Proportions and Proportional Reasoning

Mathematics Grade 7

This unit focuses on analyzing proportional relationships and using them to solve real-world and mathematical problems. It addresses *Critical Area I*, "developing understanding of and applying proportional relationships," in the *Massachusetts Curriculum Frameworks for Mathematics* for grade 7. Students extend their understanding of ratios and rate from grade 6 to develop an understanding of proportionality to solve single- and multi-step problems, including a variety of percent problems.

These Model Curriculum Units are designed to exemplify the expectations outlined in the MA Curriculum Frameworks for English Language Arts/Literacy and Mathematics incorporating the Common Core State Standards, as well as all other MA Curriculum Frameworks. These units include lesson plans, Curriculum Embedded Performance Assessments, and resources. In using these units, it is important to consider the variability of learners in your class and make adaptations as necessary.

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<tr><th>Goals</th><th colspan="2">Transfer</th></tr>
<tr><td rowspan="2">Massachusetts Curriculum Framework for Mathematics, 2011

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction (1/2)/(1/4) miles per hour, equivalently 2 miles per hour.

7.RP.2 Recognize and represent proportional relationships between quantities.

a) Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

b) Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

c) Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn</td><td colspan="2">Students will be able to independently use their learning to…

Apply mathematical knowledge to analyze and model mathematical relationships in the context of the situation in order to make decisions, make conclusions, and solve problems.</td></tr>
<tr><td colspan="2">Meaning</td></tr>
<tr><td>UNDERSTANDINGS

Students will understand that…

1. Proportions represent a constant rate of change.

2. Proportional reasoning can be applied to real world situations.

3. There are different ways to represent/model proportions. (e.g., tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships)

4. In some cases, the most informative assessment of a situation is to form ratios between pairs of quantities. (e.g., is or is not)</td><td>ESSENTIAL QUESTIONS

1. What kinds of questions can be answered using proportional reasoning?

2. In what ways, will you use proportional reasoning for personal finance/making purchasing decisions?

3. What types of questions cannot be answered using proportional reasoning? Why not?</td></tr>
</table>

d) Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate. 7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: discount, percent increase and decrease. SMP.1 Make sense of problems and persevere in solving them. SMP.2 Reason abstractly and quantitatively. SMP.3 Construct viable arguments and critique the reasoning of others. SMP.4 Model with Mathematics. SMP.6 Attend to precision. ***Massachusetts Curriculum Framework for English Language Arts and Literacy, 2011:*** 6-8.WHST.1 Write arguments focused on discipline-specific content 6-8.WHST.2.D Use precise language and domain-specific vocabulary to inform about or explain the topic. 6-8.WHST.2.F Provide a concluding statement or section that follows from and supports the information or explanation presented.	*Students will know…* 1. The concept "proportional" is defined as the equivalency of ratios. 2. Proportional reasoning involves comparisons of the relationships among ratios. 3. Unit rate can be a measure of the steepness of the related line. 4. Proportional vocabulary/language-unit rate, ratios, proportions, proportional reasoning, equivalence, discounts, percent of increase/decrease, constant of proportionality, origin (x, y plot), scale factor, complex fraction. 5. Proportional relationship may exist between variables in an equation (e.g., d/r=t) 6. A variety of ways to represent a proportion (e.g., in tables,	*Students will be skilled at…* 1. Setting up and solving proportions 2. Determining if two quantities are proportional and solve. 3. Identifying unit rate in table, graph, equation, diagrams, verbal description 4. Modeling proportions using an equation (e.g., d/r = t) 5. Plotting a set of points and interpreting the points (0, 0) and (1, r) where r is the unit rate. 6. Solving problems (discounts, percent increase, percent decrease, percent of error) 7. Writing and sharing word problems involving proportions. 8. Constructing viable arguments and critiquing others when proportions/proportional reasoning would or would not solve a problem 9. Scaling a ratio, rate, or fraction up or down with the same relative characteristics as the original. (e.g., enlarge a photo)

	graphs, equations, diagrams, and verbal descriptions of proportional relationships).	
	Evidence	
	Curriculum Embedded Performance Assessment **PERFORMANCE TASK(S):** **Title: Should We Drive or Bike?** **Goal:** Determine if the increase in gasoline consumption will allow your brother to drive to after-school practice or if you both will need to ride your bikes. **R**ole: Analysts (rider) **A**udience: The brother **Situation:** Your brother Tim drives the two of you to and from school every day in his car. Tim pays for gas using his weekly allowance. You and Tim are excited because you both just made the soccer team! Now you and Tim will have daily practice after school at the town soccer field on the other side of town. Attending practice means Tim will need to pay more for gas each week. Can Tim afford to buy the extra gas needed on his current allowance or will you both need to ride bikes to practice? **Product/ performance:** You will create a data display (e.g. story board, posters, PowerPoint, etc.) of your findings and conclusions including mathematical evidence. You will also need to make a presentation to your class. (further details provided on page 86)	

OTHER EVIDENCE:

- Pre-Assessment- Rabbit Hash
- Frayer Models for *proportion* and other unit vocabulary

- Formative Assessment- Compute unit rate and make unit conversions; specifically compute ratio rates associated with ratios and fractions (e.g., if a person walks 1/2 mile every 1/4 hour, compute the unit rate as the complex fraction (1/2)/(1/4) mph, equivalently 2 mph)
- Work sample: Create a design, choose scale factor(s) and enlarge/reduce the design
- Formative Assessment- Determine the scale factor and/or determine the missing side of a polygon given similar figures
- Group Activity: Choose a scale factor and create a floor plan of the classroom or other room of their choice
- Role-play: Comparison Shopping (differentiate -whole number % (54%) vs. rational % (62 1/8%)
- Open Response Prompt: Why did/do mathematicians create/apply proportional reasoning to solve real world problems?
- Teacher Observation
- Summative assessment

Stage 3 – Learning Plan

Summary of Key Learning Events and Instruction

Lesson 1. Unit Launch & Pre-Assessment

Lesson 2. Body Ratio Arm Span to Height (ratio and unit rate)

Lesson 3. Sunday Circulars Scavenger Hunt (mark down, percent decrease)

Lesson 4. The Big Sale (comparison shopping)

Lesson 5. Census **(**percent increase, decrease and change)

Lesson 6. Intercepting Villains (d/t=r)

Lesson 7. Are Cars Speeding in Front of School? (mile per hour)

Lesson 8. Are They Proportional? (students discuss proportional and non-proportional scenarios)

Lesson 9. You're a Rock Star! (scaling a photograph- scale factor and golden ratio)

Lesson10. Student Generated Word Problems **(**write word problems for other students to solve)

Lesson 11. Gulliver's Suit by Proportion (students conjecture making a suit by taking only one measurement)

Unit Resources

BOOKS

Cut Down to Size at High Noon by Scott Sundby and illustrated by Wayne Geehan
This parody of classic western movies teaches scale and proportion. The story takes place in Cowlick, a town filled with people with intricate western-themed hairstyles that the town's one and only barber creates with the help of scale drawings. Enter a second barber, and the town does not seem big enough for both of them! The story reaches its high point of suspense when the two barbers face off with scissors at high noon. The duel ends in a draw of equally magnificent haircuts, one in the shape of a grasshopper and the other in the shape of a train engine, and the reader learns that scale drawings can be used to scale up as well as down.

If You Hopped Like a Frog by David M. Schwartz and illustrated by James Warhola
Imagine, with the help of ratio and proportion, what you could accomplish if you could hop like a frog or eat like a shrew. You would certainly be a shoo-in for the Guinness World Records. The book first shows what a person could do if he or she could hop proportionately as far as a frog or were proportionately as powerful as an ant. At the back of the book, the author explains each example and poses questions at the end of the explanations.

WEBPAGES

MADESE Common Core
http://www.doe.mass.edu/candi/commoncore

NCTM Illuminations
http://illuminations.nctm.org/Lessons.aspx

PBS learning media
http://www.pbslearningmedia.org

Teachers' Domain
http://www.teachersdomain.org/asset/scl10_int_shadows/

Wiki Space
http://7math.wikispaces.com/Proportional+Reasoning

Ask Dr. Math
http://mathforum.org/library/drmath/sets/mid_ratio.html

Math History
http://archives.math.utk.edu/articles/atuyl/confrac/history.html

Math Wiki
http://7math.wikispaces.com/Proportional+Reasoning

Writing in Math:
Bernadette Russek, Writing to Learn Mathematics, WAC Journal, Vol. 9, pp. 36-45. (PDF file)

Using Writing in Mathematics (University of Puget Sound)

LESSON RESOURCES

Gulliver's Travel web site- http://www.literaturecollection.com/a/swift/gulliver/7/

Enlarge/reduce photo Scaling a Photograph (Resource http://www.pbslearningmedia.org/content/vtl07.math.number.rat.lpscaleup/)
Rabbit Hash handouts

Donald in Mathland video http://www.teachertube.com/viewVideo.php?video_id=35970

Bianca Gears video http://www.pbslearningmedia.org/content/vtl07.math.number.rat.lpgears/#content/4dd2ff59add2c73bce009582

Bicycles, Past and Present (optional lesson)

Lesson 1 – Unit Launch

Estimated Time: 1-2 sixty minute classes

Resources for Lesson:

- Teacher internet access
- *Rabbit Hash* handouts
- *Donald in Mathland* video
  http://www.teachertube.com/viewVideo.php?video_id=35970
- *Bianca Gears* video
  http://www.pbslearningmedia.org/content/vtl07.math.number.rat.lpgears/#content/4dd2ff59add2c73bce009582
- *Bicycles, Past and Present (optional lesson)*

Content Area/Course: Mathematics Grade: 7
Time (minutes or hours): 1-2 sixty minute classes
Unit Title: Proportions and Proportional Reasoning
Lesson 1: Unit Launch
Essential Question(s) to be addressed in this lesson:

What kinds of questions can be answered using proportional reasoning?

Standard(s)/Unit Goal(s) to be addressed in this lesson: 7.RP Analyze proportional relationships and use them to solve real-world and mathematical problems.

SMP.1 Make sense of problems and persevere in solving them

Assumptions about what students know and are able to do coming into this lesson (including language needs): ratios, rate, and unit rate

Resources for Lesson:

- Teacher internet access
- *Rabbit Hash* handouts
- *Donald in Mathland* video http://www.teachertube.com/viewVideo.php?video_id=35970
- *Bianca Gears* video http://www.pbslearningmedia.org/content/vtl07.math.number.rat.lpgears/#content/4dd2ff59add2c73bce009582
- *Bicycles, Past and Present (optional lesson)*

Lesson Sequence and Description	Teacher notes
Unit Launch Introduce the Unit- *Today we are going to begin a new unit. Last year you learned about ratio and rate. Today you will learn about proportions and proportional reasoning.* *Who can tell us about what you remember about ratios? Rate? Unit Rate?* 1) Pre-assess students using "Rabbit Hash" assignment (on pages following this lesson plan) 2) HOOK'em -Set Context/ Background by viewing either the Donald in Mathland or Bianca Gears video. If you choose to use the *Bianca Gears* video, the following questions will focus students on the mathematics of the video: • **2:07 remaining on video-** Pause so students can see the model. Ask, "When the salesman turns the handle, what is that similar to? (pedaling) What did you observe about the rotation speed of the back gear?" • **1:45 remaining-** ask students "how many teeth were on the front wheel? The back? What is the ratio between them? • **1:38 remaining-** (right after Bianca counts 9 on the back gear) ask students to predict the ratio between the back and the front. Ask, "If you pedal once, how many times will the back gear spin?" • After the video, ask the following questions: • When you change to a "higher" gear, why do you move faster? • Why do you move slower when you change to a lower gear? • Is it better to be in a higher or lower gear when going up a hill? Why? • How does the ratio change if you're in 9th gear? How many teeth would be on the back gear? There are a few different options for launching this unit. 2) The *Donald in Math Land* video could also be used and followed with the generate-your-own rectangle activity (see more detailed notes below). Suggestions using the Donald in Math Land are as follows; 1- After the video, ask "what did the Greeks compare when looking at rectangles?" (the length: width). Explain that the ratio is irrational, but can be approximated by~1.62.	Preview videos and decide which you will use: *Donald in Math Land* http://www.teachertube.com/viewVideo.php?video_id=35970 (start at about 2 mins into the video- This video is used to build a historical context for the use of ratio and proportion. or *Bianca Gears* http://www.pbslearningmedia.org/content/vtl07.math.number.rat.lpgears/#content/4dd2ff59add2c73bce009582 Specific accommodations for students who indicate through pre-assessment that they already know- use the following activity as a station Measurement: *How Many Noses Are in Your Arm?* PBS Mathline lesson http://www.pbslearningmedia.org/ search on *How Many Noses Are in Your Arm?* The video is designed for an educator audience for professional development purposes and not appropriate for students. However, there is a lesson plan (click on the *Support*

2- With a ruler, draw a horizontal foot-long line on the board. Ask, "if this line segment is a foot long, what should the length be?" The students should hopefully say 1.62. You may also ask "how many inches is 1.62 feet" (1 ft, 7.4 inches). 3- Complete the rectangle and draw a 2 foot line. Ask for the length's measurements, and then complete the rectangle. 4- Draw another 1 foot line, this time vertically. Ask for the dimensions if the 1 foot is the longer dimension. Complete the rectangle. Ask for another ratio (like 2:1). Make a 2:1 rectangle. 4- Ask students to make rectangles of their own in groups on paper. 5- When they're done, students can tape their rectangles onto the board with the other examples. 6- At the end, discuss, which do you think is most pleasing to the eye? Why do you think so many artists and musicians are attracted to the idea of the "golden ratio?" An optional activity, *Bicycles, Past and Present*, is included following this lesson. Start a Math Journal – Title it *Proportions* (to be used throughout unit for student reflection on learning), 1st entry for Homework (see below)	*Materials* tab to access the lesson plan) that can be *adapted* for use in a learning station. • ELLs- set context visually through video and where possible connect vocabulary words with pictures
Extended Learning/Practice (homework) Math Journal- Look online, in a newspaper, etc. to find at least three interesting ratios or proportions. Explain why it is a ratio or proportion and why it is important. For instance, '1 in 7 people in MA goes to bed hungry; this ratio is important because it might help raise awareness about the hungry in MA'. If you don't have access to a newspaper or the internet, describe a proportion or ratio of something that you would like to find out. For instance, 'I would like to find the ratio of stray dogs to pet dogs to see how many are on the street."	
Closure **Review outcomes of this lesson:** Beginning understanding of concepts **Preview outcomes for the next lesson:** Is there a proportional relationship between parts of your body (length of your face and the width of your face)?	

Lesson 1 Resources

The Totally True Tale of Rabbit Hash, Kentucky

Rabbit Hash is a small town located in rural Kentucky. It is named after its famous rabbit hash dish, which contains fried rabbit and potatoes. Along with having a strange name, the town is known for its crazy elections. In 1998, a dog named Goofy ran for mayor and won! In 2004, Junior, another dog, was elected for mayor. After Junior passed away, the town held another election, and here are the results:

Lucy Lou, Border Collie, 8,085 votes

Toby, Springer Spaniel, 4,596 votes

Travis, Cat, 3,721 votes

Higgins, Miniature Donkey, 2,229 votes

Macy, Ibizan Hound, 182 votes

Isabella Pearl, Boxer/Labrador mix, 580 votes

Pike, Chihuahua mix, 557 votes

Rembrandt, St. Bernard mix, 494 votes

Noggin, Clumber Spaniel, 184 votes

Molly C. Urso, Boxer, 150 votes

Peggy Lee, Poodle, 73 votes

Cletus, Bull Mastiff, 54 votes

Manson Mayer, Border Collie Mix, 8 votes

Paulette, 5 votes

Alex, human, 2 votes

Ruby, 1 vote

Questions to answer:

What percentage of the total votes did Lucy Lou win? Was it a close race? Explain.

How do Lucy Lou's votes compare to the runner-up? Write the comparison in as many different ways as you can think of.

Sources:
http://en.wikipedia.org/wiki/Rabbit_Hash,_Kentucky
http://www.rabbithashusa.com/election_vote.php
http://www.grouprecipes.com/24395/rabbit-hash.html

How to Make Rabbit Hash:

3 cups of cooked, de-boned rabbit
1 medium onion, chopped
5 medium potatoes, diced to fry
2 1/2 tsp. sage (more if desired)

Cook rabbit until tender and remove from bone.
Place diced potatoes in an iron skillet and fry until tender.
Add onions, sage, rabbit and cook for 10 min. Serve.

Questions to answer:

This recipe serves 6. If we wanted to give one serving to every voter in the election, how much of each ingredient would we need?

Rabbit: Potato:

Onion: Sage:

One tablespoon equals 3 teaspoons. How many tablespoons of sage do we need to feed every voter? One cup equals 48 teaspoons. How many cups of sage do we need to feed every voter?

How much of each ingredient do we need if we just want a single serving?

What is our rate of rabbit per serving?

What is the ratio of rabbit to potato? Does the ratio change as the amount of rabbit and potato increases?

Sources:
http://en.wikipedia.org/wiki/Rabbit_Hash,_Kentucky
http://www.rabbithashusa.com/election_vote.php
http://www.grouprecipes.com/24395/rabbit-hash.html

Lesson Sequence for *Bicycles, Past and Present*

This lesson is optional and can be used to introduce the unit, or as an additional lesson elsewhere in the unit. It is important to note that this lesson assumes prior understanding with circumference.

1. Ask students, why do bicycles have gears? What do they help you to do? Let them share their answers for a few minutes and then explain that today they will explore why gears work by learning about bicycles throughout history.

2. Put students in groups, then hand out worksheet, ruler, paper plates, and yardsticks.

3. Circulate the room while the students work on the activity. After the majority of the students have completed the first part, view *Bianca Gears*.

4. Have students finish the activity. If time, gather the class together and have them share their results. Questions you could ask:

Did your group decide to buy the boneshaker or the penny-farthing? Why?

What patterns did you notice when you filled out the table?

If you were designing a mountain bike, how many gears would you give it? What about a street bike?

Gearing Up!

Why do bicycles have gears? Why is third gear harder to pedal than first? And why do you move faster in higher gears? Well, to answer these questions, let's take a journey through time.

There were two choices for bicycles when they were first invented in 1860s France. The first was called the "boneshaker," invented in the early 1860s. It looked very similar to today's bicycle, but its wheels were made of wood and iron. This made for quite a bumpy ride! The other bicycle, invented in the early 1870s, was called a "penny-farthing" or "ordinary." It was known for having a very large front wheel- usually at least four feet in diameter! Because its front wheel was so big, the ride was much more comfortable.

Imagine that you lived in the late 1800s and had to make the decision of whether to purchase a boneshaker or a penny-farthing. Which bicycle would you choose? Well, before making your decision you should probably figure out which bike would go faster.

Let's use a small tricycle as a model. Like the boneshaker and penny-farthing, a tricycle's pedals are directly connected to the front wheel so that one pedal rotation equals one wheel rotation.

How far will the tricycle go if you pedal once? Make a prediction:

Now let's test your hypothesis. Take the paper plate and measure its diameter. What is its circumference?

Take the paper plate and roll it once around against a measuring stick. How many inches did it travel? How does that compare to its circumference?

Did you know…? Because riders were positioned almost directly over the front axle of the penny-farthing, when the rider hit a large bump they would be thrown head-first over the front over the bicycle. This is where the term "taking a header" came from!

Imagine that the paper plate was the front wheel of your tricycle. If you pedaled once, how far would you go?

If you were able to pedal 30 times per minute,

-How fast would you go in inches per minute?

-What is that in miles per hour? Remember that there are 12 inches in 1 foot and 5280 feet per mile.

So little tricycles do not go very fast, which is probably why parents let small children use them. Now, remember the tricycle and think back to the boneshaker and penny-farthing. Of the two, which do you think would go faster? Why?

What are the advantages and disadvantages of each? Which would you buy if you lived back then?

"Safety" bicycles were introduced in the late 1880s. They are similar to the models we have today. Not only did they have a new, more comfortable material for tires, but they also had gears which made them fast! But why were they faster than previous bicycles? Let's find out.

First, imagine that you have a bicycle with 27 inch diameter tires and your front gear has 36 cogs.

Now imagine that you're riding along in first gear, so your ratio is 1:1.

1) Draw a picture of the front and back gears. Think about how the two gears compare in size and number of cogs.

2) When you pedal once,
   a. How many times does the back gear spin?
   b. How many times does your back wheel spin?
   c. How many times does your front wheel spin?
   d. How far do you travel?

3) If you pedal 30 times per minute,
   a. What is your speed in inches per minute?
   b. Miles per hour?

Now imagine that you switched into **3rd gear**.

Did you know…? All man-powered vehicles traveling on the ground with one or more wheel are called velocipedes!

4) For every time you pedal,
   a. What is the ratio?

   b. How many times does the back gear spin?

   c. How many times does your back wheel spin?

   d. How many times does your front wheel spin?

   e. How far do you travel?

5) Draw a picture of the front gear and the back gear. Think about how the gears compare in size and number of cogs.

6) If you pedaled 30 times per minute in 3rd gear,
   a. What is your speed in inches per minute?

   b. In miles per hour?

Now you switch to 6th gear.

7) For every time you pedal,
   a. How many times does the back gear spin?

   b. What is the ratio?

   c. How many times does your back wheel spin?

   d. How far would you travel?

8) Draw a picture of the two gears. Think about how they compare in size and number of cogs.

9) If you pedaled 30 times per minute in 6th gear,
   a. What is your speed in inches per minute?

b. In miles per hour?

Plot all of your data in this chart:

Ratio	Number of Cogs in Front Gear	Number of Cogs in Back Gear	Distance Traveled Per 1 Pedal	Speed Traveled at 30 Pedals per Minute (In Miles per Hour)

10) Would it be harder or easier for you to pedal in 6th gear versus 3rd gear? Explain why or why not.

11) Are lower or higher gears better for traveling uphill? Why?

12) What patterns do you notice between gear ratios and your speed?

Did you know…? Bicycles contributed to women's rights! With the new safety bicycles, it was much easier for women to travel and participate in their nation's development. Susan B. Anthony wrote, "Let me tell you what I think of bicycling. I think it has done more to emancipate women than anything else in the world. It gives women a feeling of freedom and self-reliance. I stand and rejoice every time I see a woman ride by on a wheel...the picture of free, untrammeled womanhood."

13) Predict your speed if you were in 2nd gear traveling at 30 pedals per minute.

14) Predict your distance if you pedaled once if you were in 9th gear.

Sources:

http://curly.cis.unf.edu/community/gears/basics.html
http://adventure.howstuffworks.com/outdoor-activities/biking/bicycle3.htm
http://en.wikipedia.org/wiki/History_of_the_bicycle
http://en.wikipedia.org/wiki/Penny-farthing#Attributes

Lesson 2 – Body Ratio
Arm Span to Height

Estimated Time: 60 minutes

Resources for Lesson:

- Tape measures
- Graph paper
- Rulers

Content Area/Course: Mathematics **Grade: 7**

Time (minutes or hours): 1-2 sixty minute periods

Unit Title: Proportions and Proportional Reasoning

Lesson 2: Body Ratio Arm Span to Height Ratio

Essential Question(s) to be addressed in this lesson:
What kinds of questions can be answered using proportional reasoning?

Standard(s)/Unit Goal(s) to be addressed in this lesson:
7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.

7. RP.2 Recognize and represent proportional relationships between quantities.

7. RP.2a Decide whether two quantities are in a proportional relationship.

7. RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

7. RP.2d Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

SMP.4 Model with Mathematics.

SMP.1 Make sense of problems and persevere in solving them.

SMP.6 Attend to precision.

SMP.3 Construct viable arguments and critique the reasoning of others.

6-8.WHST. 2. D Use precise language and domain specific vocabulary to inform about or explain the topic.

Assumptions about what students know and are able to do coming into this lesson (including language needs):

Understanding of fractions, equivalent fractions, ratio, and plotting points on a coordinate plane

Outcome(s)

By the end of this lesson students will know and be able to:

Define the concept "proportional" as equivalency of ratios
Recognize that proportional reasoning involves comparisons of the relationships among ratios
Compute Unit Rate
Set up proportions
Determine if two quantities are proportional and solve

Instructional Resources/Tools

Tape measures
Graph paper
Rulers

Anticipated Student Preconceptions/Misconceptions

Lengths will not be proportional for different students' data.

Assessment

Lesson Pre-assessment- Formative

1. Define Ratio
2. What is the ratio of boys to girls in the class?
3. What is the ratio of girls to boys in the class?
4. What is the ratio of boys to the total number of students in the class?
5. What is the unit rate of girls to boys?
6. According to the chart below, is there a proportional relationship between the length of a person's face and width of a person's face? How do you know?

Face length in inches	Face width in inches	Ratio	Unit Rate
9 inches	6.5 inches	9/6.5	1.3 length inches per 1 width inch
8.5 inches	6.25 inches	8.5/6.25	1.3 length inches per 1 width inch

Summative (optional)

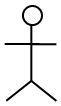
If a person walks 1/3 mile in each 1/2 hour, compute the unit rate.
1/3 mile/1/2 hour= 1/3 divide by ½= 1/3 times 2/1= 2/3 mile per hour

Lesson Sequence and Description

1. Pre Assess (see above)

2. Define Proportion: an equation that states two ratios are equivalent (MA Curriculum Framework for Mathematics. 2011. Page180) Note: Be sure students "discover" by the end of the lesson that if plotted, the line passes through (0, 0) and is linear.

3. HOOK'em! "Am I a Square?"- student explore whether their arm span and height are equal



4- Teacher will form groups of 4-6 students. Each student will measure and record a partner's arm span and height. Each group will create a table using the data collected. Along with storing the data in their own personal table, the students will also add them to a class-wide database. The teacher should leave a computer available that can be hooked up to a projector so students can then take turns adding their group's data to an excel spreadsheet.

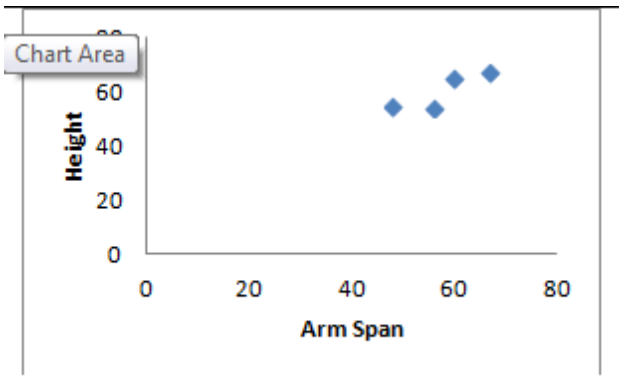
Arm Span in inches	Height in inches

5. Students will plot the points generated. Teacher checks that students labeled the axes correctly. Teacher will circulate and observe, asking students to explain what a point (48, 55) on the graph means in terms of arm span and height. Teacher will also ask one student to explain to another. For example,

Arm Span in inches	Height in inches
48	55
56	54
60	65
67	67

Teacher notes:

- Pre Assessment Questions 1-5, indicate if the students came with the assumed skill. Question 6, indicates if student s already know the content teacher is about to teach. Teach (Maybe done orally with students.
- Discuss the reasonableness of students' answers.
- Some students may compare arm length to height, while others may compare height to arm length. Discuss the difference in both the ratios.
- In number 7, teacher should help students see the linear pattern and the line passes through (0, 0), if unit rates are approximately equal. Teacher can hold a string or ruler on graph to show "line of best fit".
- ELL students - Reinforce vocabulary "Plot, ordered pairs, data, and ratio". English Language Learners' dictionary for proportions can be found at http://www.learnersdictionary.com/search/proportion



6. Within each group, students will calculate the ratios of arm span to height and compare/discuss their ratio with their partner's or group's ratios.

Arm Span in inches	Height in inches	Ratio (arm span/height)	Unit rate (decimal equivalent)

This may be a good place to stop for the day.
The teacher will have either printed out the class' data for each group or put it on each group's computer (if each group has a computer). The teacher will ask the students to plot the data in groups and look for patterns.

7. Students will do a Think-Pair-Share about any patterns they observe. Have groups share out their findings. Guiding questions are as follows:

- Do you think there is a relationship between arm length and body height?
- Does the pattern hold true for everyone or do we have any outliers?
- Why might it be helpful (or not) for humans' arm span to be the same as their height? (e.g., balance) We see that this ratio holds true for people who are 4 and 5 feet tall; what happens when

- Advanced students- Teacher will provide a different exploration opportunity- such as:
  a) Monster Scale- a large hand print found on wall determine the unit ratio of humans to a monster
  b) Height of building using shadow ratios
  c) Barbie or GI Joe dimensions to real person (doll head to feet size)
  d) Shoe size to height, graph.
  Have the student discuss the mathematical model limits, outliers

people get smaller? Taller?

- What does the origin really mean here? What is our unit rate? Do our data form a line?
- Could we create an equation to find the approximate wingspan given the height? How does our equation relate to the ratio?
- How does our equation relate to the slope of our graph? To the origin?
- Is this a proportional relationship? What does proportional mean?

Please note that students will most likely say there is not a proportional relationship because the ratios do not appear to be equivalent upon initial observation. Showing the entire class set of data will allow students to see that the unit rates are all close to 1.00, and indeed all round to the nearest whole number of 1, thus making the ratios equivalent.

Arm Span in inches	Ht. in inches	Ratio	Unit rate
48	55	48/55	0.87 arm inch per 1 height inch
56	54	56/54	1.04 arm inch per 1 height inch
60	65	65/60	0.92 arm inch per 1 height inch
67	67	67/67	1.00 arm inch per 1 height inch

Exit Slip:

Does the following represent a proportion?

6/3 and 4/2

How do you know?

What is the unit rate?

Extended Learning/Practice (homework)

Take the measurements of parent(s) or adults and sibling(s) arm and height measurements and create a table and a graph of the data.

Closure

Review outcomes of this lesson: understand unit rate and recognize a proportional relationship.

Preview outcomes for the next lesson: Percent discount/decrease

.

Lesson 3 – Sunday Circulars Scavenger Hunt

Estimated Time: 60 minutes

Resources for Lesson:

- Teacher internet access
- Sunday circulars or, alternatively, store circulars can be accessed on the web
  For example: On Line Shopper
  http://www.pricegrabber.com/shoppers+online+shopping/products
- Chart paper
- Graph paper
- Journals

Model Curriculum Unit Lesson Plan Template

Content Area/Course: Mathematics **Grade: 7** **Time (minutes or hours): 60 minutes**

Unit Title: Proportions and Proportional Reasoning

Lesson 3: Sunday Circulars Scavenger Hunt

Note: the material in this lesson was covered in grade 6. It should only be used if students don't perform well on the pre-assessment. Otherwise, you should proceed directly to Lesson 4. Alternatively, a 1-2 day activity that is similar to this activity is suggested below.

Essential Question(s) to be addressed in this lesson: In what ways, will you use proportional reasoning for personal finance?
What does percent off mean?

Standard(s)/Unit Goal(s) to be addressed in this lesson:
7.RP.3 Use proportional relationships to solve multistep ratio and percent problems
SMP.4 Model with Mathematics.
SMP.1 Make sense of problems and persevere in solving them.
6-8.WHST.1 Make arguments focused on discipline-specific content
6-8.WHST. 2.F Provide a concluding statement or section that follows from and supports the information or explanation presented.

Assumptions about what students know and are able to do coming into this lesson (including language needs):
Students know what a ratio is and that percent means out of 100. Students know how to simplify fractions, how to change percents to fractions and to decimals and how to multiply by fractions and decimals

Outcome(s)

By the end of this lesson students will know and be able to:
Recognize that proportional reasoning involves comparisons of the relationships among ratios
Compute unit rate.
Solve problems (discounts, tax, percent decrease)

Instructional Resources/Tools

Sunday circulars or, alternatively, store circulars can be accessed on the web
For example: On Line Shopper http://www.pricegrabber.com/shoppers+online+shopping/products
Chart paper, graph paper, journals

Anticipated Student Preconceptions/Misconceptions

5% = .5
Discount = price

Assessment

Pre-assessment/ Formative (This could be assigned as homework. Choose some or all from the following.)	**Summative** (optional)
1. Represent 17% as a fraction. 2. Using 17%, write a statement: ____ out ___ people prefer Crest Toothpaste. 3. Convert several percents to fractions and decimals. For example: ¾ = _____% = _____ as a decimal _____ as a fraction = 67% = _____ as a decimal _____ as a fraction = _____ as a percent = 0.05 4. ½ x ¼ = _____ 5. 1.02 x 0.07 = _____ 6. Does percent off (discount) mean the person pays less or more than the original price? 7. Sneakers are on sale at 15% off. If the original price is $150, what is the discount and what will you pay for the sneakers?	iPad Apps are on sale at 15% off. If the original price is $1.99 what is the discount and what will you pay for the App, including Mass tax (6 ¼)? Specific notes to teacher: Teacher circulates and observes students responding. Reinforce vocabulary Discount Convert Apps

Lesson Sequence and Description	**Teacher notes:**
1. Pre Assess (see above). 2. HOOK'em! Teacher will bring in Sunday Circulars, teacher will assign student to groups of 4 and students will find 3 advertisements for an item of their interest. 3. Students will circle the percent discount in each ad. 4. Students will calculate the selling price for each of the three ads and will decide where they would purchase the item based on the mathematics. 5. Students calculate and add the tax (6 ¼ %) to determine the final price to be paid at the register. 6. Students will present their three options, calculations and why they chose a particular store to the peers in their group. 7. Teacher will choose one student from each group to present his/her options and final purchase decision to the class. 8. In their math journals, students respond to the following prompts: a. How would you state percent off as a unit rate (e.g., 25%) off is unit rate of ¼ dollar "off" to 1 dollar of	• Specific accommodations for advanced students- differentiate by using a rational percent (62 1/8 %) or a fraction they have to change to a percent (e.g., 1/3 off) • Specific accommodations for struggling students- teacher pre selects ads with "easy/benchmark percents" for percent off • Apply the same concepts to problems with simple interest, gratuities, commissions, fees for personal finance to further understand.

original price)? b. You've wanted a Wii for a long time, and now that the new Wii U has come out, it is finally on sale. Each week the store takes an additional 20% off of its initial price of $250. Will the Wii ever be free? Explain your reasoning.	
Extended Learning/Practice (homework) Find 2 advertisements (that include a percent discount) of an item of their choice and determine which advertisement represents the better buy.	

Closure

Review outcomes of this lesson: Students will understand that percent off (discounts) of items can be compared to find the better buy.

Preview outcomes for the next lesson: The Big Sale –how would you decide which to buy: a dozen bagels for $4.80 or nine bagels for $2.30? Which option is a better deal?

Alternate Lesson 3 – Ratios, Percents, and the Origin of Food

Estimated Time: 60-120 minutes

Resources for Lesson:

- Teacher internet access
- Sunday circulars or, alternatively, store circulars can be accessed on the web, links given below
- Chart paper
- Graph paper
- Journals

Model Curriculum Unit Lesson Plan Template

Content Area/Course: Mathematics **Grade: 7** **Time (minutes or hours): 60 minutes**

Unit Title: Proportions and Proportional Reasoning

Lesson 3: Ratios, Percents, and the Origin of Food

Essential Question(s) to be addressed in this lesson: How can you use percentages and ratios to make sound and ethical financial decisions?

Standard(s)/Unit Goal(s) to be addressed in this lesson:
7.RP.3 Use proportional relationships to solve multistep ratio and percent problems
7.RP.2 Recognize and Represent Proportional Relationships between quantities
7.RP.2a Decide whether two quantities are in a proportional relationship.
7.RP.2c Represent proportional relationships by equations.
SMP.4 Model with Mathematics.
SMP.1 Make sense of problems and persevere in solving them.
6-8.WHST.1 Make arguments focused on discipline-specific content
6-8.WHST. 2.F Provide a concluding statement or section that follows from and supports the information or explanation presented.

Assumptions about what students know and are able to do coming into this lesson (including language needs):
Students know what a ratio is and that percent means out of 100. Students know how to simplify fractions, how to change percents to fractions and to decimals and how to multiply by fractions and decimals

Outcome(s)

By the end of this lesson students will know and be able to:
Recognize that proportional reasoning involves comparisons of the relationships among ratios
Solve problems (discounts, tax, percent decrease)
Write equations to describe proportional relationships.

Instructional Resources/Tools

Sunday circulars for grocery stores and farmer's market price list
Chart paper, graph paper, journals

Anticipated Student Preconceptions/Misconceptions

5% = .5
Discount = price

Assessment

Pre-assessment/ Formative (This could be assigned as	**Summative** (optional)

homework. Choose some or all from the following.)	
1. Represent 17% as a fraction. 2. Using 17%, write a statement: ____ out ___ people prefer Crest Toothpaste. 3. Convert several percents to fractions and decimals. For example: ¾ = _____% = _____ as a decimal _____ as a fraction = 67% = _____ as a decimal _____ as a fraction = _____ as a percent = 0.05 4. ½ x ¼ = _____ 5. 1.02 x 0.07 = _____ 6. Does percent off (discount) mean the person pays less or more than the original price? 7. Sneakers are on sale at 15% off. If the original price is $150, what is the discount and what will you pay for the sneakers?	iPad Apps are on sale at 15% off. If the original price is $1.99 what is the discount and what will you pay for the App, including Mass tax (6 ¼)? Specific notes to teacher: Teacher circulates and observes students responding. Reinforce vocabulary Discount Convert Apps

Lesson Sequence and Description	**Teacher notes:**
1) Ask students to brainstorm different places where you can purchase groceries. Ask students to categorize places, such as convenience stores, mega-stores (i.e. Walmart, Costco, Target), traditional grocery stores, and farms/farmers' markets. 2) Ask students to list different food groups and write them on the board: vegetables, fruits, grains, meats, dairy. Ask them to predict where each type of food would be the cheapest and most expensive. 3) Divide the students into groups and assign each a different food group. Distribute the misc. circulars and ask students to choose a few items from their food group. Make sure that the items they choose are located in the farmer's market handout and the majority of the circulars. Have them determine: a. Which store has the steepest discount on that particular food item? b. Which has the least discount? c. Considering the MA sales tax of 6 ¼ percent, which store has the lowest cost? What is the cost? d. What is the percent difference between the highest and lowest price? What is the ratio? e. Ask them to discuss the pros and cons of shopping locally, at a regional grocery store, and at a national/international store.	• Specific accommodations for advanced students- differentiate by using a rational percent (62 1/8 %) or a fraction they have to change to a percent (e.g., 1/3 off) • Specific accommodations for struggling students- teacher pre selects ads with "easy/benchmark percents" for percent off • Apply the same concepts to problems with simple interest, gratuities, commissions, fees for personal finance to further understand.

f. As a group, have them prepare a brief presentation that discusses what they would buy, how they calculated the cost, and why the group chose the particular store, using mathematical data that they've collected.
g. Other discussion questions: why do you think farmer's market products are more expensive (if they are)? What other factors did your group consider when deciding which to purchase from? What can we do to eat locally?

Optional Extension for the following day:

4) In their same groups, ask students research where their food item is typically grown. Alternatively, you can use these estimates:
   - Most poultry is raised in the south. For instance, Tyson Chicken's headquarters is in Arkansas, about 1,400 miles from MA.
   - Most beef cattle are raised in the Midwest, particularly Kansas, Nebraska, and Iowa. Nebraska is about 1,600 miles from MA.
   - Most pigs are raised in Iowa, about 1,250 miles away from MA.
   - There are many dairies located around New England, particularly in Maine. Maine is about 150 miles away.
   - Cheeses are frequently produced in New England as well (i.e. Cabot cheese). Vermont is also about 150 miles away.
   - Most produce is grown in California, which is about 3,100 miles away. Bananas and citrus fruits are grown in Central America. Using Costa Rica as an approximation, it is about 4,200 miles away.
   - Corn is produced in Iowa and Illinois, about 1,100 miles away. Wheat is grown in North Dakota (1,900 miles) and Kansas (1,600 miles).
   - Eggs are also frequently produced in Iowa and the South.
5) An 18 wheeler truck gets about 7 miles per gallon. As a class, write an equation to determine the number of miles traveled per gallon. Then ask students to find
   a. How many gallons of gas does an 18 wheeler use to bring the item to the grocery store?
   b. How many gallons of gas does an 18 wheeler use to bring the item to the farmer's market? (It might be nice to note here that small farmers typically don't

use 18 wheelers to travel to farmer's markets.)

6) Ask students what they know about Carbon Dioxide (CO_2) and make sure they know that humans breathe out carbon dioxide and that it is a greenhouse gas- that it traps heat in Earth's atmosphere and scientists believe it is harmful to the environment. Then ask students to write an equation to calculate how much CO_2 is released during transport, using the statistic that 19 lbs of CO_2 are released per gallon of gas used. Then ask:
    a. How much CO_2 is released during the grocery store item's transport?
    b. How much CO_2 is released during the farmer's market item's transport?
7) Ask students to compare the CO_2 released from the grocery store trip to the farmer's market trip. Ask questions such as: What is the ratio of carbon emissions between farmer's market and grocery store? What is the "percent off"? How many of the farmer's market food products could you purchase for the same amount of emissions as one supermarket product? How does that compare to the ratio between the price of supermarket and farmer's market?
8) If time, students can graph the price and carbon emissions ratios to have additional visuals for their posters.
9) Ask students to prepare and present a poster using yesterday's data about price with today's data about carbon emissions. Have their feelings changed about where they would like to shop? What other options can they think of for eating locally? What steps could they take to encourage healthy eating in their neighborhood?
10) More follow-up questions (if time): The 19 lbs of carbon stat looks at how much CO_2 is released during transportation, but what does it ignore? What other costs could be associated with large-scale farms? Are there any costs that aren't easily measurable? How do you think that would affect our models and how could we incorporate them? What else should we consider?

Optional extension:

1 pound of carbon dioxide fills up the space of a 2 ½ foot diameter ball (imagine a small exercise ball). Ask students if their item's carbon emissions would fit inside the classroom (this requires students to calculate the volume of the room and the ball).

Extended Learning/Practice (homework)	
Let's say that the local grocery store had a 20% off sale on blueberries, which usually sell for $3.99, plus sales tax of 6 ¼ percent. You could buy them at the farmer's market for $3.50, go to a pick-your-own blueberry place and buy them for $2.50 (but you have to pick them by hand). You could also buy a blueberry plant on sale for 15% off. The plant is normally $8.95 and has a fee of $2.95 for shipping. How much does each cost? What is the percent difference between the most and least expensive? What would you buy? What factors went into your decision?	

Closure

Review outcomes of this lesson: Students will understand that percent off (discounts) of items can be compared to find the better buy.

Preview outcomes for the next lesson: The Big Sale –how would you decide which to buy: a dozen bagels for $4.80 or nine bagels for $2.30? Which option is a better deal?

Here are some local farms that visit your farmer's market. If you shop at the market every week, you can get a **frequent shopper's 15% discount.**

Rocky Stone Organic Farms, Purple, Massachusetts

This town is located 18 miles from the farmer's market.

Potatoes….$1.85/lb

Sweet Potatoes…$1.85/lb

Kale….$3.00/bunch

Onions….$1.75/lb

Spinach…. $3/20 oz

Carrots…$3.00/bunch (5-6 large carrots)

Zucchini…$0.85/lb

Butternut Squash…$1.15/lb

Apples…$1.50/lb

Tomatoes…$2.50/lb

Cucumbers…$1.30 per cucumber

Bell Pepper…$1.50/lb

Frontier Valley Creamery, Boonies, Massachusetts

This town is located 31 miles from the farmer's market.

Mozzarella…$6/lb

Gouda…$4.50/8 oz

Cheddar…$4.75/8 oz

Butter Crackers… $3.99/4.4 oz

Summer Sausage… $5.99/12 oz

Giggling Brook Farm and Dairy, Carterham, Massachusetts

This town is located 43 miles from the farmer's market.

Eggs…$3.00/dozen

Beef…$8.75/lb

Small Turkey (10-15 lbs)…$69

Boneless Pork Loin...$13.99/lb

Fresh Ham…$6.99/lb

Smoked Boneless Ham…$12.99/lb

Beef Tenderloin…$23.99/lb

Beef Sirloin…$12.99/lb

Leg of Lamb, 13.99/lb

Whole Chicken: $4.20/lb

Milk…$3.10/half-gallon

Beef Brisket…$9.99/lb

Beef Hot Dogs…$5.35/lb

MacPhearson Brothers Nursery and Sundries
Nashoba, Massachusetts

This town is located 3 miles from the farmer's market.

Maple Syrup…$13.50/pint

Apple Butter…$4.10/10oz

Pumpkin Butter…$4.10/10oz

Watermelon Pickle…$4.10/10oz

Honey…$4.90/8oz

Strawberry Jam…$4.10/10oz

Raspberry Jam…$4.10/10oz

Foxfire Barbeque Sauce, mild…$4.25/16 oz

Foxfire Barbeque Sauce, medium…$4.30/16 oz

Foxfire Barbeque Sauce, Tongue-Scorching…$4.50/16 oz

Flour…$5.10/lb

Cornmeal…$5.10/lb

Mabel's Bakery
Druey, MA

The bakery is located 1.5 miles from the farmer's market.

Bread…$4.00

Bagels…$1.00/bagel

Muffins…$2.60/muffin

Cinnamon Rolls…$2.50/roll

Brownie…$2.50/brownie

Large Cookie…$2.00

Granola…$3.25/4 oz

Circulars Links:

Other Resources:

Map Displaying Where Cattle Are Raised
http://www.factoryfarmmap.org/#animal:all;location:US;year:2007

Articles-good for additional information

"Where does all that food come from?" Discovery.com, aimed at adults but suitable for kids.
http://news.discovery.com/history/us-history/american-thanksgiving-dinner-121120.htm

"Where Does Your Chicken Come From?" Discovery.com, aimed at and appropriate for adults.
http://news.discovery.com/earth/where-does-your-chicken-come-from-120106.htm

Videos-

Sierra Club- Animated video on the true cost of food. All ages.
http://www.sierraclub.org/truecostoffood/movie.asp

PBS- Eating Local (This video is fairly accessible but some larger words might need to be defined and/or discussed) All ages.
http://www.pbs.org/food/features/the-lexicon-of-sustainability-local/

PBS- "Pigs Fly" This short animation shows the Green family discussing how far their food traveled. The site also has more resources about sustainability. Aimed for kids.
http://meetthegreens.pbskids.org/episode14/

Lesson 4 – The Big Sale

Estimated Time: 2 sessions, 60 minutes each

Resources for Lesson:

- Computers with Internet access
- Access to PBS Learning Media:

http://edcar-cdn.pbs.org/u/pr/WPSU/Math%20Interactive%20%26%20Lesson%20Plan%20Big%20Sale_4f5b0540-485c-4abc-8f73-7430563d83a8/BigSale_LP_update.pdf

http://wpsu.org/games/load_market.swf

(Perhaps use the student response systems for selection, needing only one computer and projector)

- Create a worksheet entitled, "Which Option is a Better Deal?"
- Snack sized boxes of raisins

Content Area/Course: Mathematics **Grade: 7** **Time (minutes or hours): 2 sessions, 60 minutes each**

Unit Title: Proportions and Proportional Reasoning

Lesson 4: **The Big Sale*** (adapted from PBS Learning Media)

E**ssential Question(s) to be addressed in this lesson**: In what ways will you use proportional reasoning for personal finance? How does an understanding of equivalent ratios help us to analyze and make conclusions about a real world situation?

Standard(s)/Unit Goal(s) to be addressed in this lesson:
7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.
SMP.4 Model with mathematics.
SMP.6 Attend to precision.
SMP.3 Reason abstractly and qualitatively.
6-8.WHST.1 Make arguments focused on discipline specific content
6-8.WHST.2.F Provide a concluding statement or section that follows from and supports the information or explanation presented.

Assumptions about what students know and are able to do coming into this lesson (including language needs):
Recognize vocabulary (ratio, proportion, means, extremes)
Understand that proportional reasoning involves comparisons of the relationships among ratios.
Compute unit rate
Determine if two quantities are proportional and solve.

Outcome(s)

By the end of this lesson students will know and be able to:

Construct viable arguments and critique others when proportions/proportional reasoning would or would not solve a problem

Instructional Resources/Tools

Computers with Internet access
Access to PBS Learning Media:
http://edcar-cdn.pbs.org/u/pr/WPSU/Math%20Interactive%20%26%20Lesson%20Plan%20Big%20Sale_4f5b0540-485c-4abc-8f73-7430563d83a8/BigSale_LP_update.pdf

http://wpsu.org/games/load_market.swf
(perhaps use the student response systems for selection, needing only one computer and projector)

Create a worksheet entitled, “Which Option is a Better Deal?”

Snack-sized boxes of raisins

Assessment

Pre-Assessment/Formative	**Summative** (optional)
1. Define • Ratio • Proportion 2. While shopping, you are presented with the option of buying a dozen bagels for $4.80 or buying nine bagels for $2.30. Which option is a better deal?	
Lesson Sequence and Description	**Teacher notes:**
Day 1 1. Pre-Assess (see above) students. 2. HOOK'em! You can eat the raisins if you can tell me the better buy. 3. Teacher will present students with the warm-up worksheet: "Which Option is a Better Deal?" - While shopping at the grocery store, you are presented with the option of buying a dozen cookies for $4.20 or buying nine cookies for $3.15. Which option is a better deal? Note Unit rate $0.35 for both, make sure students "discover". 4. Once students have completed the warm-up, the teacher will pull the class back together for a discussion. "Which option did you decide is the better deal and more importantly *how* did you decide it was the better deal?"Why? (Choose 2-4 students to respond, depending on time) 5. Teacher will explain the scenario- While grocery shopping, there are always many different options, of the same type of food that we must choose from. We have the ability to choose from twelve versus twenty-four cans of soda, eight versus twelve ounces of peanut butter, and it is up to us to determine which option gets us more for our money." For Example $4.69 for 3 cans of peas or $13.25 for 9 cans of peas? Which is the better buy? Why? 6. Determine the unit rate (cost per box of raisins) and EAT! **Day 2** "Which option did you decide is the better deal and more importantly *how* did you decide it was the better deal?" 1. Choose students to share their answer from Day 1 2. Share the mathematical explanation below "When deciding which option is a better deal, we are making a comparison of two scenarios and also a comparison of two quantities. A comparison of two quantities is known as a ratio, and can be set up using a colon, using the word "to", or most often by using a fraction. Then, when we have two	• Teacher will have a bag of snack-sized raisin boxes ready for this unit. You will need the two size and prices of the bags and how many raisin boxes are in each bag. • Allow students the opportunity to complete the warm-up on their own, while making sure that they can explain to you their reasoning for why they believe the deal they have chosen is the "better deal." Teacher circulates and supports. Continually questioning, probing deeper and gradually releasing responsibility to students. • Suggestions for struggling students: o Group them with another student. o Have the two students discuss the situations and explain to one another how to come to a conclusion or where their confusion falls, monitor discussion and intervene if needed. o Use whole number prices for differentiation purposes.

equivalent ratios, we have a proportion. A proportion is a statement between two equivalent ratios. Both a ratio and a proportion are helpful for us to use in order to determine the best options when we are buying groceries, or any product."

a. "In this situation, what are the ratios comparing?" (student responses)

b. "What does the proportion show us?" (student responses)

Students learned about rates in 6th grade, so there isn't a need to explicitly define ratio. Instead, ask: "How did you decide which the better deal was?" (Students should respond: "We computed the unit rate and compared them.)" "What other ways can we show the comparison?"

3. Ask students the following question, "It is extremely important for us to be able to write ratios for the two given situations and then set up a proportion to determine if the ratios are equal. If they are not equal, then we know that one option is better than the other. Let's say that the two scenarios are not equal. Then, how could we determine which option is better mathematically?"
4. Present students with the comparison scenario below:

   While at the grocery store, you are given the option of buying 12 ounces of ketchup for $2.79 or 20 ounces for $4.29. Which option is the better deal?

   When would it not be a good deal? (e.g., too much for your size family)

   Allow students a few minutes to work together to figure out which option is the better deal. Once most students have completed the task, pull the class back together to discuss their findings.

5. Ask students the following questions,
   a. "Are the two options in this scenario equal? *How* do you know?" (students "turn and talk", teacher randomly selects a student to respond by drawing from popsicle sticks, which have all the students names on them, teacher asks if anyone has a different method)
   b. "If we know that the two scenarios are not equal, how can we determine which is the better deal?" (student response- see above)
6. Have students visit the website http://wpsu.org/games/load_market.swf to access "The Big Sale Interactive" and begin to practice using proportions to determine the better deal when presented with two options. Have students record their work so that they may revisit any errors and so that they can prove their conjectures.

7. As students are working on the Big Sale tasks, teacher monitors students' performance. Visit each student and have them explain their thinking and assist students who may be struggling to correctly identify which option is the better deal. You will be able to see the percentage of questions that the students are correctly answering in the bottom right corner of the screen. Sample questions to ask students while they are working: "Why is it necessary to determine which option is the better deal?" *(it is necessary because you want to receive the most product for the least amount of cost)* "How do you determine which option is the better deal?" *(by setting up a proportion to determine if the situations are equivalent and then determining the price per unit)* Once most students have had the opportunity to solve 5 - 10 deals (or have successfully solved a predetermined percentage of deals), teacher will pull the class back together and review. 8. After finding the unit rate, e.g., ketchup, ask the students how much it would cost to buy 64 oz of ketchup, and then 1024. Ask them if they can express the cost in equation form 9. Teacher will review with students why it is necessary to determine the better deal when grocery shopping and how we can mathematically determine the better deal. In order to ensure understanding, teacher will have students create a scenario of their own. Have them write down the situation and then solve it to determine the better deal. These problems should be collected and then could be used as a warm-up/review the following class period. 10. Math Journal Entry: How are unit rates applicable in your life? Why? (e.g., cell phone provider selections)	For # 7 on Day 2 preview the website http://wpsu.org/games/load-market.swf o The computer lab or mobile cart will be needed to carry out this activity. Student response systems can also be used. o Observation during presentations of student work noting supports of their conclusions, independent work, student interaction, and internet activity.\ o Continually questioning, probing deeper and gradually releasing responsibility to students. • Teacher may, once students have the concept of proportionality, begin to model methods for efficiently determining (e.g., Means/Extremes)

Extended Learning/Practice (homework)

Have students research prices at two local grocery stores.

Have them compare prices on the same product to find the better deal.

Closure

Review outcomes of this lesson: Can you determine the best deal using ratios and proportions?

Preview outcomes for the next lesson: Do you think these same concepts would be used in understanding Census?

Lesson 5 – Census Percent Increase and Decrease

Estimated Time: 2 sessions, 60 minutes each

Resources for Lesson:

- Laptop/computer
- Projector

Content Area/Course: Mathematics **Grade: 7** **Time: 2 sessions, 60 minutes each**

Unit Title: Proportions and Proportional Reasoning

Lesson 5: Census Percent Increase and Decrease*
(*adapted from Digits) www.DIGITS.us.com

Essential Question(s) to be addressed in this lesson: Comparisons are helpful for making plans, predictions, and decisions. Percents are one way to make comparisons. When is it most convenient to use percents? How can you use a percent to represent change?

Standard(s)/Unit Goal(s) to be addressed in this lesson:
7.RP.2, Recognize and represent proportional relationships between quantities.
7.RP3, Use proportional relationships to solve multistep ratio and percent problems
SMP.4 Model with Mathematics.-variety of ways to model proportional relationships
SMP.2 Reason abstractly and quantitatively
SMP.3 Construct viable arguments and critique the reasoning of others
6-8.WHST.1 Make arguments focused on discipline specific content

Assumptions about what students know and are able to do coming into this lesson (including language needs):
Vocabulary (percent of change, percent of increase/decrease)
Proportional reasoning involves comparisons of the relationships among ratios recognize and represent proportional relationships between quantities.
How to set up proportions.

Outcome(s)

By the end of this lesson students will know and be able to:
Solve problems (discounts, percent increase and decrease)

Instructional Resources/Tools

Laptop/computer
Projector

Anticipated Student Preconceptions/Misconceptions

Misconceptions - a greater amount of change results in a greater percent of change. Teachers should emphasize that the percent of change depends on the ratio of the amount of change to the original quantity.

A common error that students make is to use the wrong quantities to find the ratio.

Another common misconception is that a 20% increase followed by a 20% decrease results in the original quantity. For example, if an item that costs $100 is increased by 20% the new price is $120. If then there is a decrease of 20%, the price is now $96.00.

Assessment

<table>
<tr><th>Pre-Assessment/Formative</th><th>Summative (optional)</th></tr>
<tr><td>1. Define
percent of change
percent of increase/decrease

2. Two friends argue about which of their little town's populations grew the most between 2000 and 2009. Write an argument to support each friend's point of view below. Explain your reasoning mathematically.

<table>
<tr><td colspan="3">U.S. Census Bureau Data</td></tr>
<tr><td>Population Data</td><td>Little Falls, MN</td><td>Little Falls, WI</td></tr>
<tr><td>2009</td><td>8,067</td><td>1,540</td></tr>
<tr><td>2000</td><td>7,719</td><td>1,334</td></tr>
</table>
</td><td></td></tr>
</table>

Lesson Sequence and Description

Day 1

1. Pre Assess students.
2. Have students "turn and talk" to each other while he/she listens to the discussions.
3. HOOK'em! Who Counts? You Count!
http://www.census.gov/schools/pdf/materials/cis_lesson_58US.pdf
4Introduce the concept of using a percent to describe how much something has increased or decreased. Teacher emphasizes that you only need to know two quantities to find percent of change: the original quantity and the amount of change.

Ask students the question: what information is important to know about a city's population over time? (Students: whether it is growing and how fast). Teacher: In the preassessment we looked at rate of growth, but how could we express that as a percentage? By what **percent** did Little Falls grow? (Turn and talk.)

After a few minutes, ask students to regroup. Students might say that Little Falls grew by 105%, and to that, the teacher can say, "remember yesterday when we calculated a 20% increase on X? In groups, find out what Little Falls' population would be if it increased by 105%." If students say it increased by 1.05%, then the teacher could say the same thing. If the students struggle, the teacher can say, "for every one person in 2000 there was 1.05 in 2009. So what was the increase?" (1.05-1=0.05, or 5%.) The teacher will then ask the students to find the percent increase in their town.

For Example
Springfield Massachusetts

Population, 2010	153,060
Population, 2000	152,082
Population, percent change, 2000 to 2010	0.6% (Increase)

Lawrence, Massachusetts

Population, 2010	76,377
Population, 2000	72,043
Population, percent change, 2000 to 2010	6.0% (Increase)

5. This should probably be on another day, but:

Teacher: "We also have a formula that we can use to find the percent

Teacher notes

During Turn and Talk about Pre-assessment question #2:

Possible examples of students' responses:
Friend A: Little Falls, MN, grew by 348 people,
8,067 - 7,719 =348, while Little Falls, WI, grew by only 206 people, 1,540 -1,334 =206. So, Little Falls, MN, grew more in population.

Friend B: In Little Falls, WI, for every 1 person in 2000, there were about 1.15 people in 2009. 1,540 /1,334 is about 1.15. In Little Falls, MN, for every 1 person in 2000 there were about 1.05 people in 2009. 8,067 /7,719 is about 1.05. Little Falls, WI, grew at a greater rate.

Some students may only mention the number of people. Challenge these students to consider the large difference in populations between the two towns. Ask students which town they would say grew the most if both towns grew by the same number of people. Other students my use ratios to find the relative change per person. They can express these ratios as fractions and find the greatest fraction. They may have an easier time comparing the ratios in decimal or percent form.

- Teacher will preview resources for HOOK'em activity which is a link to the 2010 Census Teacher materials for Grades 5-8 pages 10 and 15.

- Link for 2010 census data by state, county, by Link for 2010 census data by state, county, city/town is available at http://quickfacts.census.gov/qfd/states/25/2534550.html

- For struggling students use friendly numbers.

change: $\frac{\text{new}-\text{old}}{\text{old}} * 100 = \%$ change. Why does this equation work? How can we prove that it will give us the correct percentage?" Ask students to work in groups- students can test misc. quantities to show why it works, make observations, etc. However, by connecting the formula to the rate pre-assessment, it will help students make a stronger connection between rate and percent increase. It also incorporates proofs!

Have students discuss: "Why is the equation for percent increase the same as the one for percent decrease?"

For Example

	Michigan	USA
Population, 2010	9,883,640	308,745,538
Population, 2000	9,938,444	281,421,906
Population, percent change, 2000 to 2010	-0.6% (decreased)	9.7%

- Not all problems need to be done. Pick and choose some that you find would work with your students.

Day 2

1. Share the following problem with students:
A student has been working out for two months.
Identify the percent increase in each situation.

Month 1 record:
15 push-ups 100 crunches
Month 2 record:
30 push-ups 114 crunches

(Solution:
100% increase 14% increase

Students should be able to notice that the absolute error in both problems 2a and 2b are the same but the percent error is not. Draw attention to the correct or measured circumference (the denominator).

Ask students the following questions:

Before solving the problem, have students discuss:
• What is meant by *percent increase*?
• Does a greater amount of increase always mean a greater percent of increase?

While solving the problem, have students discuss:
• How will you find the percent of change?

After solving the problem, has students discuss the following:
• Your friend says that the number of push-ups increased by 200%. Explain the error in reasoning.
Another follow-up: your friend says that the number of push-ups

increased by 50%. Explain the error in reasoning.
How can you check that your answer is correct???

2. Share the problem with students:

a. Jane made a birthday crown for her little sister Sally. Jane guessed that the circumference of Sally's head was 25 inches. But when Sally tried on the crown it slipped down to her neck. Jane measured the circumference of Sally's head and found that it was really 20 inches. What was the percent error in the estimation? What does the percent error mean? Explain.

b. Jane made a birthday crown for her little sister Sally. Jane guessed that the circumference of Sally's head was 20 inches. But when Sally tried on the crown it got stuck at the crown of her head and started ripping. Jane measured the circumference of Sally's head and found that it was really 25 inches. What was the percent error in the estimation? What does the percent error mean? Explain?

How is this problem similar to or different from the push up problem?

The difference between the percent error formula and percent change formula might be confusing for students. It will be important for the teacher to highlight that when you find the percent change you divide by the old value, and when you find the percent error you divide by the actual value, not the estimate.

4. Teacher will share problem with students:

The force of gravity on the Moon is different from the force of gravity on Earth. This means that an object has a different weight on the Moon than it does on Earth. By what percent does an astronaut's weight decrease on the Moon?

Your Weight on Earth	Your Moon Weight
154 lbs.	26 lbs.

Solution:
(Approximately 83.44%)

Ask these questions:
Before solving the problem

- How is solving this problem similar to solving the previous problem?

Teacher Note(s)
For #4: link for calculating weight on moon versus weight on Earth
http://www.vat19.com/brain-candy/your-weight-on-the-moon.cfm

Math Journal
How can you use a percent to represent change?

Answer: A percent represents change by comparing the amount of change to the original quantity. The amount of change is the part. The original quantity if the whole. The percent is the percent of change as an increase or

- How is it different?

While solving the problem
• How can you use percent of change to show decreases in weight?
After students solve the equation, ask them the following questions:

• If the astronaut traveled back from the moon to the Earth and weighs 154 lb again, by what percent does her weight increase on the Earth? Why isn't the answer the same as the answer to the Example? (The astronaut's weight increases by about 503.9% on Earth. The percent is different from the Example because the original quantity used in the formula is different.)

Write an equation so that you can figure out anyone's weight on the moon given their weight on the earth

Extended Learning/Practice (homework) (*adapted from digits)

Give a real-world example of when it might be useful to calculate a percent of change. Would you expect the percent of change to be a percent increase or a percent decrease?

Closure

Review outcomes of this lesson: Define % of Change.
Preview outcomes for the next lesson: What is a rule? What is an equation? What is a formula?

Lesson 6 – Intercepting Villains

Estimated Time: 2 sessions, 60 minutes each

Resources for Lesson:

- Laptop/computer
- Projector
- Access to: http://mass.pbslearningmedia.org/content/vtl07.math.measure.rate.lprace/#

  (here you will access the QuickTime video, Intercepting the Wicked Witch handout, Assessments A and B, and the Answer Key)

Content Area/Course: Mathematics **Grade: 7** **Time: 2 sessions, 60 minutes each**

Unit Title: Proportions and Proportional Reasoning

Lesson 6: Intercepting Villains* (*adapted from PBS Learning Media) Access to: http://mass.pbslearningmedia.org/content/vtl07.math.measure.rate.lprace/#

Essential Question(s) to be addressed in this lesson: What kinds of questions can be answered using proportional reasoning?
How can you use proportional reasoning to determine how fast your broom must go to reach Motherboard before the Wicked Witch?!

Standard(s)/Unit Goal(s) to be addressed in this lesson:
7.RP.2.B Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
7.RP.2.C Represent proportional relationships by equations
SMP.4 Model with Mathematics.
6-8.WHST.1 Make arguments focused on discipline specific content

Assumptions about what students know and are able to do coming into this lesson (including language needs):
Vocabulary (ratio, rate, variables, equations, scale and interval)
How to solve problems (discounts, percent increase and decrease)

Outcome(s)

By the end of this lesson students will know and be able to:
Recognize that unit rate can be a measure of the steepness of the related line
See that a proportional relationship may exist between variables in an equation (e.g., d/r=t)
See that relationship may exist between variables in an equation.
Model proportions using an equations (e.g., d/r=t)

Instructional Resources/Tools

Laptop/computer
Projector
Access to: http://mass.pbslearningmedia.org/content/vtl07.math.measure.rate.lprace/#
(here you will access the QuickTime video, Intercepting the Wicked Witch handout, Assessments A and B, and the Answer Key)

Anticipated Student Preconceptions/Misconceptions

When students are completing the Intercepting the Wicked Witch handout, make certain they draw a straight line segment from (0, 0) with a slope of 50 cybermeters per second. The line will "end" at (120, 6000). Be certain that students do not mistake the line segments on the distance-time graph for the actual path traveled.

Assessment

Formative	Summative (optional)
1. Define a. Variables b. Equations c. Scale d. Interval 2. Who wins a 10 mile race if one is traveling at 5 miles per hour and one is traveling at 7 miles per hour? How do you know?	Assessment Level A (proficiency) – students are asked to use the rate-time-distance equation, d= rt, to complete a table of times and rates for different distances.

Lesson Sequence and Description	Teacher notes:
Day 1: 1. Pre Assess (see above). 2. HOOK'em -Teacher will read the following to his/her students, "The CyberSquad needs to figure out how to catch Wicked before she attacks Motherboard. Wicked travels on a broom at a constant speed of 50 cybermeters per second. Motherboard is located at a distance of 400 cybermeters away. The CyberSquad leaves x seconds after Wicked. How fast must their broom go so that they can reach Motherboard before, or at the same time, as Wicked?" 3. Distribute the Intercepting the Wicked Witch handout. 4. Ask students to complete the handout and discuss results. 5. Play the A Race to Motherboard QuickTime video. Tell the students that as they watch the video clip, they should compare the CyberSquad's solution to their own. **Day 2:** 6. Discuss the students' solutions, as well as the ones show in the video clip. Be sure students are able to work with multiple forms of the equation d=rt (r=d/t and t=d/r). Chart data Plot points Additional questions: • Describe the graph? (a straight line) What quantities vary proportionally in this situation? • What is the value of the constant of proportionality/unit rate? Write it in as many ways as you can. • What does that value represent in the context of our problem? 7. In your math journal, use the following equation d=rt where r=60 mph to write a word problem.	• Teacher previews lesson. • Here are some suggestions for providing context to the lesson and focusing students on important moments in the video. • We often measure how fast something can go. For example, most cars can travel at a speed of 60 miles per hour. This means that if the car drives at that same rate for an hour, it will travel 60 miles. What other speeds have you heard or read about? What sort of measurement is involved in finding out the speed? What measurement tools would be needed? • Link to video found at http://mass.pbslearningmedia.org/content/vtl07.math.measure.rate.lprace/# • Download to your computer before class and preview it. • As you watch the video, listen carefully to hear the speed of Wicked's broom. Compare that speed to the test model broom the CyberSquad finds. Try to determine who will get there first and explain why you think that. • For ELLs show the transcript with the

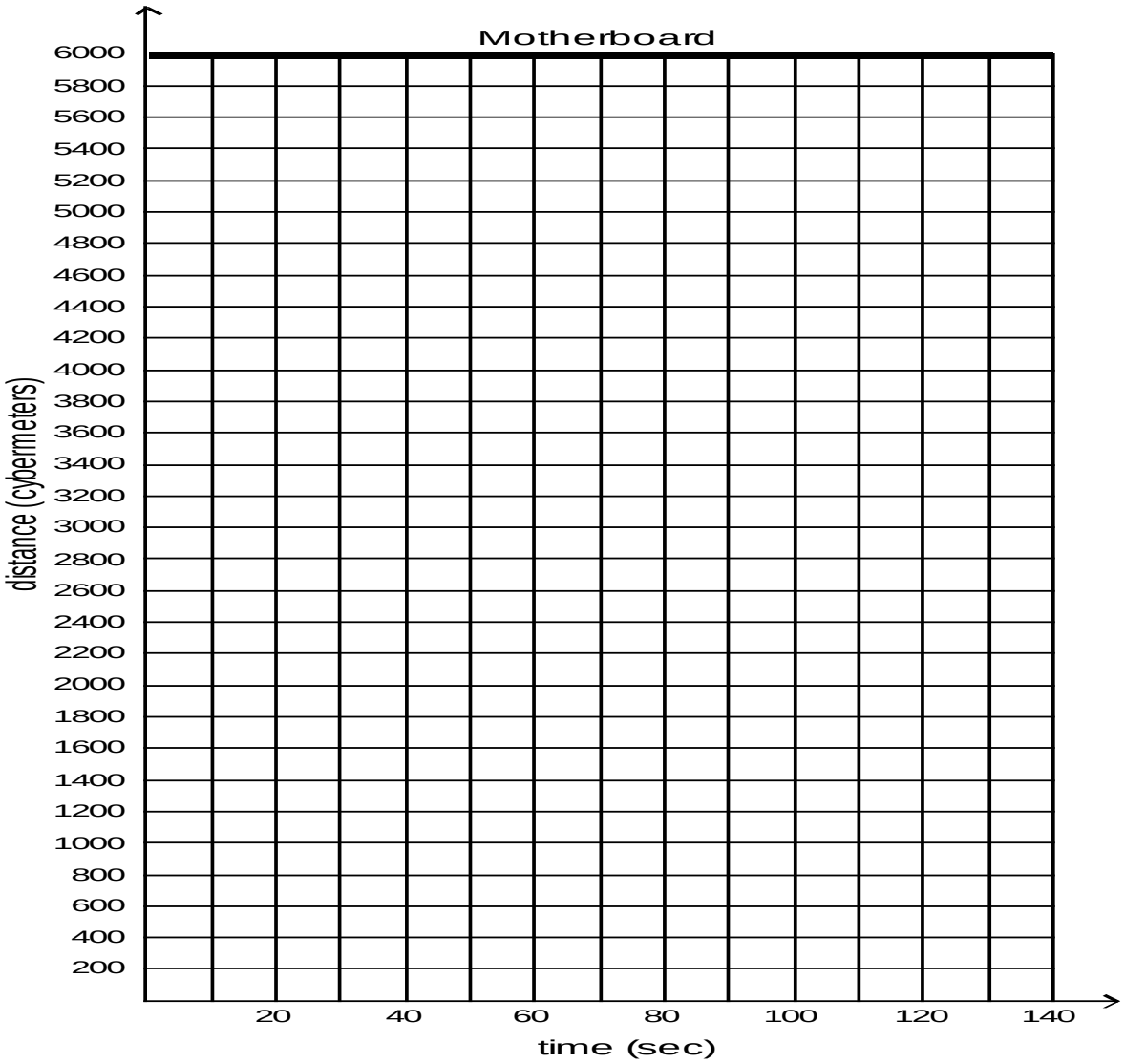
Extended Learning/Practice (*adapted from digits)	video and or get it translated.
Three teams train turtles for the Third Annual Turtle Trot, a 30-foot race. If the turtles trot at their training pace, which turtle will win the race? By how many minutes? Explain your reasoning. Team 1 Turtle Team 2 Turtle Team 3 Turtle 18 feet in 6 minutes 12 feet in 4 minutes 10 feet in 2 minutes	

Closure

Review outcomes of this lesson: What were the speeds of the two brooms? How much faster was the test model broom that the CyberSquad had than Wicked's broom? Control Central was 6000 cybermeters away. If it was only 3000 cybermeters away, do you think the CyberSquad would have arrived there first? Why or why not? Do you think you could figure out your running speed? How would you do it?

"Intercepting Wicked the Witch" Handout
Page 1

1. If Motherboard is located at a distance of 6000 cybermeters from Wicked, and Wicked travels at 50 cybermeters per second, plot her distance-time graph to reach Motherboard.



""Intercepting Wicked the Witch" Handout
Page 2

1. Using the rate-time-distance equation ($d = rt$), substitute 6000 cybermeters for d and 50 cybermeters per second for r, and then solve for t. Describe how this answer can be seen on the graph.

2. Wicked's broom travels at 50 cybermeters *per* second and he (she?) is traveling 6000 cybermeters. How do you figure out how long it takes Wicked to travel the distance? Write an equation that will let you figure out any distance, rate or time, using d for distance, r for rate, and t for time. Make sure to test your equation to make sure it gives you the correct answer!

3. If the CyberSquad locates a broom that travels 100 cybermeters per second, how long will it take them to reach Motherboard? How much later than Wicked can they leave in order to reach Motherboard at the same time as, or earlier than, Wicked?

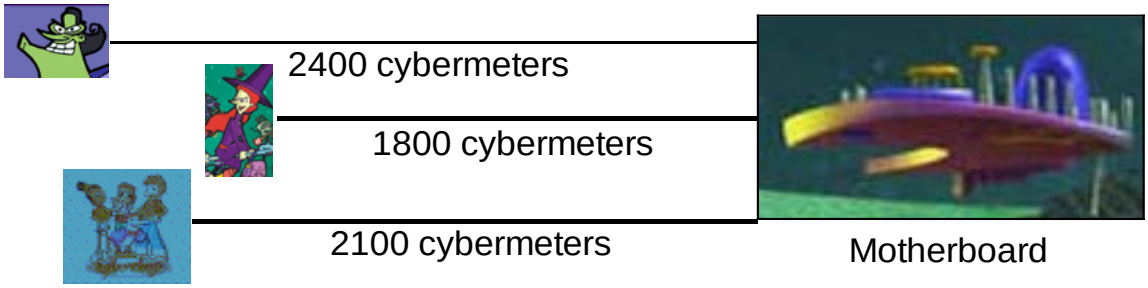
4. The CyberSquad's original broom travels at 10 cybermeters per second. How long will it take them to reach Motherboard? Will they be able to catch Wicked? They later found another broom that travels twice as fast as Wicked's broom. Now how long will it take?

5. On page 1 of this handout, plot one possible distance-time graph in which the CyberSquad reaches Motherboard before, or at the same time as, Wicked. The CyberSquad will use the broom that travels 100 cybermeters per second.

Intercepting Villains Using the Right Rate

Assessment A

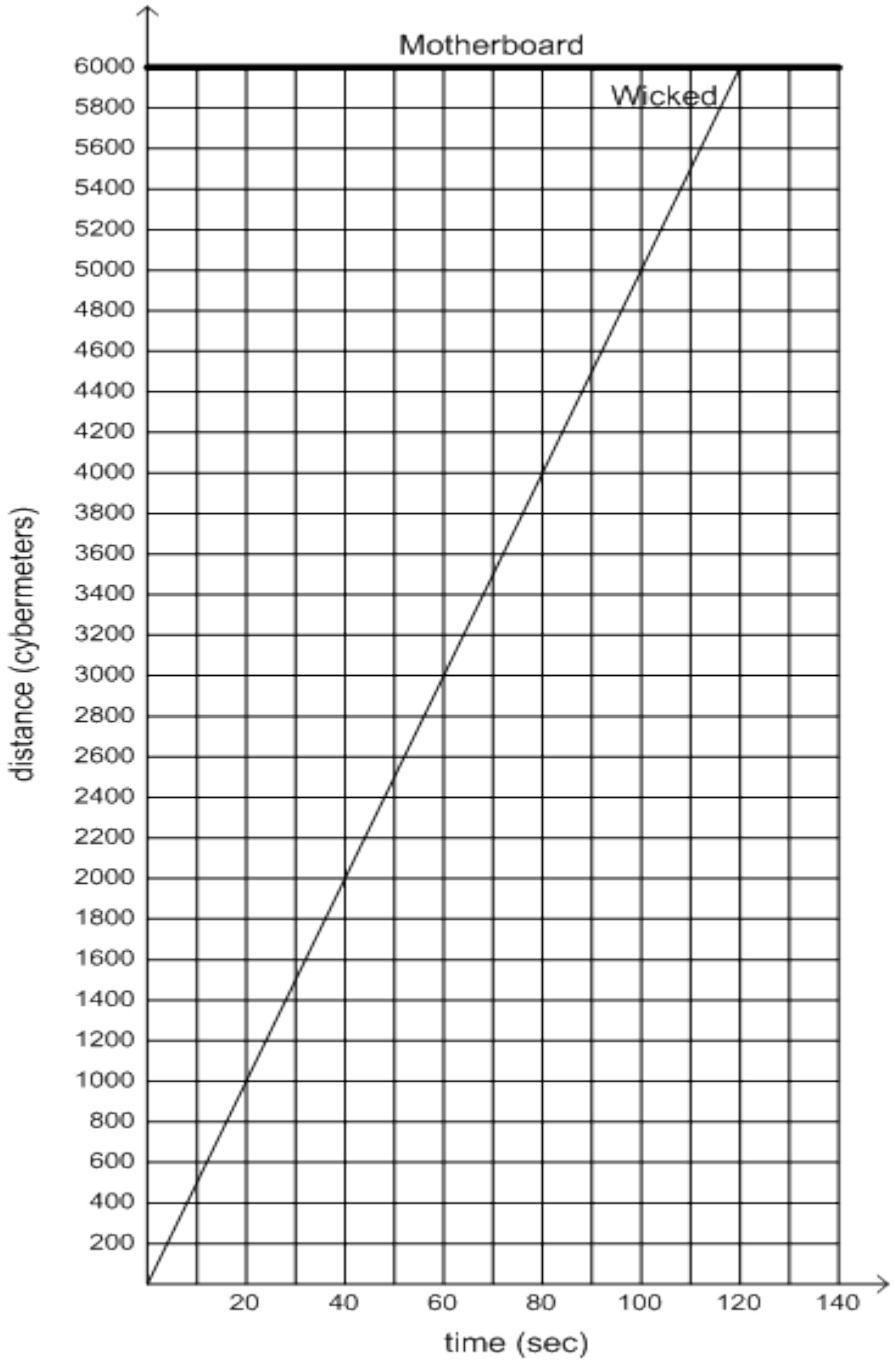
Suppose Wicked, Hacker, and the CyberSquad are different distances from Motherboard (as shown in below). We know that Hacker is 2400 cybermeters from Motherboard and he travels at a constant speed of 80 cybermeters per second in his spacecraft. Fill in the table below with the distance, rate, and time conditions that show everyone reaching Motherboard in the same amount of time.



	Time (seconds)	**Rate** (cybermeters/second)	**Distance** (cybermeters)
Hacker			
Wicked			
CyberSquad			

Intercepting Villains Using Rate (answer key)

1. **Handout 1: "Protecting Motherboard from Wicked"**



2. $d = rt$

   6000 cybermeters = (50 cybermeters/sec) * t

   (6000 cybermeters) / (50 cybermeters/sec) = t

   120 seconds = t

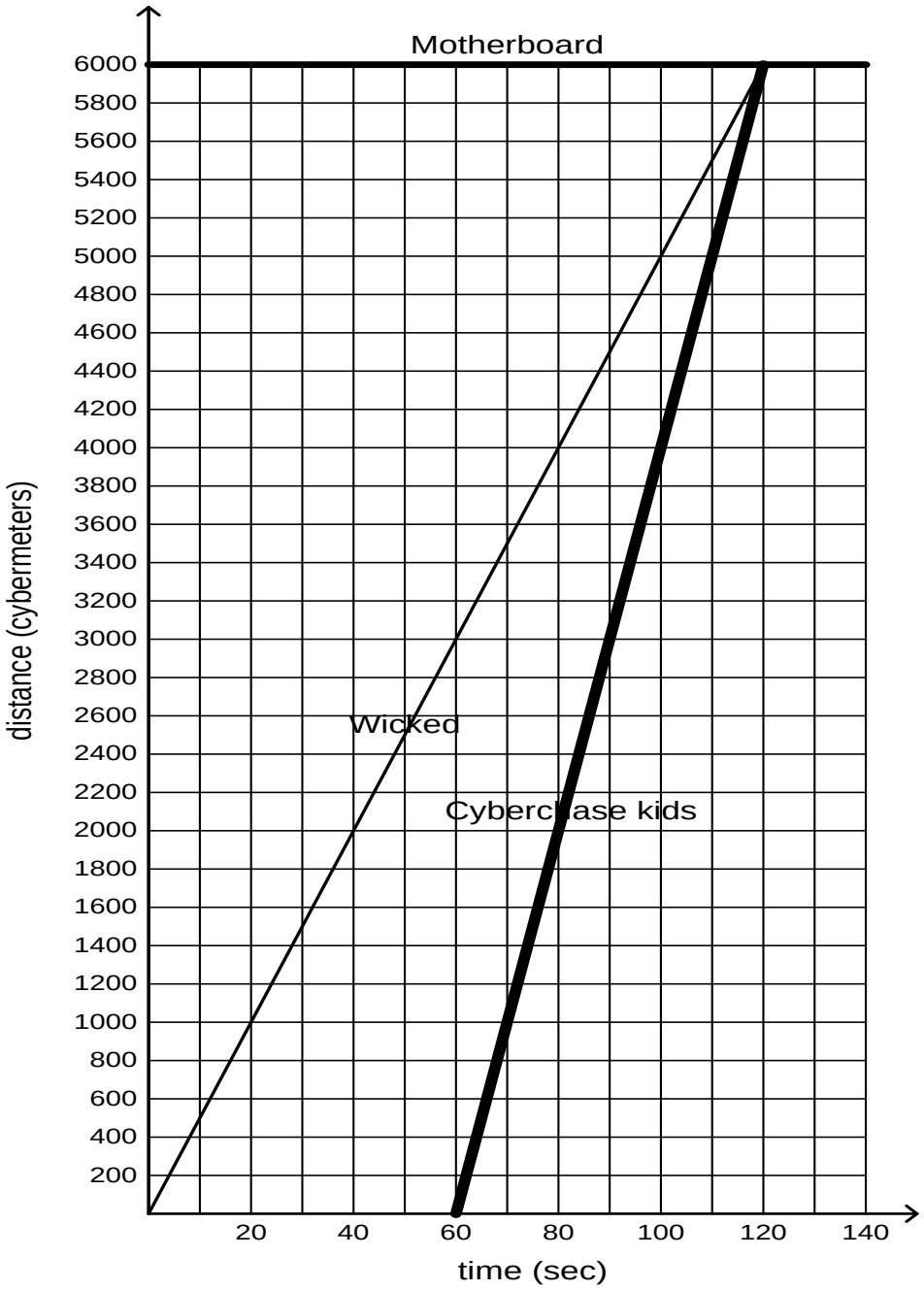
You can see this on the graph because Wicked's final position is 6000 cybermeters at time = 120 seconds. That is, when Wicked is 6000 cybermeters from her initial starting position, she has traveled for a total of 120 seconds.

3. If the Cyberchase kids can travel at 100 cybermeters/sec it will take them:
   t = (6000 cybermeters) / (100 cybermeters/sec) = 60 seconds

Since it would only take them 60 seconds to reach Motherboard, or half the time it would take Wicked to reach Motherboard. So, the Cyberchase kids could leave 60 seconds after Wicked and still reach Motherboard at (or before, if they left more than 60 seconds earlier) the same time.

Intercepting Villains Using Rate

4.



Assessment A

Hacker, Wicked, and the Cyberchase Kids must all leave and arrive at the same time, so their total time of travel would be identical.

	Time (seconds)	**Rate** (cybermeters/second)	**Distance** (cybermeters)
Hacker	30	80	2400
Wicked	30	60	1800
Cyberchase Kids	30	70	2100

Lesson 7 – Are Cars Speeding in Front of the School?

Estimated Time: 3 sessions, 60 minutes each

Resources for Lesson:

- Stop watches
- Tape measures

Content Area/Course: Mathematics **Grade(s): 7** **Time (minutes or hours): 3 sessions, 60 minutes each**

Unit Title: Proportions and Proportional Reasoning

Lesson 7: Are Cars Speeding in Front of the School?

Essential Question(s) to be addressed in this lesson:
1. What kinds of questions can be answered using proportional reasoning?
   For Students: "Are cars speeding in front of the school?"

Standard(s)/Unit Goal(s) to be addressed in this lesson:
7.RP.2b Identify the constant of proportionality (unit rate in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
7.RP.2.C Represent proportional relationships by equations
SMP.1 Make sense of problems and persevere in solving them.
SMP.2 Reason abstractly and quantitatively
SMP.3 Construct viable arguments and critique reasoning of others
6-8.WHST.1 Make arguments focused on discipline-specific content

Assumptions about what students know and are able to do coming into this lesson (including language needs):
Can form ratios, check for equivalence, and have a basic understanding of unit rate.

Outcome(s)

By the end of this lesson students will know and be able to:
Recognize that proportional relationships may exist between variables in an equation (e.g., d/r=t)
Utilize a variety of methods to represent a proportion (e.g., in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships).
Model proportions using an equation (d/r = t)
Construct viable arguments and critique others when proportions/proportional reasoning would or would not solve a problem.

Instructional Resources/Tools

Stop watches
Tape measures

Anticipated Student Preconceptions/Misconceptions

Students believe speed is measured only in miles per hour

Assessment

Pre-assessment/ Formative	Summative (optional)
Quiz 1. What is the ratio of students who are the only child in class to students who have siblings? 2. Are 11/25 and 33/75 equivalent ratios?	Write a persuasive letter to the principal regarding whether or not cars are speeding in front of the school that includes appropriate data, math, and reasoning

3. If I am traveling at 50 miles per hour, what would be the ratio? What would the ratio be if it was stated in feet per second? Are these proportional? 4. If a car were traveling 50 ft. in 3 secs in front of our school are they speeding? How do you know?	

Lesson Sequence and Description	Teacher notes:
Day 1 1. Complete Pre Assessment 2. Determine the speed limit in the school zone 3. Mark a measured about 1000 ft (about334 yards) in front of school 4. In teams of three (1) at beginning of length to flag starting stop watch, 2) at end of length with stop watch, and 3) recorder) collect the time with a stop watch for 12 cars as they pass that 1000 ft (about 334 yard) mark. **Day 2** 1. In small groups of three compare data, discussing data error, average times Questions to ask: Why do we take the average? Are there any data points that skew your data? What are possible sources of error? (length of time, data size, effect of drivers seeing students with stopwatches) How could we reduce our error? (Increased length of time/distance, take more samples, hidden cameras). 2. Using averaged times collected, individually students calculate 4 of the 12 times into miles per hour 3. Regroup and share miles per hour info for all 12 cars 4. Determine how many were speeding **Day 3** 1. Write persuasive essay to principal: a. Table with 12 car data ft/sec and m/h b. Evidence of speeding or not (percent of cars speeding versus not, percent over the speed limit, etc.) c. Suggestion for next steps (e.g., not speeding- praise citizens, speeding -get cop to catch them and give ticket)	• Specific accommodations for students with disabilities, ELLs, advanced students- group students so each has a task for their abilities • Instructional practices that support academic language development- persuasive essay

Extended Learning/Practice (homework)

Clock the cars passing in front of your homes, determine if they are speeding and share results.

Closure

Review outcomes of this lesson: Apply Unit Rate (mph) to a real world situation.

Preview outcomes for the next lesson: Explore the question are there non proportional problems in the real world?

Lesson 8 – Are They Proportional?

Estimated Time: 1-2 sessions, 60 minutes each

Resources for Lesson:

- Graph paper
- Pencils
- Rulers

Content Area/Course: Mathematics **Grade: 7** **Time: 1-2 sessions, 60 minutes each**

Unit Title: Proportions and Proportional Reasoning

Lesson 8: Are They Proportional?

Essential Question(s) to be addressed in this lesson:
What kinds of questions can be answered using proportional reasoning?
What types of questions cannot be answered using proportional reasoning? Why not?

Standard(s)/Unit Goal(s) to be addressed in this lesson:
7.RP.2a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
SMP.4 Model with Mathematics.

Assumptions about what students know and are able to do coming into this lesson (including language needs):
Vocabulary (proportional relationship, equivalent ratios, constant of proportionality, proportion, dependent variable, independent variable)
There are a variety of ways to represent a proportion

Outcome(s)

By the end of this lesson students will know and be able to:
Determine whether two quantities represent proportional relationships and solve.
Construct viable arguments and critique others.

Instructional Resources/Tools

Graph paper
Pencils
Rulers

Anticipated Student Preconceptions/Misconceptions

Misconception - just because a relationship can be linear, does not mean the relationship is proportional.

Assessment

Pre-Assessment/Formative	**Summative** (optional)
Have students discuss their current understanding of the following vocabulary in a pair-share: unit rates equivalent ratios Proportion proportional relationship constant of proportionality independent variable dependent variable	

Lesson Sequence and Description

HOOK'em

Two babysitters were making play dough. The play dough recipe they were using called for 4 cups of salt and 10 cups of flour. They were trying to figure out that if the recipe were to be increased to use 6 cups of salt, how many cups of flour would be needed? One of them said that 12 cups of flour would be needed (incorrect additive thinking) and the other said that 15 cups of flour would be needed (correct multiplicative thinking). Get students to discuss who was correct? What was the reasoning?

Task #1:

Determine whether the following relationship is proportional:

A 10-inch candle burns at a constant rate of 1 inch per hour. Make a table, graph, and write the equation. Is this relationship proportional? Why or why not?

Task #2:

Do the diameter and the circumference of all circles form a proportion? Use 3.14 for π in the equation c=πd to find six ordered pairs for the diameter (x) and the circumference (y) lengths of six circles. List them in a table. Plot the ordered pairs in the table. Is this relationship proportional? Why or why not?

Task #3:

The parents are planning for a party for the teachers at Harking Middle School. They are planning to bake cookies for the party, but they need to make sure they have enough cookies, so they will have to increase the recipe proportionally. Mrs. Difloures created Table 1 to determine the right amount of each ingredient. Mrs. Bryant created table two.

Which table shows a proportional relationship and the correct amount of each ingredient

Table 1 Original Recipe	Butter 2 sticks	Sugar 1 cup	Brown Sugar ¼ cups	Chocolate Chips 8 ounces
Teacher Recipe	6 sticks	3 cups	2/4 cups	24 ounces
Table 2 Original Recipe	Butter 2 sticks	Sugar 1 cup	Brown Sugar ¼ cups	Chocolate Chips 8 ounces
Teacher Recipe	5 sticks	4 cups	1 cup	11 ounces

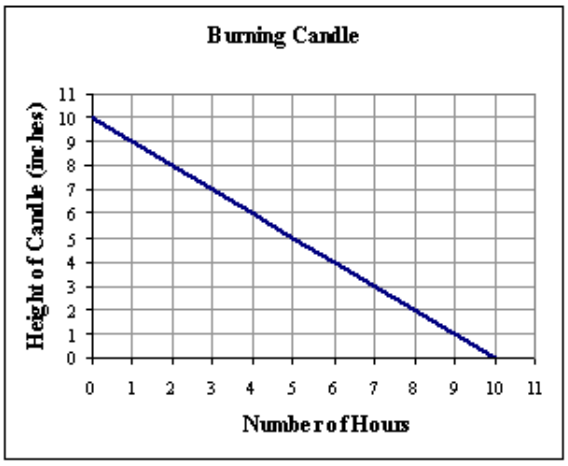
Teacher notes:

- Push the discussion toward understanding that ratios and proportions involve multiplicative comparisons. Equal ratios result from multiplication (or division), not from addition (or subtraction).
- To determine the height of the candle multiply the number of hours that the candle burns by 1 inch per hour, and subtract the product from the candle's initial height (10 inches).
- Rule in Equation: If *y* represents the height of the candle after *x* hours of burning, then the relationship can be expressed as an equation in the form $y = mx + b$, where *m* represents the rate at which the candle burns (1 inch per hour) and *b* represents the initial height of the candle (10 inches).
- Height of Candle = Rate at which it Burns · Number of Hours Burned + Initial Height
  $y = -1 \cdot \boldsymbol{x} + 10$
  OR… $y = -1x + 10$ or $y = 10 - 1x$
  (Note: Not Proportional does not go throughout point (0,0))
- Table for Burning Candle

Number of hours candle burns (x)	Height of candle in inches (y)
0	10
1	9
2	8
3	7
4	6
5	5
6	4
7	3
8	2
9	1
10	0

- A direct proportion (or variation) has the form y = kx; indirect proportion (also called inverse proportion -- these are synonyms) has the form y = k/x.; but indirect proportion is NOT linear. It means that y is directly proportional to the reciprocal of x, rather than to x itself.

Graph



- For differentiation, use whole numbers or simpler numbers.
- Homework
    - Set 1 Proportional
    - Set 2 Proportional
    - Set 3 not Proportional
    - Set 4 not Proportional
    - Set 5 Proportional

Extended Learning/Practice (homework)

Graphing Proportional Relationships

For each set of numbers, determine if the relationship is proportional or non proportional.
Then graph each set of numbers on a coordinate grid.

Set 1:
x -1 0 1 2 3
y -2 0 2 4 6

Set 2:
x 2 4 6 3 5
y 4 8 12 6 10

Set 3:
x 0 1 2 3 4
y 1 2 3 4 5

Set 4:
x 1 2 3 4 5
y 3 5 7 9 11

Set 5:
x -2 -3 -4 -5 -6
y 2 3 4 5 6

What do you notice about the graphs of the proportional relationships compared to the non-proportional relationships?

Lesson 9 – You're a Rock Star!

Estimated Time: 2 sessions, 60 minutes each

Resources for Lesson:

- Teacher internet access
- http://www.pbslearningmedia.org/content/vtl07.math.number.rat.lpscaleup/

Content Area/Course: Mathematics **Grade(s): 7** **Time (minutes or hours): 2 sessions, 60 minutes each**
Unit Title: Proportions and Proportional Reasoning
Lesson 9: You're a Rock Star! (Scaling a photograph)
Essential Question(s) to be addressed in this lesson:
What kinds of questions can be answered using proportional reasoning?
For Students -"If you had a photograph of your favorite sport star/singer that measured 12 inches tall x 8 inches wide and you wanted to enlarge it to be a 6-foot likeness to hang on your wall, how wide would that picture become? (from PBS lesson)

Standard(s)/Unit Goal(s) to be addressed in this lesson:
7.RP.3 Use proportional relationships to solve multistep ratio and percent problems
6-8. 2.F Provide a concluding statement or section that follows from and supports the information or explanation presented.

Assumptions about what students know and are able to do coming into this lesson (including language needs):
Can measure to nearest 1/16th of an inch and define scale factor.

Outcome(s)

By the end of this lesson students will know and be able to:

- Describe, utilize and solve problems with scale factor and complex fractions/ratios.
- Recognize that proportional reasoning involves comparisons of the relationships among ratios
- Compute unit rate
- Set up proportions
- Determine if two quantities are proportional and solve
- Scale a ratio, rate, or fraction up or down with the same relative characteristics as the original. (e.g., enlarge a photo)

Instructional Resources/Tools (What does the complexity of these texts or sources demand of the students?)

http://www.pbslearningmedia.org/content/vtl07.math.number.rat.lpscaleup/

Anticipated Student Preconceptions/Misconceptions

That all sides of a photo are positive integers for example: 3" x 5"

Assessment

Pre-assessment/ Formative	**Summative** (optional)
1. Measure your book to the nearest 1/16th of an inch 2. Define the scale factor a. Determine the scale factor and/or determine the missing side of a polygon given similar figures 3. Journal entry Measure length and width of several photos, Create a ratio	**Assessment: Level A** (proficiency): Students are asked to calculate the heights of several aspect-preserving enlargements and reductions. **Assessment: Level B** (above proficiency): Students are asked to calculate the dimensions of

Convert to a decimal Note in your journal any patterns you see	enlargements and reductions of a 3 x 5 (height x width, in inches) photograph.

Lesson Sequence and Description	Teacher notes:
Overview Students are asked to figure out the dimensions of enlargements of rectangular photographs (and some reductions), based on the percentage of the enlargement. (Utilizes a PBS learning media lesson. http://www.pbslearningmedia.org/content/vtl07.math.number.rat.lpscaleup/) 1. Student complete pre-assessment 2. Students research Golden Ratio Calculator (http://www.blocklayer.com/GoldenRatio.aspx), 3. Students discuss the patterns 4. Student view the video In this video segment from Cyberchase, Bianca takes on a new job in a print shop. Her first assignment is to enlarge a photograph of the King of Sloovoonia. The enlargement is supposed to be a life-size image of the six-foot tall king. Working with percentages, Bianca makes a few failed attempts before she finally creates an enlargement that is the proper height and width. Points to note in the video: a. What did Bianca do wrong on her first attempt? What would a good estimate be for the height:width of the king? b. What did Bianca do wrong on her second attempt? What would a good estimate be for the height: width of the king? c. Stop the video before the 100% image prints and ask students to predict what the paper would look like. d. Create a design, choose scale factor(s) and enlarge/reduce the design e. Group Activity: Choose a scale factor and create a floor plan of the classroom. In this video segment from Bianca takes on a new job in a print shop. Her first assignment is to enlarge a photograph of the King of Sloovoonia. The enlargement is supposed to be a life-size image of the six-foot tall king. Working with percentages, Bianca makes a few failed attempts before she finally creates an enlargement that is the proper height and width. Common picture sizes with ratios: 5 by 3 ½ = 6 by 4 =	For differentiation, use whole number of inches for measurements. Common picture sizes with ratios: 5 by 3 ½ = 1.42 6 by 4 = 1.5 11 by 8 ½ = 1.29 14 by 11= 1.27

11 by 8 ½ = 14 by 11= In your journal, note any items that do not fit the pattern and describe why you think that might be.	
Extended Learning/Practice (homework) Measure food boxes in your cupboard, enter info in your journal, set-up ratio, calculate decimal equivalent, look for patterns	

Closure

Review outcomes of this lesson: Summarize ratio to patterns and generalizations

Preview outcomes for the next lesson: Writing your own proportion word problems to be solved by others.

Lesson 10 – Student Generated Word Problems

Estimated Time: 2 sessions, 60 minutes each

Resources for Lesson:

Optional: Writing in Math: Bernadette Russek, Writing to Learn Mathematics, WAC Journal, Vol. 9, pp. 36-45. (PDF file) and/or Using Writing in Mathematics (University of Puget Sound)

Content Area/Course: Mathematics **Grade(s): 7** **Time (minutes or hours): 2 sessions, 60 minutes each**

Unit Title: Proportions and Proportional Reasoning

Lesson 10: Student Generated Word Problems

Essential Question(s) to be addressed in this lesson:
What kinds of questions can be answered using proportional reasoning?

Standard(s)/Unit Goal(s) to be addressed in this lesson:

7.RP Analyze proportional relationships and use them to solve real-world and mathematical problem.

SMP.1 Make sense of problems and persevere in solving them.

SMP.4 Model with Mathematics.

Assumptions about what students know and are able to do coming into this lesson (including language needs):
Can explain/demonstrate ratio, proportion, unit rate, and multiple ways to represent proportionality

Outcome(s)

By the end of this lesson students will know and be able to:
Use proportional vocabulary/language-unit rate, ratios, proportions, proportional reasoning, equivalence, discounts, percent of increase/decrease, similarity, constant of proportionality, origin (x, y plot), scale factor, complex fraction.
Recognize that there are a variety of ways to represent a proportion (e.g., in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships).
Write and share word problems involving proportions.

Instructional Resources/Tools

Writing in Math:
Bernadette Russek, Writing to Learn Mathematics, WAC Journal, Vol. 9, pp. 36-45. (PDF file)
Using Writing in Mathematics (University of Puget Sound)

Anticipated Student Preconceptions/Misconceptions

Mathematics problems have only one right answer.
That you do not write in mathematics.

Assessment

Pre-assessment/ Formative	Summative (optional)
1. Give an example of ratio in a complete sentence. 2. Is this a proportion? 34/40 and 67/83 3. If you are traveling at 60MPH, what is the unit rate? 4. Create a table and then a rule for the following situation: For every mile in the taxi it cost you $5.00 5. Write a word problem for the proportion: 4 : 8 = 2.5 : 5	Student groups (4) share and solve each other's proportional word problems.

Lesson Sequence and Description	Teacher notes:
Day 1 1. Students complete pre-assessment. 2. Teacher models writing a word problem For example 4 : 8 = 2.5 : 5 might yield the following word problem What is the better buy? 4 pounds of ground beef at $8.00 or 2 ½ pounds of ground beef at $5.00? 3. Students, in pairs, are given 2 or more sample proportions, and they write word problems using the proportions. **Day 2** 1. Students switch and in new pairs solve each other's word problems from day 1, identifying the unit rate, writing the rule, and creating corresponding equations. For example: 4 : 8 = 2.5 : 5 Unit rate - 1 pound of ground beef for $2.00 Rule -multiply the number of pounds of ground beef by $2.00 equals the cost. Equation: Cost = 2 times the number of pounds of ground beef. C = 2P 2. Select (e.g., pull Popsicle sticks with student names for random selection) a student and the person who solved their problem to share the problem and solution. (repeat 3-4 times) 3. Students, individually, look for (internet, real world, work place) examples of possible proportions and write word problems in your math journal, and then solve them to determine if they are proportional.	• For pre-assessment and throughout the lesson, the teacher circulates and observes. • Pre-assessment questions 1-4, tell him/her if the students came with the assumed skill. Question 5, tells him/her if they already know the content he/she is about to teach. • Differentiate by giving more proficient students fractional or decimal quantities. • For students that are already proficient: Use the following information to answer the questions below: 1 food calorie = 1,000 gram calories (g-cal) 1 g-cal = 0.001162 watt-hours 1 watt-hour = 860.42 g-cal A typical sandwich provides about 250 food Calories. How many gram calories is that? (250,000 gram calories) • Specific support for ELLs- additional modeling, pairing with more proficient students in the language who are able to translate and explain.
Extended Learning/Practice (homework) Talk to your parents/family about ways they use proportions and write an entry in your math journal. If you have access to the internet at home and have parent's permission begin researching examples of proportions.	

Closure

Review outcomes of this lesson: How did writing word problems today, add to your understanding of ways to model proportional relationships (e.g., in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships)

Preview outcomes for the next lesson: Tailors measure clients to make clothes, how many measurements do you think they make?

Lesson 11 – Gulliver's Suit by Proportions

Estimated Time: 2 sessions, 60 minutes each

Resources for Lesson:

- Teacher internet access
- Gulliver's Travel web site- http://www.literaturecollection.com/a/swift/gulliver/
- Tape measures
- Coordinate Graph charts

Content Area/Course: Mathematics **Grade(s): 7** **Time (minutes or hours): 2 sessions, 60 minutes each**

Unit Title: Proportions and Proportional Reasoning

Lesson 11: Gulliver's Suit by Proportions

Essential Question(s) to be addressed in this lesson:

What kinds of questions can be answered using proportional reasoning?

For Student's "How does proportional reasoning help you tailor a suit for a giant like Gulliver?"

Standard(s)/Unit Goal(s) to be addressed in this lesson:

7.RP.2.A Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

7.RP.2.B Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

SMP.1 Make sense of problems and persevere in solving them.

6-8.WHST.1 Make arguments focused on discipline specific content

Assumptions about what students know and are able to do coming into this lesson (including language needs):
Can form ratios, can check for equivalence, have a basic understanding of unit rate
Recognize that proportional reasoning involves comparisons of the relationships among ratios

Outcome(s)

By the end of this lesson students will know:
A variety of ways to represent a proportion (e.g., in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships)

Instructional Resources/Tools

Gulliver's Travel web site- http://www.literaturecollection.com/a/swift/gulliver/7/
Tape measures
Coordinate Graphs charts

Anticipated Student Preconceptions/Misconceptions

The ratios will not be proportional.

Assessment

Pre-assessment/ Formative	Summative (optional)
Quiz 1. What is the ratio of boys to girls in class? 2. Are 3/5 and 2/7 equivalent ratios?	Presentation of their argument for or against the number of measurements needed to tailor a suit. • Include mathematics of their measurements • Include charts and can correctly point out the relationships

3. Does this graph represent a unit rate?	• Accurately determine if there is a unit rate • Refers to the literary piece to justify their conclusion as to whether they could make a suit in this fashion

Lesson Sequence and Description	Teacher notes:
Day 1 Read section from Gulliver's Travels that has the Lilliputians measuring his thumb to create him a suit if clothes "… Then they measured my right thumb, and desired no more; for by a mathematical computation, that twice round the thumb is once round the wrist, and so on to the neck and the waist, and by the help of my old shirt, which I displayed on the ground before them for a pattern, they fitted me exactly…." http://www.literaturecollection.com/a/swift/gulliver/7/ Have student measure and record their thumb circumference, chart results, Discuss is there a proportion? Discuss connection to unit rate. **Day 2** Now, using the Lilliputians "theory" - double their thumb measurements and compare to their wrist - double their wrist measurements and compare to their neck - double their neck measurements and compare to their waist Discuss if it would be possible or not to make a suit from a single measurement. In your math journal, write your opinion about the Lilliputians' theory. Is it possible? Why or why not?	• Tailors often use only a few measurements.
Extended Learning/Practice (homework) Speak with at least 4 family and friends and see if they believe that this can be done and why. Record their answers.	

Closure

Review outcomes of this lesson: Apply ratio and proportion to literature and real world

Preview outcomes for the next lesson: Culminating Performance Task.

Curriculum Embedded Performance Assessment (CEPA)

Should We Drive or Bike?

Estimated Time: 2 sessions, 60 minutes each
Content Area/Course: Mathematics **Grade(s): 7**

Time (minutes or hours): 2-3 sessions, 60 minutes each

Unit Title: Proportions and Proportional Reasoning

Lesson Title: CEPA

Title: Should We Drive or Bike?

Goal: Determine if the increase in gasoline consumption will allow your brother to drive to after-school practice or if you both will need to ride your bikes.

Role: Analysts (rider)

Audience: Your brother

Situation: Your brother Tim drives the two of you to and from school every day in his car. Tim pays for gas using his weekly allowance. You and Tim are excited because you both just made the soccer team! Now you and Tim will have daily practice after school at the town soccer field on the other side of town. Attending practice means Tim will need to pay more for gas each week. Can Tim afford to buy the extra gas needed on his current allowance or will you both need to ride bikes to practice?

Product/ performance: You will create a data display (e.g. story board, posters, PowerPoint, etc.) of your findings and conclusions including mathematical evidence. You will also need to make a presentation to your class.

Task 1: Visit gasbuddy.com or your local gas station and determine the current cost of 1-10 gallons of gas using the information you got from gasbuddy.com or your local gas station (This information will be used to complete tasks 2 and 3 and can be done prior to completing rest of the CEPA).

- Present the information graphically and in a table or spreadsheet. What can you conclude?
- Write an equation on the relationship between the two variables in the graph. Determine the cost of 13 1/3 gallons of gas using the equation. Could you determine the cost using the graph? Could you determine the cost by using the table? Explain your reasoning using mathematical evidence.

Task 2

A: Your parents give Tim a fixed allowance each week that covers the cost of gas for your weekly commute. He also gets an additional four dollars for other expenses. His car's fuel tank capacity is 13.2 gallon and it uses 1/16 of a tank of gas roundtrip every day going to and from school.

•Determine Tim's allowance using the current cost of gas.
•Use a table or spreadsheet and graph to present the information on the daily usage of gas in a week.
•Is there another way you could determine the total weekly roundtrip consumption of gas? Explain your reasoning.

B. Recently both you and Tim made the soccer team. You now need to attend after-school practice every day at the soccer field across town. Attending the daily practice would use an additional 0.3 gallons of gas every day. Could you attend practice every week on Tim's current allowance? Create a table or spreadsheet and graph to support and explain your reasoning using a table, graph, etc.

C. Compare the graphs from 2A and 2B. What is different or the same? Why? Justify your reasoning using mathematical evidence. Write a description of both graphs, including their shapes. Provide information about their relationship to the origin and what it means.

Task 3

A. One day on the way home from school, you and Tim are at a junction where there are two gas stations (A and B) when the dashboard light starts flashing. The car is almost out of gas! Tim knows his car and estimates that he needs to buy at least 2/3 of a gallon of gas to make it safely home. Tim has only 2 ½ dollars. He determines that he cannot afford to buy gas at station A. At station B though, he can buy the minimum amount of gas he needs with all of the cash he has!

•How did he make this decision?
• What was the advertised gas selling for at station B?
• What can you say about the price of a gallon of gas at station A?

B: After soccer season ends, you and Tim will return to the regular driving schedule. Suppose that there is another oil spill in the Gulf and it is predicted that that gas prices will increase by 6 percent every week for the next 13 weeks.

• Calculate the price for a gallon of gas for 13 weeks. Begin with the price of a gallon of gas before the spill (same as in Task 1). Present the information in a graph and spreadsheet or table.
•What can you say about the relationship of the two variables? How do you know?
• What is the percent change in the price per gallon of gas from the pre-spill price to the fourth week after the spill?

C. When will it become necessary for you and Tim to stop driving to school each day and look for alternate transportation? Justify your reasoning with mathematical evidence.

Enrichment (writing assignment)

D. Do you think it is realistic for the price of a gallon of gas to keep rising at 6% every week? Why or why not? What do you think will happen? Explain your reasoning.

Analytic Scoring Rubric
Should We Walk or Bike?

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CEPC Analytic Scoring Rubric Should We Walk or Bike?				
CATEGORY	4	3	2	1
Analysis	Analysis for when gas prices become unreasonable is accurately analyzed with full consideration of all scenario parameters.	Analysis for when gas prices become unreasonable is accurately analyzed with evidence of consideration of all scenario parameters.	Analysis for when gas prices become unreasonable shows evidence of some inaccuracies mathematical reasoning.	Analysis for when gas prices become unreasonable shows evidence of numerous inaccuracies in mathematical reasoning.
Mathematical Concepts	Work shows evidence of in-depth understanding of proportions and proportional relationships	Work shows evidence of full understanding of proportions and proportional relationships	Work shows evidence of partial understanding of proportions and proportional relationships	Work shows evidence of limited understanding of proportions and proportional relationships
Precision of Mathematical Language	Complex mathematical language is accurately used, e.g. proportions, ratio, etc.(refer to stage 1 for additional vocabulary) throughout the presentation to communicate about mathematical reasoning	Appropriate mathematical language is accurately used, e.g. proportions, ratio, etc.(refer to stage 1 for additional vocabulary) in much of the presentation to communicate mathematical reasoning	Some mathematical language, e.g. proportions, ratio, etc.(refer to stage 1 for additional vocabulary) accurately used to communicate mathematical reasoning	Limited or no mathematical language is accurately used to communicate mathematical reasoning
Mathematical Accuracy	All work is shown and contains little or no errors in calculations, equations, graphs, and tables.	All work is shown and mostly accurate, containing a limited number of mathematical errors in calculations, equations, graphs, and tables.	Most work is shown but contains a number of inaccuracies in calculations, equations, graphs, and tables.	Work is limited, missing and/or contains numerous mathematical errors in calculations, equations, graphs, and tables.
Argument	Reasoning for when gas prices become unreasonable is included in the presentation and is convincing, thoughtful, and backed up with extensive evidence.	Reasoning for when gas prices become unreasonable is included in the presentation, and is convincing and backed up with appropriate evidence.	Some reasoning for when gas prices become unreasonable is included in the presentation. Reasoning is somewhat convincing and backed up with some evidence.	Reasoning for when gas prices become unreasonable may or may not be included in the presentation. The reasoning is not convincing and may be backed up with little or no evidence.