1\frac{1}{2}$ -inch by $1\frac{1}{2}$ -inch by $3\frac{1}{4}$ -inch rectangular prism and contains 800 calories. How many calories are in a pre-formed “pat” of butter measuring 1 inch by 1 inch by $\frac{3}{8}$ inch? Express your answer to the nearest whole number.

54

Rectangle MNOP has length $4x + 9$ and width $4x - 3$. What is the absolute difference between the length and width of rectangle MNOP?



61

What is the value of $(2 \times 6^3 + 6^2) - 7 \times 6^2$?

70

What is the value of the following expression?

$$2^{0^{12}} - 2^{2^{10}}$$

Lo

Logic

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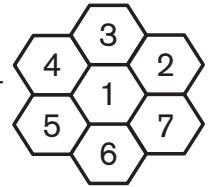
Get the answers on [pg. 38](#).



Problem #

3

In the figure shown, if no cell may be visited more than once and not every cell must be visited, how many paths start in cell 1 and end in cell 7?



22



Kelly has 6 identical white socks and 5 identical black socks in a drawer. If she selects without looking, how many socks must she take from the drawer to be assured of a matching pair?

37

Kylie writes a 1 after A, a 2 after B, and so on, writing a single digit in counting order after each of the first nine letters of the alphabet. When she reaches 10, she writes the 1 after J and the 0 after K. When she reaches the Z, the 1 from 18 follows it; then, she cycles back to the start of the alphabet, and the 8 follows the A. After what letter does Kylie write the 6 in 26?

63

Fido has to climb five stairs. If he steps on at least three of the five stairs, but never climbs more than three stairs in one step, in how many possible ways can Fido climb the stairs?

98

What is the maximum number of distinct intersections of 30 different coplanar circles?

130

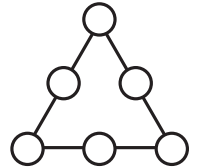
Tony chooses a positive integer k . After his friends make the following five statements, Tony says that exactly two of them are correct. What is the least possible value of k ?

- Bruce guesses that k is a multiple of 15.
- Steven guesses that k is a multiple of 18.
- Thor guesses that k is a multiple of 20.
- Clint guesses that k is a multiple of 28.
- Natasha guesses that k is a multiple of 60.

Problem #

167 Tryouts were held for three positions on the school basketball team: center, guard and forward. There were 4 players who tried out for center, 10 for guard and 10 for forward. These numbers include 1 player who tried out for center and guard, 3 who tried out for center and forward, and 4 who tried out for guard and forward. All six of these counts include 1 player who tried out for all three positions. If 17 players, in all, tried out for the team, how many players tried out only for guard?

199 The numbers 1, 2, 3, 4, 5 and 6 are to be placed in this figure, one number per circle, so that the sums of the numbers on each side of the triangle are the same. How many distinct solutions are there, not including rotations and reflections?




Me

Measurement

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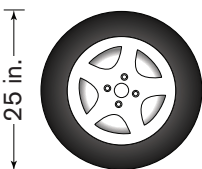
Problem #

11 How many pieces that are exactly 5 inches long can Sue cut from a string that is 7 feet long?

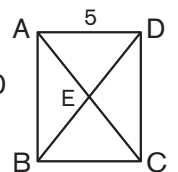
15  The mass of a fluorine atom is 3.16×10^{-23} g. What is the mass of 1,000,000,000 fluorine atoms? Express your answer in scientific notation to three significant figures.

29 Kevin takes a bus from home to school. The bus travels 8 miles west, then turns and travels 8 miles north, then turns and travels 7 miles west to the school. If the bus were able to travel directly from Kevin's house to the school, along a straight path, how much shorter would the trip be?

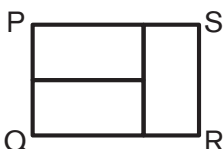


35  The tires on a certain car are 25 inches in diameter. If the car is moving at a constant speed of 65 miles per hour, how many rotations per second is the front left tire making? Express your answer as a decimal to the nearest tenth. (1 mile = 5280 feet)

45 In rectangle ABCD, shown here, $AD = BC = 5$ units. Diagonals AC and BD, each of length 10 units, intersect at E. What is the degree measure of $\angle AEB$?



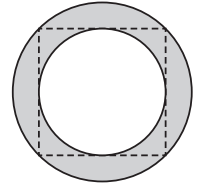
50 When leaned against a vertical structure, a straight ladder can be used safely if its top is no more than 4 feet above the base of the structure for every foot that the bottom of the ladder is away from the base. How high can a 22-foot ladder safely reach up a vertical structure? Express your answer as a decimal to the nearest tenth.

69  The figure shows rectangle PQRS composed of three congruent rectangles. If the area of PQRS is 1536 cm^2 , what is its perimeter?

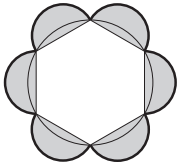
Problem #

- 75 The width of a rectangle is one-third of its length. If the perimeter of the rectangle is 136 cm, what is its area?
- 83 Squares A and B have at least one point in common. The area of square A is 225 cm^2 and the area of square B is 16 cm^2 . What is the maximum distance between the centers of the squares? Express your answer as a common fraction in simplest radical form.
- 86 Oberon and Lance sit directly opposite each other at a large round table. Arthur sits at the same table, 20 feet from Oberon and 21 feet from Lance. What is the diameter of the table?

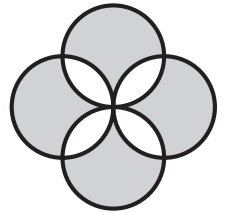
- 119 The figure shows a square inscribed in a circle of radius 12 inches, and another circle inscribed within that square. What is the area of the shaded region between the two circles? Express your answer in terms of π .



- 125 A regular hexagon is inscribed in a circle of radius 1 unit. On each side of the hexagon, a semicircle is constructed with the side of the hexagon as a diameter, as shown. What is the perimeter of the figure formed by these semicircles? Express your answer as a decimal to the nearest tenth.



- 146 Lucky draws a four-leaf clover by shading portions of four overlapping circles of radius 2 cm as shown. What is the area of the shaded regions?



- 173 The standard gravitational acceleration of an object near Earth's surface is $g_n \approx 32 \text{ ft/s}^2$. Kingda Ka is a roller coaster at the Six Flags amusement park in Jackson, New Jersey. It accelerates from a stop to 128 mi/h in 3.5 seconds. What is the acceleration of the roller coaster as a multiple of g_n ? Express your answer as a decimal to the nearest tenth.



Number Theory

These playbook problems are not in sequential order.

Get the answers on [pg. 38](#).



Problem #

- 30 What is the result when Ellen starts with the integer 123,456 and performs the following sequence of operations: subtract 6, divide by 10, subtract 5, divide by 10, subtract 4, divide by 10, subtract 3, divide by 10, subtract 2, divide by 10?
- 32 A jar contains some number of pennies. When pennies are removed 2, 3, 4, 5, 6 or 8 at a time, one penny is left over. There are no pennies left over when they are removed 7 at a time. What is the least number of pennies that could be in the jar?
- 42 How many positive integers from 1 to 100, inclusive have an even number of positive divisors?
- 46 What is the value of n if $4! + 5! = n!3!$?

Problem #

- 52** Consider the set of all possible two-digit numbers that can be created using an unlimited supply of 1s, 3s, 7s and 9s. What is the greatest absolute difference between any two primes in this set?

72

Multiply →			
↓	3	2	6
	4	1	4
	12	2	24

Multiply →			
↓	18	a	
	b	4	
			5184

In the first grid, numbers were multiplied vertically and horizontally until a value was found for the shaded box. For instance, $3 \times 2 = 6$ and $4 \times 1 = 4$ were the results from the first two rows, and then $6 \times 4 = 24$ in the third column. The partially completed second grid follows the same rules, and both a and b are positive integers. What is the least possible value of $a + b$?

- 78** If A represents a digit in the equation $0.0A = \sqrt{0.0049}$, what is the value of A ?
- 81** The sum of eleven consecutive integers is 11. What is the least of these eleven integers?
- 85** What is the sum of the positive integer factors of 2017?
- 100** What is the sum of all the prime numbers less than 500 with only 3s and 4s as digits?
- 106** The proper factors of a positive integer are all of the distinct positive integer factors of the number except the number itself. An abundant number is a positive integer whose proper factors sum to a value greater than the number. Which abundant number less than 50 has the greatest proper factor sum?
- 107** If $(AAA)^3 = A6,926,0A7$, what digit does A represent?
- 117** What digit is in the units place in the product $3^{17} \times 7^{23}$?
- 120** What is the value of the sum $321_5 + 321_4$ when written in base 3?
- 135** What is 10111010_2 when written in base 8?
- 138** What is the greatest integer n such that $n!$ has n digits?
- 139** Adam has a triangle with vertices labeled 1 through 3. Jayvon has an octagon with vertices labeled 1 through 8. Each boy starts at position 1 and counts consecutive vertices on his polygon, continuing in the same direction, until he has reached 120 and is back at the vertex labeled 1. Percy did the same activity with his polygon, and he also finished at the vertex labeled 1. If Percy's polygon is not a triangle or an octagon, what is the sum of all the possible numbers of sides his polygon might have?
- 149** What single digit does D represent when $2D \times D51 = 807D$?
- 153** The product of the 3-digit number ABC and its reverse, CBA , is 140,209. If A , B and C each represent a different digit, what is the value of $A + B + C$?
- 156** What common fraction is equivalent to $0.\overline{327}$?
- 159** What is the greatest integral value of n for which $32!$ has 2^n as a factor?
- 166** What is the greatest four-digit palindrome that is divisible by 7 and 8?
- 174** To the nearest whole percent, what percent of all positive integers are not multiples of 2, 3, 4, 5 or 6?

Problem #

- 185** Given the expression $\frac{n^2 - 9}{n^2 - 4}$, for how many positive integers n from 1 to 2016, inclusive, is the GCF of the numerator and denominator greater than 1?
- 206** The number 3638 has a digital sum of $3 + 6 + 3 + 8 = 20$ and a digital product of $3 \times 6 \times 3 \times 8 = 432$. What is the absolute difference between the least and greatest four-digit numbers that each have a digital sum of 20 and a digital product of 432?
- 215** For how many four-digit numbers is the sum of the digits equal to the product of the digits?
- 218** Karla writes down six different prime numbers in increasing order. She notices that the product of the first three prime numbers she has written is equal to the sum of the last three prime numbers she has written. What is the least possible value of the last prime number Karla wrote?

BASES STRETCH

The **base 10** number system, the number system we are most familiar with, uses the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Numerals with these digits in the ones, tens, hundreds and higher places express specific numerical quantities. In base 10, the number 245, for example, is composed of 2 hundreds, 4 tens and 5 ones. That is, $2(10^2) + 4(10^1) + 5(10^0) = 200 + 40 + 5 = 245$.

A **base b** number system uses the digits 0, 1, ..., $b - 1$. Numerical quantities are expressed with these digits in the b^0 , b^1 , b^2 and higher places. In base b , if $b \geq 6$, the numeral 245_b represents the number $2(b^2) + 4(b^1) + 5(b^0)$. In base 8, for example, $245_8 = 2(8^2) + 4(8^1) + 5(8^0) = 2(64) + 4(8) + 5(1) = 128 + 32 + 5 = 165$.

Bases greater than 10 use letters to represent the digits greater than 9. For example, the 12 digits used in base 12 are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A and B. The numeral 10 in base 12 has 1 twelve and 0 ones. That is, $10_{12} = 1(12^1) + 0(12^0) = 1(12) + 0(1) = 12 + 0 = 12$.

- 241** What is the representation of 24_9 in base 10?
- 242** What is the representation of 24_8 in base 10?
- 243** What is the representation of 24_7 in base 10?
- 244** What is the representation of 24 in base 9?
- 245** What is the representation of 24 in base 8?
- 246** What is the representation of 24 in base 7?
- 247** What is the representation of 4991 in base 12?
- 248** What is the representation of $3BB_{12}$ in base 6?
- 249** If $523_b = 262$, what is the value of b ?
- 250** If $441_b = n^2$ and $351_b = (n - 2)^2$, for some $b < 10$, what is the value of n ?



Percents + Fractions

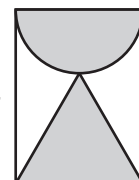
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Get the answers on [pg. 38](#).



Problem

- 5 What is the absolute difference between $\frac{1}{2}$ and $\frac{1}{3}$? Express your answer as a common fraction.
- 36 Nish trains to run a half marathon, a distance of 13.1 miles. Her training starts in week 1 with two 2-mile runs and one 4-mile run. Each week thereafter, the distances of her runs increase by 10% over the previous week's distances; therefore, she runs 8.8 miles in week 2. In which week of training does she first exceed 13.1 miles for the week?
- 60 What is $40\% \times \frac{2}{3} \times 24 \div 0.8$?
- 65 A grocery store is required to charge customers an 8% sales tax on certain items. However, some purchases at the store, such as food products, are not subject to sales tax. During a certain month, the store sold \$400,000 worth of groceries, not including the sales tax. If the store also collected \$10,000 in sales tax that month, then what was the total amount (in dollars) of the store's sales that month that were not subject to sales tax?
- 101 Mac has 25 marbles, of which 20% are red. Thayer has 20 marbles, of which 75% are not red. What is the absolute difference between the numbers of red marbles they have?
- 137 Recently, the manufacturer changed how Leon's favorite pens are sold. The price of a box of pens has been reduced by 10%, and there are now 25% fewer pens per box. What is the percent change in the cost per pen?
- 177 To create a new flag design, Howard paints a semicircle and an equilateral triangle inscribed in a rectangle as shown. What percent of the flag does the painted area cover? Express your answer to the nearest whole number.
- 192 What is the absolute difference between $1.\overline{18}$ and $2.\overline{36}$? Express your answer as a common fraction.



FRACTIONS STRETCH

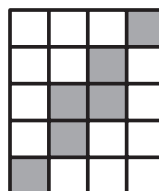
Solve the following problems. Express any non-integer answer as a common fraction.

221 What fraction of 100 is 25?

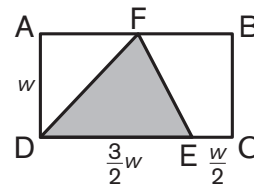
222 What fraction of $\frac{3}{8}$ is $\frac{9}{16}$?

223 What is the value of $\sqrt{\frac{3}{11} \div \frac{11}{12}}$?

224 What fractional part of this grid of 20 unit squares is shaded?



Problem #



225 What fraction of the area of rectangle ABCD is the area of inscribed triangle DEF?

226 On a number line, what common fraction is $\frac{3}{4}$ of the way from $\frac{1}{2}$ to $\frac{3}{4}$?

227 What is the reciprocal of $\frac{1}{2 + \frac{1}{3}}$?

228 What common fraction is equal to $0.\overline{75}$?

229 If $\frac{1}{\frac{1}{n + \frac{1}{3}} + \frac{1}{\frac{1}{3} + n}} = \frac{5}{12}$, what is the value of n ?

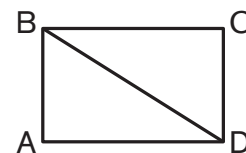
230 If $\frac{2x}{x-3} - 2 = \frac{4}{x+2}$, what is the value of x ?

Pg

Plane Geometry

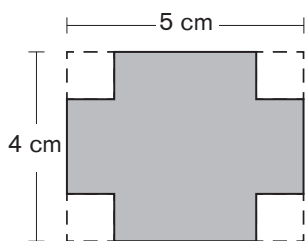
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Problem #



6 If the perimeter of rectangle ABCD is 34 cm and $AB = 5$ cm, what is the perimeter of $\triangle ABD$?

38



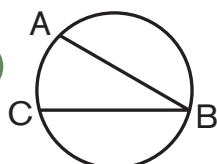
A dodecagon is formed when a 1-cm by 1-cm square is removed from each corner of a 4-cm by 5-cm rectangle as shown. What is the area of the dodecagon?

48

Paige cuts a square out of a circular pizza. The corners of the square lie on the circumference of the pizza. To the nearest whole number, what percent of the pizza is left when Paige removes the square?



77



The circle shown has a diameter of 12 inches, $m\angle ABC = 30$ degrees and $AB = BC$. What is the length of minor arc AC? Express your answer in terms of π .

Problem #

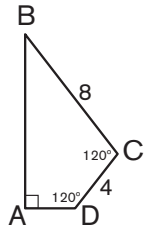
96



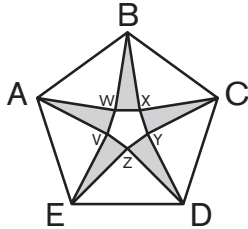
The hypotenuse of a 5-12-13 right triangle is the diameter of a semicircle containing the right angle vertex, as shown. What is the total area of the shaded regions? Express your answer to the nearest whole number.

109

In quadrilateral ABCD, $m\angle C = m\angle D = 120$ degrees, $m\angle A = 90$ degrees, $BC = 8$, $CD = 4$. What is the area of ABCD? Express your answer in simplest radical form.

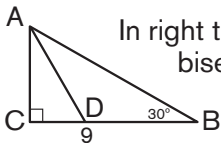


115



In the figure, regular pentagons ABCDE and VWXYZ have the same center. Each side of pentagon ABCDE is the hypotenuse of an isosceles right triangle. In each right triangle, the vertex opposite the hypotenuse is a vertex of pentagon VWXYZ. Each side of the smaller regular pentagon VWXYZ is also the base of one of the shaded acute isosceles triangles. What is the degree measure of the vertex angle of each shaded triangle?

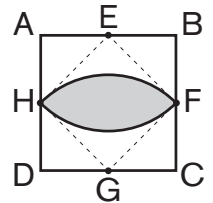
124



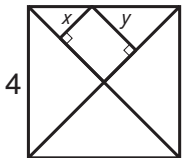
In right triangle ABC, $m\angle B = 30$ degrees and $BC = 9$ cm. If D is on side BC so that segment AD bisects acute $\angle A$, what is DC?

129

Square ABCD, shown here, has side length 2 meters, and E, F, G and H are midpoints of the sides. The curved lines are arcs of circles with centers at E and G. What is the area of the shaded region? Express your answer as a decimal to the nearest hundredth.



132

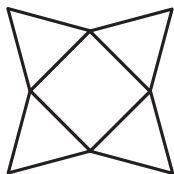


In the figure, the segments of lengths x and y lie on perpendiculars to the diagonals of a square of side length 4. The sum $x + y$ can be written in the form \sqrt{z} . What is the value of z ?

143

Two sides of a regular pentagon are doubled and a new pentagon is formed. By what percent is the perimeter increased?

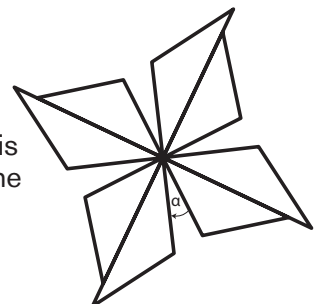
148



The net of a square pyramid, shown here, is a square with an equilateral triangle on each of its sides. The side length of the square can be expressed as $6x - 6$ or $2x + 14$, for the same value of x . When the net is folded to form a square pyramid, its surface area can be expressed in simplest radical form as $a^2(\sqrt{b} + c)$. What is the value of $a + b + c$?

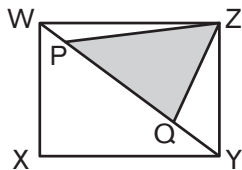
154

A fan design has four pairs of similar isosceles triangles that create four blades, as shown. In each pair of triangles, the base of the smaller triangle is a segment of the base of the larger triangle, and the measure of the vertex angle of each triangle is twice the sum of the measures of its base angles. What is the degree measure of the angle labeled α ?



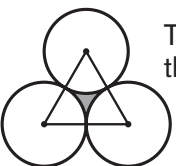
Problem #

- 157** Parallel lines l , m and n are in a plane with line m a distance of 1 cm from each of the lines l and n . Line l is tangent to a circle that has radius 3 cm. Lines m and n intersect the circle, and the four points of intersection are connected to form a trapezoid. If the area of the trapezoid is expressed in the form $\sqrt{a} + \sqrt{b}$ cm², what is the value of the product ab ?

- 163**  The area of rectangle WXYZ is 90 cm². P and Q are points on diagonal WY such that $3(WP + QY) = 2PQ$. What is the area of triangle PQZ?

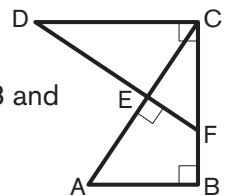
- 169** A triangle exists with side lengths $2x$, $3x + 7$ and $6x - 5$ for how many integer values of x ?

- 171** A circular pizza is cut along four diameters into eight identical sectors. If the total perimeter of each sector is 10 inches, what is the area of the whole pizza? Express your answer as a decimal to the nearest tenth.

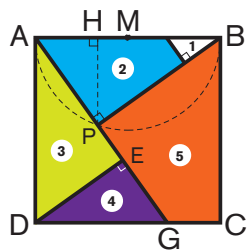
- 180**  Three circles, each having radius 4 units, are externally tangent to each other. A triangle joins the centers of the circles. What is the area of the shaded region within the triangle but outside the circles? Express your answer as a decimal to the nearest tenth.

- 181** In square WXYZ, point V is the midpoint of side YZ, and the area of $\triangle XYV$ is $\frac{4}{5}$ unit². What is the area of square WXYZ? Express your answer as a common fraction.

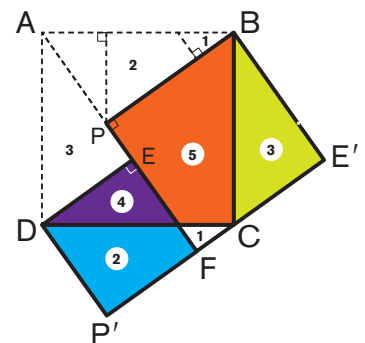
- 189** In pentagon ABCDE, with right angles ABC, BCD and AEF, as shown, $AE = 14$, $DE = 18$ and $EF = 8$. What is the length of side BC? Express your answer in simplest radical form.

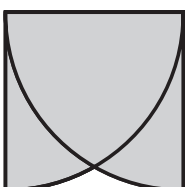


- 193** A square is dissected into five pieces as shown on the left, with $AH = \frac{1}{3}AB$ and $AM = MP = MB$. The



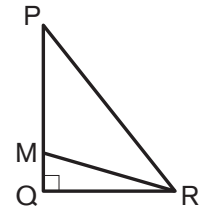
pieces are shaded and numbered. The pieces are rearranged so as to form two squares, one with twice the area of the other. The rearrangement of the pieces is shown on the right, overlaid on top of the original square. If square DEFP' in the second picture has a side length of 1 inch, then what is the side length of the original square? Express your answer in simplest radical form.



- 197**  In the square shown, the side lengths are 6 cm, and the intersecting arcs are quarter-circles. The area of the shaded region, expressed in simplest radical form in terms of π , is $a\pi + b\sqrt{c}$ cm². What is the value of $a + b + c$?

Problem #

- 217** In right triangle PQR, shown here, M is on PQ such that $PM = MR$. If $PQ = 12$ units and $QR = 9$ units, what is the value of MQ ? Express your answer as a common fraction.



ANGLES AND ARCS STRETCH

SECANT	a line that intersects the circle at two points
CHORD	a line segment whose endpoints are two points on the circle
TANGENT	a coplanar line that intersects the circle at a single point of tangency
CENTRAL ANGLE	an angle with its vertex at the center of the circle
INSCRIBED ANGLE	an angle with its vertex on the circle and whose sides are chords of the circle
MAJOR ARC	an arc of the circle with measure greater than or equal to 180°
MINOR ARC	an arc of the circle with measure less than 180°

ANGLE AND ARC MEASURES

In the figures below, observe how the degree measure of $\angle AXB$ decreases as the distance between the vertex of the angle and the center of the circle increases.

$m\widehat{AB} = 80^\circ$ $m\widehat{CD} = 80^\circ$ $m\angle AOB = 80^\circ$	$m\widehat{AB} = 80^\circ$ $m\widehat{CD} = 40^\circ$ $m\angle AXB = 60^\circ$	$m\widehat{AB} = 80^\circ$ $m\widehat{CD} = 0^\circ$ $m\angle AXB = 40^\circ$	$m\widehat{AB} = 80^\circ$ $m\widehat{CD} = 40^\circ$ $m\angle AXB = 20^\circ$
Figure I	Figure II	Figure III	Figure IV

- In Figure I, angles AOB and COD are central angles of circle O that intercept arcs AB and CD, respectively. The degree measure of a central angle and the arc it intercepts are equal.

$$m\angle AOB = m\widehat{AB} \text{ and } m\angle COD = m\widehat{CD}$$

- In Figure II, vertical angles AXB and CXD, formed by the intersection of chords AC and BD inside circle O, intercept arcs AB and CD, respectively. The degree measure of vertical angles formed by two chords intersecting inside a circle is half the sum of the measures of their intercepted arcs.

$$m\angle AXB = m\angle CXD = \frac{1}{2}(m\widehat{AB} + m\widehat{CD})$$

- In Figure III, $\angle AXB$ is inscribed in circle O. The degree measure of an inscribed angle is half the measure of the intercepted arc.

$$m\angle AXB = \frac{1}{2}m\widehat{AB}$$

- In Figure IV, $\angle AXB$, formed by the intersection of two secants at point X outside of circle O, intercepts arcs AB and CD. The degree measure of an angle formed by two secants, two tangents or a secant and a tangent is half the difference of the measures of its intercepted arcs.

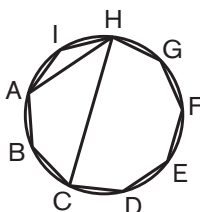
$$m\angle AXB = \frac{1}{2}(m\widehat{AB} - m\widehat{CD})$$

ANGLES AND ARCS STRETCH (continued)

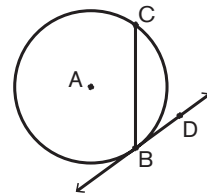
It may appear that there are four different formulas for calculating the four types of angles. But in each case, the measure of the angle in question is, essentially, the average of the measures of the intercepted arcs. In Figure IV, note that, with respect to $\angle AXB$, \widehat{AB} appears concave, while \widehat{CD} appears convex. So the measure of $\angle AXB$ can be thought of as the average of 80° and -40° .

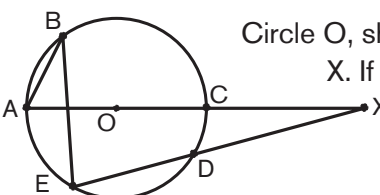
Solve the following problems by using what you've learned about angles and arcs. Express any non-integer value as a decimal to the nearest tenth.

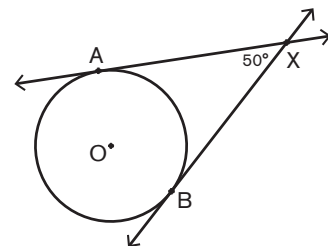
Problem

231  Regular nonagon ABCDEFGHI is inscribed in a circle, as shown. What is $m\angle AHC$?

232 In circle A, shown here, \overleftrightarrow{BD} is tangent to the circle at B, and major \widehat{BC} has measure 230° . What is $m\angle CBD$?



233  Circle O, shown here with chords AB and BE, has secants AC and DE that intersect at X. If $m\angle ABE = 35^\circ$ and $m\angle AXE = 15^\circ$, what is the measure of \widehat{CD} ?



234 In this figure, lines AX and BX are tangent to circle O at A and B, respectively. If $m\angle AXB = 50^\circ$, what is the measure of major \widehat{AB} ?

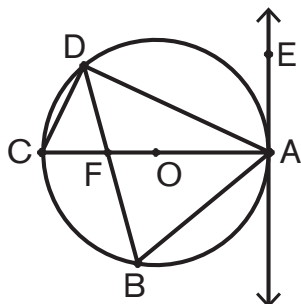
Use the figure at the right for questions 235 through 238.

235 What is $m\angle ABD$?

236 What is $m\widehat{AB}$?

237 What is $m\angle BAE$?

238 What is $m\angle CFD$?

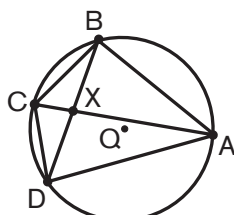


\overleftrightarrow{AE} is tangent to circle O

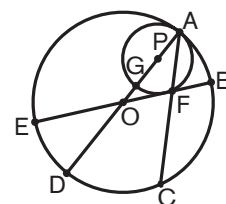
$$\overleftrightarrow{AE} \perp \overleftrightarrow{AC}$$

$$m\angle BDC = 40^\circ$$

$$m\widehat{AD} = 125^\circ$$

239  Quadrilateral ABCD is inscribed in circle Q, as shown, with diagonals intersecting at X. If $m\widehat{AB} = 110^\circ$, $m\widehat{BC} = 60^\circ$ and $AB = BD$, what is $m\angle CXD$?

240 Circle P is internally tangent to circle O at A, as shown. \overleftrightarrow{AC} and \overleftrightarrow{BE} intersect at F, which is also the point of tangency between \overleftrightarrow{BE} and circle P. \overleftrightarrow{AD} and \overleftrightarrow{BE} are diameters of circle O, and \overleftrightarrow{AG} is a diameter of circle P. If $m\widehat{CD} = 50^\circ$, what is the measure of minor \widehat{BC} ?





Probability, Counting + Combinatorics

These playbook problems are not in sequential order.

Get the answers on [pg. 39](#).



Problem #

7



Harvey has a fair eight-sided die that has a different number from 1 to 8 on each side. If he rolls this die twice, what is the probability that the second number rolled is greater than or equal to the first number? Express your answer as a common fraction.

16

A restaurant offers a dinner special in which diners can choose any one of 3 appetizers, any one of 4 entrées, any two different side dishes out of 5 and any one of 6 desserts. How many different meals are possible?

18

Four chips are distinctly labeled with the digits 2, 3, 1 and 7, one chip for each digit. Two chips are drawn at random without replacement and placed in the order in which they are drawn, from left to right, to form a two-digit number. What is the probability that the two-digit number is a prime number? Express your answer as a common fraction.



27

A number is randomly selected from the integers 1 through 25, inclusive. What is the probability that the number chosen is divisible by 2, 3, 4 or 5? Express your answer as a common fraction.

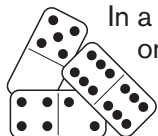
28

Using only 1s and 2s, in how many different ordered sequences can Siddarth write a sum that equals 5? For example, a sum of 3 can be written $1 + 2$, $2 + 1$ or $1 + 1 + 1$.

34

How many distinct positive four-digit integers can be formed using the digits 1, 2, 3 and 4 each once, such that no adjacent digits differ by more than 2?

47



In a standard set of dominoes, a face of each domino has a line through the center, with 0 to 6 dots on each side of the line. Each possible combination of dots is used exactly once, one combination per domino. What is the probability that a randomly selected domino will have the same number of dots on both sides of the line? Express your answer as a common fraction.

55

The faces of a cube are randomly and independently painted either red or blue with equal likelihood. What is the probability that the cube has all blue faces? Express your answer as a common fraction.

57

Kavon has a fair eight-sided die with each side having a different one of the digits 1 through 8. He rolls the die twice and writes down, in order, the results to form a two-digit number. What is the probability that his two-digit number is prime? Express your answer as a common fraction.

59

Colorado used to issue license plates with the format of two letters (excluding Q) followed by four digits from 0 through 9. Later the state switched to a format of three letters (excluding Q) followed by three digits. What is the ratio of the number of possible old-style plates to the number of possible new-style ones? Express your answer as a common fraction.



66

A school of 100 fish swims in the ocean and comes to a very wide horizontal pipe. The fish have three choices to get to the food on the other side: swim above the pipe, through the pipe or below the pipe. If we do not consider the fish individually, in how many ways can the entire school of fish be partitioned into three groups with each group choosing a different one of the three options and with at least one fish in each group?

68

In how many different ways can the letters of CHAIRS be arranged?

Problem #

- 76** How many three-letter permutations can be made using letters from ALASKA?
- 91** On Saturday, three different football games are televised at noon and four different games are televised at 8 p.m. On Sunday, five different games are televised at noon. If Amanda watches one Saturday game at noon and another at 8 p.m. and one game at noon on Sunday, how many different combinations of games can she watch?
- 97** A group of 12 tourists will split up for two tours. A tour guide will lead one group on a hike. Another tour guide will lead the other group on a safari. If at least one tourist goes with each guide, in how many different ways can the tourists split up for the two tour groups?
- 110** Zzyzx Road is in California near Nevada. How many five-letter arrangements of the letters in the English alphabet follow Zzyzx alphabetically?
- 112** Cora has five balls—two red, two blue and one yellow—which are indistinguishable except for their color. She has two containers—one red and one green. If the balls are randomly distributed between the two containers, what is the probability that the two red balls will be alone in the red container? Express your answer as a common fraction.
- 116** A 12-foot by 12-foot square bathroom needs to be tiled with 1-foot square tiles. Two of the tiles are the wrong color. If the tiles are placed randomly, what is the probability that the two wrong-colored tiles share an edge? Express your answer as a common fraction.
- 126** For how many three-digit positive integers is the sum of the digits of the integer equal to 9?
- 127** A toy manufacturer produces blue yo-yos and red yo-yos simultaneously at the same rate. During production, yo-yos of each color exit the assembly line in random order. What is the probability that the next four yo-yos that exit are all the same color? Express your answer as a common fraction.
- 133** A box contains 26 slips of paper, each showing a different letter of the alphabet. If two slips of paper are drawn from the box at the same time, what is the probability that both letters appear in the word ALGEBRA? Express your answer as a common fraction.
- 145** A *repeating* integer is one in which a sequence of digits occurs two or more times to make the entire number. The 4-digit number 4242 is a repeating integer. How many numbers are six-digit repeating integers?
- 147** Eve's cousin, Fin, lives in a different country. According to a postcard Eve got, Fin plans to visit the U.S., but Eve can't tell the exact date of Fin's visit, because of the way the date is written. Eve doesn't know if the date format used in Fin's country is M/D or D/M, where M and D are different and represent the two-digit month and two-digit day, respectively. For how many dates in the year would both interpretations of the date written on the postcard result in a valid date?

150



The numbers 2 through 9 on a telephone keypad, like the one shown, are associated with the letters of the alphabet. Each person in a particular office is assigned a phone extension based on the first three letters of his or her last name. For instance, John DOE has the extension 363, and Marvella JOHnson has the extension 564. How many unique three-digit extensions can be assigned using the digits 2 through 9?

155

Twelve couples participate in a fitness retreat. One strength-building exercise requires participants to form teams of three so that the two people who make up a couple are not on the same team. How many different teams of three can be formed in this manner?

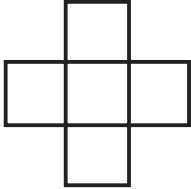


160



Storm places coins, having a total value of at least \$1.00, in a bag. The coins may include pennies, nickels, dimes and quarters, but no more than three of any single denomination. How many different combinations of coins can Storm place in the bag?

Problem #

- 161** Three students each flip three fair coins. What is the probability that all three students get the same number of tails? Express your answer as a common fraction.
- 162** Twins Taylor and Tyler were born on 05/02/07. This date is referred to as a *sum date* because the sum of the month and day is equal to the two-digit year: $05 + 02 = 07$. How many years in the 21st century will have a sum date in each month during that year?
- 165** The five squares of the diagram shown are to be colored orange, yellow, green, blue and indigo, with exactly one color per square. Two colorings are the same if one is just a rotation of the other (but not if the diagram must be flipped over). How many distinct colorings are there?
- 
- 175** Seventy-five bingo balls, each with a different positive integer from 1 through 75, are placed in a cage. A random ball is selected, its number is announced, and the ball is returned to the cage. This process occurs a total of 20 times. What is the probability that at least one ball is selected more than once? Express your answer as a decimal to the nearest hundredth.
- 176** Tina and Tricia play on a softball team. Tina has 8 hits out of 20 times at bat, and Tricia has 6 hits out of 16 times at bat. Based on their past performance, what is the probability that both girls will get a hit the next time they bat? Express your answer as a common fraction.
- 183** Xera and Yeta use this method to decide who will sit in the front passenger seat of the car. Xera throws a standard six-sided die, after which Yeta picks a card from a standard deck of 52 cards, with replacement. They continue to take turns die-throwing and card-picking until either Xera wins by rolling a four or Yeta wins by picking a card with a four. What is the probability that Yeta wins? Express your answer as a common fraction.
- 187** A box contains 15 slips of paper, each bearing a different natural number from 1 to 15, inclusive. If three of these slips are randomly drawn, one at a time, without replacement, what is the probability that three consecutive numbers are drawn in increasing order? Express your answer as a common fraction.
- 194** How many arrangements of all eight letters in TRESPASS do not have S as the final letter?
- 195** Four vertices of a regular octagon are chosen at random. What is the probability that a square can be made by connecting the vertices? Express your answer as a common fraction.
- 202** How many positive three-digit integers have one digit equal to the average of the other two digits?
- 203** A bag contains only red marbles and green marbles, two of which are to be drawn without replacement. There are at least two marbles of each color in the bag. If the probability of both marbles being red is half the probability of both marbles being green, then what is the minimum possible number of marbles in the bag?
- 205** A math club has 16 members. The coach wants to select three boys and three girls to represent their school at a tournament. There are six times as many ways to choose the girls as there are ways to choose the boys. What is the ratio of girls to boys in the club? Express your answer as a common fraction.
- 207** Jebediah has two coins in his pocket. One is a fair coin, while the other has heads on both sides. He pulls one coin out at random and flips it three times. If the coin lands heads all three times, what is the probability that it is the fair coin? Express your answer as a common fraction.
- 211** Eight points on a circle are labeled. How many chords can be drawn connecting any two of these points?
- 212** Tad draws three cards at random, without replacement, from a deck of ten cards numbered 1 through 10. What is the probability that no two of the cards drawn have numbers that differ by 1? Express your answer as a common fraction.

Problem #

214 If three distinct integer lattice points are randomly selected from the interior of the circle defined by $x^2 + y^2 = 8$, what is the probability that they are the vertices of a triangle? Express your answer as a common fraction.

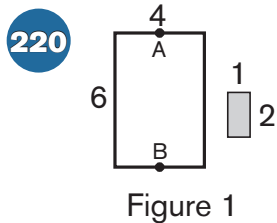
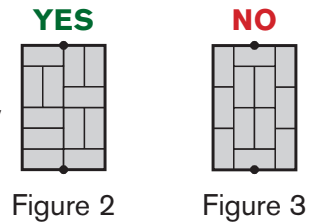



Figure 1 shows a 2×1 gray rectangle and a 6×4 white rectangle with the midpoints of two sides labeled A and B. If the larger rectangle cannot be rotated, in how many ways can 12 of the gray rectangles be arranged inside the white rectangle so that none crosses a segment from A to B? One such arrangement is shown in Figure 2, but not in Figure 3.



Ps

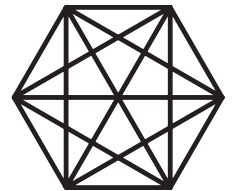
Problem Solving (Misc.)

These playbook problems are not in sequential order.  Get the answers on [pg. 39](#).

Problem #

- 9** When the integer n is squared, the result is less than 150. What is the sum of all possible values of n ?
- 19** The letters A, B, ..., Z are equally spaced in order on a number line, with A at 0 and Z at 25. What is the average of the two numbers that are 4 units from the letter M?
- 39** Jesse added all but one of the first ten positive integers together. The sum was a perfect square. Which one of the first ten positive integers did Jesse not include?

62 The figure shown consists of a regular hexagon and all of its diagonals. How many triangles in the figure have at least two congruent sides?



152 Cody's ZIP code is a five-digit number whose digits are all different. In this number, there are two pairs of adjacent digits in which the digits differ by 1. There is a pair of adjacent digits in which one digit is 4 times the other. There is a pair of adjacent digits whose sum is 10, as well as a pair of non-adjacent digits whose sum is 10. The sum of all five digits is a multiple of 10. If the leftmost digit of the number is 7, what is Cody's ZIP code?

182 Ali gives Stan a closed box that contains at least one of each token worth 5, 11 or 19 points. Ali says that the tokens have a combined value of 56 points. How many tokens are in the box?

191 The digital clock shown can display up to four digits to represent the hour and the minute. For how many minutes in a 12-hour period does the digit 0 appear on the clock?



208 What is the minimum number of people that must be in a room to ensure that there are three people who, when considered pairwise, all know each other or three people who, when considered pairwise, all do not know each other?

216 A cube has one red, one green, one yellow and three blue faces. How many distinct cubes satisfying this description are possible?

These playbook problems are not in sequential order.

Get the answers on [pg. 39](#).



Problem

2



The length of 5 small paper clips is equal to the length of 2 large paper clips. The length of 8 small paper clips is equivalent to the length of how many large paper clips? Express your answer as a mixed number.

17

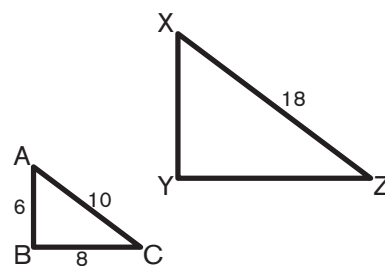
It takes 15 machines 15 minutes to make 500 raviolis. The machines produce raviolis at a steady rate. How long would it take 75 of these machines to make 6000 raviolis?

23

For what value of m does $\frac{3}{m} = \frac{27}{72}$?

26

Right triangle ABC has side lengths 6, 8 and 10 units. Right triangle XYZ, with hypotenuse of length 18 units, is similar to $\triangle ABC$. What is the ratio of the area of $\triangle ABC$ to the area of $\triangle XYZ$? Express your answer as a common fraction.



31



When one New Zealand dollar, NZ\$1.00, was worth US\$0.77, how much was US\$1.00 worth in New Zealand dollars?

40

A bag contains 20 purple and 40 green marbles. How many purple marbles need to be added so that $\frac{5}{12}$ of the marbles will be green?

41

A cube of cheese has edge length 1 inch and weighs 0.6 ounce. What is the edge length, in feet, of a cube of the same cheese that weighs 64.8 pounds? (1 pound = 16 ounces)

67

If Reid is traveling at a speed of 44 ft/s, how many miles will he travel in an hour given that 1 mile = 5280 feet?

71

Amy's favorite lotion costs \$3.00 for 4 fluid ounces. At that same rate, what would she expect to pay for a quart of lotion? (1 quart = 32 fluid ounces)

93

At noon, Randy's family left the Texas-Oklahoma border, traveling north on I-35. At noon, Marco's family left their home in Minnesota, 1029 miles from the Texas-Oklahoma border, traveling south on I-35. If Randy's family is traveling 45 mi/h and Marco's family is traveling 53 mi/h, how many hours will it take for the two families to pass each other? Express your answer as a decimal to the nearest tenth.

104

Four oranges cost a total of 90 cents. At this rate, what is the cost of 3 dozen oranges?

105

When fully matured, a grape contains 80% water. After the drying process, called dehydration, the resulting raisin is only 20% water. What fraction of the original water in the grape remains after dehydration? Express your answer as a common fraction.

108

A Vermont syrup maker has 100 liters of a mixture that is $\frac{1}{4}$ maple syrup and $\frac{3}{4}$ base. She wants to add enough maple syrup to bring the ratio of maple syrup to base up to 1:1. If she has to evaporate 90% of the maple sap to get the maple syrup to add to that mixture, how many liters of maple sap does she need to start with?

Problem #

- 122 Nathan ran 2.5 miles at a pace of 7 minutes 36 seconds per mile. If he wishes to complete the entire 5-mile run at an average pace of 7 minutes 24 seconds per mile, what should his pace be for the next 2.5 miles? Express your answer as a decimal to the nearest tenth.
- 140 On Mars a day is called a sol. Mars has a 668-sol year with a 7-sol week. If a regular Martian year has 95 weeks, and a leap year is one week longer, what fraction of the years are leap years? Express your answer as a common fraction.
- 142 Becca and Varun are walking side by side at the same constant speed. Becca steps onto a moving walkway and continues to walk at the same speed, while Varun walks alongside, maintaining his speed. When Becca reaches the end, Varun has covered only two-fifths of the length of the walkway. What is the ratio of the walkway's rate to Becca and Varun's walking speed? Express your answer as a common fraction.
- 178 The inhabitants of the planet Rundia run footraces similar to those run on Earth. However, Rundians measure the distances that they run in *bars*, where one bar measures four-fifths of a meter, and they measure time in *ticks*, where there are 100 ticks in one minute. Rundian sprinter Sejes Wesno can run a 100-bar race in 11.53 ticks. The great Earth sprinter Usain Bolt ran a 100-meter race in 9.58 seconds. If Wesno is k times as fast as Bolt, what is the value of k ? Express your answer as a decimal to the nearest hundredth.
- 190 Pump P can fill a water tank in 12 hours, and Pump Q can fill the same tank in 15 hours. The two pumps started filling the tank at the same time and worked together until the tank was 60% full. At that point, Pump P was turned off, and Pump Q continued to fill the tank until it was completely full. How many hours did it take to completely fill the tank?
- 209 Tire pressure is directly proportional to temperature on a temperature scale where zero degrees is absolute zero. Given that temperatures in degrees Celsius (C) and degrees Fahrenheit (F) are related by the formula $F = \frac{9}{5}C + 32$, and absolute zero is -273.15°C , by what percent does tire pressure decrease when the temperature drops from 80°F to 40°F ? Express your answer to the nearest whole number.
- 213 Twelve people have sheared $\frac{1}{3}$ of a field of pine trees in 7 days. How many more people need to be added to the crew to shear the rest of the trees in the field in the next 6 days?



Sequences, Series + Patterns

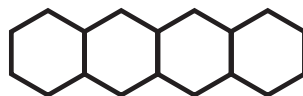
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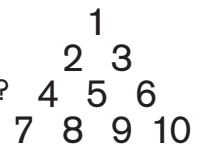


Problem #

- 13 A hexagonal table that is by itself seats 6 people, one person at each side. A row of hexagonal tables is created by pushing together a certain number of hexagonal tables so that a side of one table meets a side of the next table, in the way shown here. If 50 people can sit at the row of tables that was created, how many tables are in the row?



- 73 The consecutive counting numbers are written in a triangular table, as shown, with one more number in each successive row. What is the sum of the numbers in the row that contains 25?



- 74 In the arithmetic sequence 12, w , x , y , z , 47, what is the value of y ?

Problem #

- 84** In the number sequence 3, 5, 2, ..., after the first two terms, the n th term is defined as $a_n = a_{n-1} - a_{n-2}$. For example, $a_3 = a_2 - a_1 = 5 - 3 = 2$. What is the sum of the first 200 terms of this sequence?
- 94** What is the 41st digit after the decimal point in the decimal expansion of $\frac{1}{27}$?
- 114** An arithmetic sequence of integers has 20 as the first term and 56 as the last term. How many different sets of integers form such a sequence?
- 131** Caynan wrote a sequence of consecutive integers beginning with -37 . If the sum of the integers he wrote is 200, what is the greatest integer in the sequence Caynan wrote?
- 164** There are 50 equally spaced points marked on a circle. Sara numbers them clockwise from 1 to 50. Starting at 1, she then draws congruent, connected segments between points that are 8 spaces apart, moving clockwise from the end of the previous segment. For example, the first segment is drawn from 1 to 9, and the second from 9 to 17. What is the sum of the numbers on the endpoints of the 23rd segment drawn?
- 200** Xena runs halfway across a field in 1 minute. The next fourth of the field takes her $\frac{2}{3}$ of a minute to cross, the next eighth takes $\frac{4}{9}$ of a minute, and so on, with each half of the previous distance taking $\frac{2}{3}$ of the previous time. How many minutes does it take Xena to cross the field?




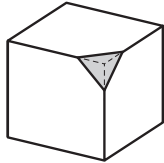
Solid Geometry

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Problem #

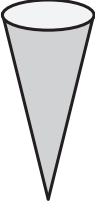

- 90** Donny needs to find the surface area of a dozen donuts so he can make enough glaze. He doesn't know how to calculate the exact surface area of a donut, so he makes an approximation based on a roll of 1-inch-wide masking tape, which has the same volume as the donut. The outer radius of the tape is 2.5 inches, and the inner radius is 1.5 inches. What surface area did Donny estimate for a dozen donuts? Express your answer in terms of π .
- 
- 99** A sphere, a cone and a cylinder all have the same height and radius. The sphere and cone are completely filled with water. If the amount of water in the cylinder is the same as the total of the amounts in the sphere and cone, what percent of the cylinder's volume is filled?
- 103**  In the figure shown, a triangular pyramid has been cut off the corner of the cube so that an equilateral triangle face is formed. If each corner of the cube is cut off in this manner, what is the maximum sum of the number of faces, edges and vertices on the new polyhedron?
- 128** Ginger wants to make bubble tea in a cylinder-shaped cup with inside measurements of diameter 6 cm and height 12 cm. After she places 48 identical spherical tapioca bubbles into her empty cup, exactly 100π mL of liquid will fill the cup right to the top. Given that $1 \text{ mL} = 1 \text{ cm}^3$, what is the radius of each tapioca bubble? Express your answer as a decimal to the nearest tenth.
- 134** In Swimmington, where Maxwell lives, the charge for water usage is 0.15 cent per gallon. Maxwell has a cylindrical pool of height 4.5 feet and diameter 24 feet. What is the cost for Maxwell to fill his pool so that the water surface is 3 inches below the top of the pool, given that $1 \text{ gallon} = 231 \text{ in}^3$? Express your answer to the nearest whole number.

Problem #

136 Solid metal spheres with diameter $\frac{1}{6}$ inch are dropped into a rectangular prism tank, where they sink to the bottom. The tank is 10 inches wide by 15 inches long by 8 inches deep, and the water level is currently 3 inches. How many spheres does it take to raise the water level 1 inch? Express your answer to the nearest hundred.

151 Keaton wants to build a rectangular prism with volume 2016 in^3 so that the length of each edge is a whole number of inches. What is the least possible sum of the three dimensions of the prism he builds?

168 When ice melts and becomes water, its volume decreases by 8%. A cylindrical block of ice completely fills a container with a height of 10 cm and a radius of 4 cm. When all of the ice melts, what will be the height of the water in the cylinder? Express your answer as a decimal to the nearest tenth.

184  When a cone's height is decreased by a factor of four, to maintain the same volume, the radius must be increased by a factor of two, or 100%. When the cone's height is decreased by a factor of three, by what percent must the radius be increased to maintain the same volume? Express your answer to the nearest whole number. 

198 A rectangular prism with length \geq width \geq height has positive integer dimensions and a volume of 60 units^3 . How many different prisms are there that meet these conditions?



Statistics + Data

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Problem #

21 What is the average of the integers from 13 to 31, inclusive?

51 Jamie's scores on the first five tests in his algebra class were 81, 75, 86, 98 and 92. After three more tests the median of his test scores was 88. What is the greatest possible value for the lowest score on these three tests?

82 The mean of x and y is 12 and the mean of y and 12 is $\frac{z}{2}$. What is the mean of x and z ?

95 For the integers 15, 17, 11, 13, x , y , the mode, median and mean form an increasing arithmetic sequence, in that order. If $x \leq y$, what is the greatest possible value of y ?

118 What is the geometric mean of 14 and 126?

121 A set of six different positive integers has a median and mean of 6. If the largest number in the set is 12, what is the largest possible sum for the three largest numbers?

186 The mean, the median and the mean of all modes of the integers 7, 4, 5, 6, 5, x are equal. How many possible values are there for x ?