2014 ■ School Competition ■ Booklet

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GENERAL INSTRUCTIONS

This section contains instructions, rules and procedures for administering the MATHCOUNTS School Competition. It is important that the coach look upon coaching sessions during the academic year as opportunities to develop better mathematics skills in all participants. Therefore, the coach is encouraged to postpone the selection of those students who will be competing at the Chapter Competition until just prior to the event in February. *Selection of the students need not be based on the results of a school competition*. For this reason, schools may deviate from these rules in administering the School Competition. However, experience with the official rules may aid the students who compete at the chapter, state and national levels.

Individual scores are kept for each participating student, and a team score is calculated for each team. Before beginning the competition, divide participants into teams of four students and designate a captain for each team.

At the end of each round of the competition, collect all competition booklets, problems and scratch paper.

- 1. Use of notes or other reference materials (including dictionaries) is not permitted.
- 2. **The Target and Team Rounds assume the use of a calculator.** Calculator use is permitted in these rounds only. Any calculator that does not contain a QWERTY (i.e., typewriter-like) keypad is permitted. Calculators that have the ability to enter letters of the alphabet but do not have a keypad in a standard typewriter arrangement are acceptable. Smart phones, laptops, iPads®, iPods®, personal digital assistants (PDAs), and any other "smart" devices are not considered to be calculators and may not be used during competitions. Students may not use calculators to exchange information with another person or device during the competition.
- 3. Talking and signals are permitted only during the Team Round.
- 4. Before the competition, coaches should review the rules for acceptable forms of answers with students. They can be found in the "Forms of Answers" section of this booklet and in the *MATHCOUNTS School Handbook* (page 51).

SPRINT ROUND INSTRUCTIONS

- 1. Distribute scratch paper.
- 2. Distribute Sprint Round booklet, and instruct each student to print his/her name in the allotted space.
- 3. Read aloud instructions appearing on the front cover of the booklet while students read instructions silently.
- 4. Instruct students to begin. Start timing.
- 5. After 37 minutes, give a three-minute warning. After 40 minutes, say, "Stop, pencils down" and instruct students to close competition booklets.

TARGET ROUND INSTRUCTIONS

- 1. Distribute scratch paper.
- 2. Distribute the first/next pair of Target Round problems, and instruct each student to print his/her name in the allotted space.
- 3. Read aloud the instructions appearing on the cover of the first pair of problems.

- 4. Instruct students to begin. Start timing.
- 5. Give a ten-second warning at 5 minutes 50 seconds, and, after 6 minutes say "Stop, pencils down." Collect all papers.
- 6. Repeat steps 2, 4 and 5 for each of the next three pairs of problems.

TEAM ROUND INSTRUCTIONS

- 1. Place each team (of four students) in a room with at least five feet of unoccupied space between teams. Distribute scratch paper.
- 2. Distribute Team Round booklet to each person, and instruct team captain to print team name and team members' names on his/her booklet. This becomes the team's official answer booklet.
- 3. Read aloud the instructions printed on the cover of the booklet while students read the instructions silently.
- 4. Instruct students to begin. Start timing.
- 5. After 17 minutes, give a three-minute warning. After 20 minutes, say "Stop, pencils down" and instruct students to close competition booklets.

COUNTDOWN ROUND INSTRUCTIONS

The Countdown Round is a mandatory component at the National Competition, and it is used to determine the final rank of the top competitors. At the chapter and state levels, the use of the Countdown Round officially is at the discretion of the state coordinator. When used officially, the Countdown Round will adhere to the rules presented below. The instructions may be modified as necessary at the school level. *This round is available in PowerPoint format for the School Competition. See the cover of this School Competition Booklet for instructions on obtaining a copy.*

- 1. Based on scores in the Sprint and Target Rounds, rank all students and select the top 25%, up to ten students, to compete in the Countdown Round.
- 2. Seat the two lowest-ranked students so they are in clear view of the moderator. Each competitor should be given scratch paper and sharpened pencils before the round begins. Invite the competitors to introduce themselves and test their buzzers.
- 3. Read the following statement to all students who will be competing in the round:

I will read a problem to you as it is being projected. You may use the scratch paper and pencil in front of you to calculate your answer to the problem. You are not allowed to use calculators during this round.

You will have a maximum of 45 seconds to solve the problem after it appears on the screen. You will be given a ten-second warning before time expires. As soon as you have solved the problem, press your buzzer. [Schools may have alternate methods of determining order of finish and should adjust directions to students accordingly.] I will call on the first person who signals. Do not announce your answer until I call on you. Each time you wish to answer, you must signal, though you may not answer more than once for any question. If you do not signal before you answer, your answer will be disqualified. If you answer after signaling but before I call on you, your answer will be accepted, but I ask that you please wait until you hear your name so that there is no confusion.

Once I call on you, you will have three seconds to begin your answer. Your opponent may continue working while you are responding.

If you answer correctly, you will score one point in the round. If you answer incorrectly, your opponent will have the remainder of the 45 seconds to press his/her buzzer for an opportunity to answer the problem and score a point in the round.

Whoever answers the most of the three problems correctly—not necessarily two out of the three—will progress to the next round to compete for the next place. If you are tied after three questions, we will use the sudden victory rule, which I will describe if it occurs.

[Note that the above procedure does not necessarily require a student to answer two out of the three problems correctly. For instance, a student answering only one problem of three will progress to the next round if his/her opponent has not correctly answered any questions in the round.]

It is very important that rules be followed exactly. If you answer without signaling your buzzer, your answer will be disqualified.

- 4. Conduct the round as described above. After the winner of each round is identified, dismiss his/ her opponent, and ask the next written competition place-holder to be seated to participate in the next round. Invite the new competitor to introduce himself or herself and test his/her buzzer.
- 5. If a sudden victory situation occurs, read the following statement to the students:
 - Since you are tied at the end of three problems, I must declare a sudden victory situation. I will now continue to read problems to both of you. Rules for answering problems remain the same as before. The first one of you to answer a problem correctly will progress to the next round.
- 6. Repeat procedure until the champion is identified. However, just before the 4th-ranked student competes in his/her first round, announce that for the final four rounds, the first student to answer three problems correctly will progress to the next round.
- *Rules for the Countdown Round change for the National Competition.

SCORING

- 1. Acceptable form for answers is as follows:
 - a. All answers must be expressed in simplest form. A "common fraction" is to be considered a fraction in the form $\pm \frac{a}{b}$, where a and b are natural numbers, and GCF(a,b)=1. In some cases the term "common fraction" is to be considered a fraction in the form $\frac{A}{B}$, where A and B are algebraic expressions, and A and B do not share a common factor. A simplified "mixed number" ("mixed numeral," "mixed fraction") is to be considered a fraction in the form $\pm N\frac{a}{b}$, where N, a and b are natural numbers, a < b and GCF(a,b)=1.
 - b. Ratios should be expressed as common fractions, unless otherwise specified.

- c. Radicals must be simplified. A simplified radical must satisfy the following conditions:
 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction.

 Numbers with fractional exponents are *not* in radical form.
- d. Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., "How many dollars...," "How much will it cost...," "What is the amount of interest...") should be expressed in the form of (\$)a.bc, where a is an integer and b and c are digits. The only exceptions to this rule occur when a is zero, in which case it may be omitted, and when b and c are both zero, in which case they may both be omitted.
- e. Units of measurement are not required in answers, but must be correct if given. When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lbs 4 oz will not be accepted. Similarly, if a problem asks for the number of cents and 25 cents is the correct answer, \$0.25 will not be accepted.
- f. Do not make approximations for numbers (e.g. π , $\frac{2}{7}$, $\sqrt{2}$) in the data given or in solutions unless the problem says to do so.
- g. Do not do any intermediate rounding (other than the "rounding" the calculator performs) when computing solutions to problems. All rounding should be done at the end of the calculation process.
- h. Answers expressed in scientific notation should be in the form $a \times 10^n$ where a is a decimal, $1 \le |a| < 10$ and n is an integer.
- i. An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole-number answers should be expressed in their whole-number form. Thus, 25.0 will not be accepted for 25.
- j. The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.
- 2. Specific instructions stated in a given problem should take precedence over any general rule or procedure.
- 3. Scores are kept for individuals and teams. The **individual score** is the sum of the number of Sprint Round questions answered correctly plus twice the number of Target Round questions answered correctly. *The maximum possible individual score is 46.* The **team score** is calculated by dividing the sum of the team members' individual scores by four, and then adding twice the number of Team Round questions answered correctly. *The maximum possible team score is 66.*

Because the School Competition is not the only mechanism available to determine which students should advance to the Chapter Competition, ties on the School Competition needn't be broken. At the Chapter, State and National Competitions, however, ties among individuals or teams will be broken by comparing the scores of specific rounds.

Sprint Round Answers

- 1. 17 candy bars
- **2.** 25
- **3.** 65 °F
- **4.** 2675
- **5.** 4 triangles
- **6.** 7
- **7.** 100 yards
- **8.** 4
- **9.** 16,200 pounds
- **10.** 5

- **11.** 11 fractions
- **12.** 5
- 13. $\frac{13}{6}$
- **14.** $192\frac{1}{3}$
- **15.** 0.3 or .3
- **16.** \$60 or 60.00
- **17.** (4, 3)
- **18.** 21
- 19. $\frac{2}{5}$
- **20.** 3 dimes

- **21.** 10 drops
- **22.** 5 ways
- **23.** 93 games
- **24.** 85
- **25.** 20 units
- **26.** 8 in^2
- **27.** 200,000 people
- **28.** 55 coins
- **29.** $-\frac{1}{2}$
- **30.** 64 units²

Target Round Answers

- **1.** Saturday
- **2.** \$394,184,000 or \$394,184,000.00
- **3.** 16 paths
- 4. 6 minutes
- **5.** 6.25%
- 6. $\frac{1}{21}$
- 7. 483 cm²
- **8.** 4.1 cm

Team Round Answers

- **1**. 100
- **2.** 11,259
- **3.** 14.2 cm
- **4.** 21
- 5. 12 triangles

- **6.** 30 feet
- **7.** 176 feet
- **8.** 14,256 ft²
- **9.** 157 mi/h
- 10. $\frac{180}{13}$ units

2014 ■ School Competition ■ Sprint Round Problems 1–30

Name			

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

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1.	candy bars
1.	

At a candy store Alexis purchased 3 candy bars for \$1.50. At this rate, how many whole candy bars can she buy with \$8.75?



2. _____

If $x \boxtimes y = (x + y)^2$ for positive integers x and y, then $4 \boxtimes 5 = (4 + 5)^2 = 9^2 = 81$. What is the value of $2 \boxtimes 3$?

3.



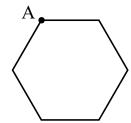
Consider the weather data for 10 days shown in the table below. John identified the mode of the daily high temperature and the mode of the daily low temperature, in degrees Fahrenheit, based on this data. What is the mean of those two numbers?

	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Sun.	Mon.	Tues.	Wed.
High	72	74	70	69	72	74	78	79	72	70
Low	61	58	60	58	63	58	63	58	60	59

4. _____

Karina wrote the addition problem shown here on the board. However, one digit is incorrect. When written correctly, the number containing the incorrect digit is what four-digit integer?

5. _____ triangles

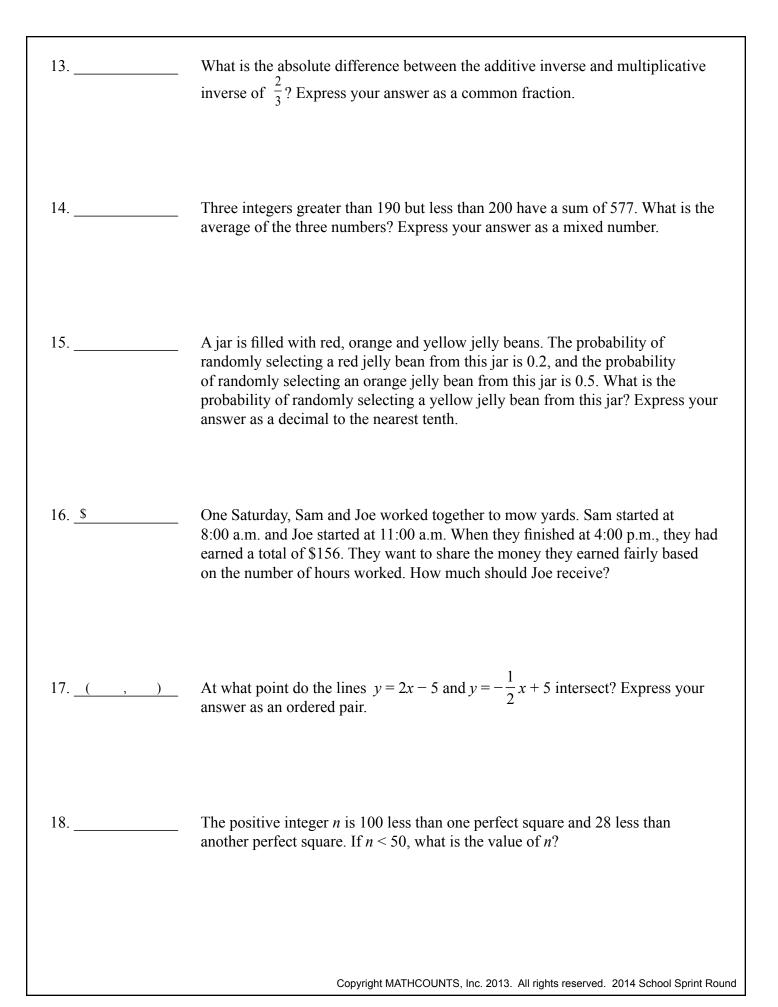


Point A is a vertex of a regular hexagon. When all possible diagonals from point A are drawn in this hexagon, how many triangles are formed?

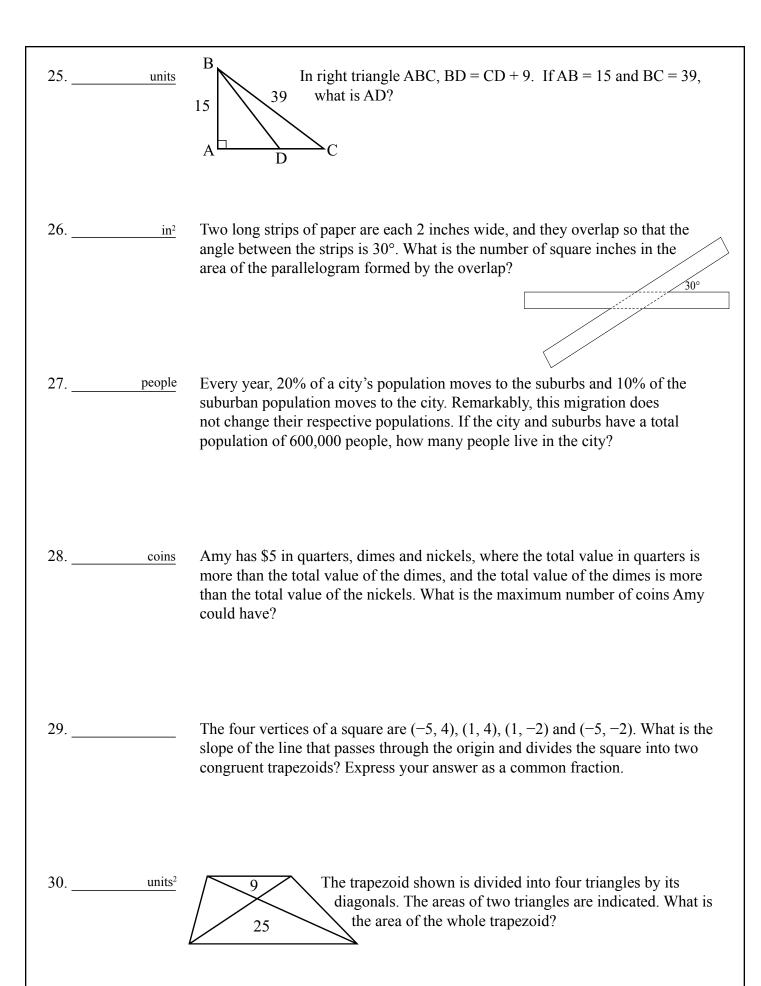
6. _____

The function y = 3x + 6 is graphed on a coordinate plane. The y-coordinate of a point on the line is 27. What is the x-coordinate of that point?

7yards	Two men stand back-to-back and walk in opposite directions for 40 yards each. Each of them then turns left and walks another 30 yards each. In yards, how far are the two men from one another?						
8.	Jacinta rolled a standard six-sided die repeatedly until she got every number from 1 to 6. She stopped rolling the die when she'd gotten each number at least once. The line plot shows how many times each number occurred. What number occurred on her last roll? Number of Times 1 through 6 Rolled X X X X X X X X X X X X X						
9. pounds	If 1 bag of seed covers exactly 90 square meters, and each square meter produces 60 pounds of oats, how many pounds of oats will be produced if 3 bags are planted?						
10	What is the sum of the two integers between which $\frac{\sqrt{17}}{2}$ lies?						
11. <u>fractions</u>	How many common fractions strictly between 0 and 1 have a denominator of at most 6 when written in lowest terms?						
12	What is the sum of the exponents when $\frac{(x^4y^2)(x^3y)^3}{(x^3y^2)^2(x^2y)}$ is written in simplest form with all exponents non-negative?						



19	In one middle school, 300 students play sports and 200 students are on the Honor Roll. If 120 of the students on the Honor Roll also play sports, what is the probability that a randomly chosen student who plays a sport is also on the Honor Roll? Express your answer as a common fraction.
20. dimes	Tailynn has 12 common US coins that add to 83¢. She has exactly five coins of one type. How many dimes does she have?
21. <u>drops</u>	A veterinary assistant needs to prepare a mixture of a vitamin solution for parakeets. The ratio is 1 drop of vitamin oil to 25 drops of water. If the assistant needs to prepare 260 drops of the combined solution, how many drops of the vitamin oil will she need?
22. <u>ways</u>	How many ways can the integers 1 through 6 be arranged in the 2-by-3 grid shown such that each row and each column have their numbers in increasing order?
23. games	A baseball team has won 15 games and lost 9. If these 24 games represent $\frac{1}{6}$ of the games played during the entire season, how many more games must the team win in order to win $\frac{3}{4}$ of their games for the season?
24	An arithmetic progression is formed by five distinct prime numbers. What is the least possible sum of those five numbers?



2014 ■ School Competition ■ Target Round Problems 1 & 2

Name			
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DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of eight problems, which will be presented in pairs. Work on one pair of problems will be completed and answers will be collected before the next pair is distributed. The time limit for each pair of problems is six minutes. The first pair of problems is on the other side of this sheet. When told to do so, turn the page over and begin working. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the problem sheets. If you complete the problems before time is called, use the time remaining to check your answers.

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1.	day thereafter, is previous day. The	ran one mile. The 15% greater than to day that the distants is what day of the	he distance nce Sam ru	he ran the	
2. \$	The table shows t	the number of coin			
Σ. Ψ	352,800,000 nick facility that year.	els (worth \$17,640 In dollars, what is cility in Philadelpl	0,000) were the total va	produced	at the Denver
Δ. Ψ	352,800,000 nick facility that year.	els (worth \$17,640 In dollars, what is	0,000) were the total va	produced	at the Denver
Δ. Ψ	352,800,000 nick facility that year. that year at the fa	els (worth \$17,640 In dollars, what is cility in Philadelph	0,000) were the total vania?	e produced alue of all o	at the Denver coins produced

2014 ■ School Competition ■ Target Round Problems 3 & 4

Name			
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3. paths

Starting with the G on top and only moving one letter at a time to one of the two closest letters in the row beneath it, how many different paths from top to bottom spell GREAT?

G
RR
EEE
AAAA
TTTTT

At Podunk Regional Airport, 3% of flights are delayed by one hour, 2% are delayed by two hours, and 1% are delayed by three hours. The other 94% of flights are on time. What is the average delay, in minutes, across all flights at this airport?

2014 ■ School Competition ■ Target Round Problems 5 & 6

Name			

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5	Where Kyle lives, there is a 4.75% tax rate, so a dresser priced at \$96 would cost \$100.56 with tax. Where Kendra lives, a \$96 item costs \$102 with tax. What percent is the tax rate where Kendra lives? Express your answer to the nearest hundredth.
6.	A code consists of four different digits from 1 to 9, inclusive. What is the probability of selecting a code that consists of four consecutive digits but not necessarily in order? Express your answer as a common fraction.

2014 ■ School Competition ■ Target Round Problems 7 & 8

Name		

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7.	cm ²	A jeweler can bend a piece of wire to make a circle with an area of 615.44 cm². What is the maximum area the jeweler could enclose with the same length of wire in the shape of a square? Express your answer to the nearest whole number.
8.	cm	Circle P has radius 10 cm. Two perpendicular radii are drawn, and a smaller circle is drawn tangent to both radii and the larger circle, as shown. What is the radius of the smaller circle? Express your answer as a decimal to the nearest tenth.

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2014 ■ School Competition ■ Team Round Problems 1–10

Team Members	, Captain
•	

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 10 problems which the team has 20 minutes to complete. Team members may work together in any way to solve the problems. Team members may talk to each other during this section of the competition. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. The team captain must record the team's official answers on his/her own competition booklet, which is the only booklet that will be scored. If the team completes the problems before time is called, use the remaining time to check your answers.

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1.	The integers 1 to 8, inclusive, are arranged into four pairs of distinct numbers, using each integer exactly once. The product of each pair is calculated, and then the sum of the four products is found. What is the maximum possible value of the sum?
2.	What is the largest positive integer such that each digit is at least the sum of all the digits to its left?
3. <u>cm</u>	A rectangular box is 9 cm wide, 11 cm tall and 20 cm long. In centimeters, what is the diameter of the smallest circular opening through which the box will fit? Express your answer as a decimal to the nearest tenth.
4	In the table shown, a and b each represent an integer from 1 to 9, inclusive. The sum of each row and each column is given. What is the value of ab?
	Total
	a 8 b 5 23
	2 a 2 b 14
	b b a a 20
	b 8 5 a 23
	Total 19 26 17 18
5. triangles	How many triangles of any size are there in this figure?

6. <u>feet</u>	A small rubber ball always bounces to half the height from which it fell. If one bounce is considered to be the downward drop and the upward rebound, how far will the ball travel (up and down combined) in 10 bounces if it is dropped from a height of 10 feet? Express your answer to the nearest whole number.
7. <u>feet</u>	Police can use the formula $s = \sqrt{24d}$ in accident investigations to determine the speed of a vehicle when the brakes are applied on asphalt in dry conditions. The s represents the speed of the vehicle, in miles per hour, when the brakes are applied, and d represents the stopping distance of the vehicle, in feet. Using this formula, how many feet would a car traveling 65 mi/h need in order to come to a stop on asphalt in dry conditions? Express your answer to the nearest whole number.
8. <u>ft²</u>	In a particular rectangular pool, it is possible to swim a mile by swimming the length 40 times or along the perimeter (the pool's edge) 11 times. There are 5280 feet in a mile. In square feet, what is the area of the region bounded by the edge of the pool?
9. <u>mi/h</u>	Jinting's speed in a seven-person drag race was 153 mi/h. Jinting's speed was 24 mi/h faster than Xavier's. Kasimu's speed was 14 mi/h slower than Misha's. Pierre's speed was 10 mi/h faster than Yoko's and was 38 mi/h faster than Xavier's. Becca's speed was 17 mi/h slower than Kasimu's. Yoko's speed was 15 mi/h faster than Becca's. In miles per hour, what was the speed of the person who finished fourth?
10 units	In rectangle ABCD, shown here, \overline{CE} is perpendicular to \overline{BD} . If BC = 15 and DC = 36, what is CE? Express your answer as a common fraction.

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