We have studied different methods to add and subtract integers (positive and negative whole numbers). But often, before we begin to add or subtract, we need to clean up an expression by reducing the number of signs it has.

Example:	6 + (–4)	→	Adding a negative number is the same as subtracting.	→	Rewrite as: 6 – 4
	5 – (–7)	→	Subtracting a negative number is the same as adding a positive one.	→	Rewrite as: 5 + (+7) and then as 5 + 7

A memory device I teach is that "minus minus equals plus plus." In other words, whenever you see back-to-back negative signs (usually separated by a parenthesis), draw two vertical lines through them to turn them into plus signs.

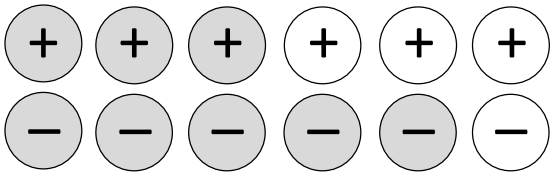
Once there is a single sign for each number, there are several methods to combine them, including:

- Integer coins
- Number lines
- Scoreboards

Integer coins

Imagine there are two different set of coins, positive and negative. Shade three positive coins to represent the number +3, and five negative coins to represent –5.

This could be written as –5 + 3 = _____ 3 – 5 = _____ or 3 + (–5) = _____



Ask these questions	Answers
(1) Are there more positive coins or negative coins?	negative
(2) How many more?	2
Therefore the answer is:	–2

–4 – 6 would result in four negative coins, then six more negative coins (with no positive coins) for a total of ten negative coins. Are there more positive coins or negative? Negative. How many more? Ten. Therefore the answer is –10.

See "Integer Coins Solved" in Unit One of www.esamath7.weebly.com

Of course, it isn't practical to use coins for this problem: 251 – 472 = _____. But you can visualize the two rows of coins, with 472 on bottom and 251 on top. And answer the questions, are there more positive or negative? Negative. And, how many more? 221. The rule then becomes that when the signs are opposite, you subtract the smaller number from the larger, and keep the sign of the larger. When the signs are the same, you add the numbers and keep the sign. Tell whether each of these numerical equations would result in a positive or negative solution by circling the sign:

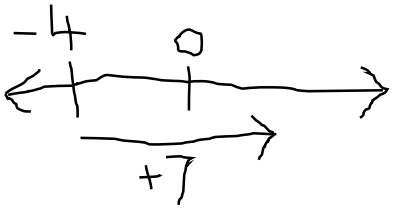
–5 + 13	+ *or* –	13 – (–5)	+ *or* –	10 – 82	+ *or* –	3 – (–84)	+ *or* –
612 + 19	+ *or* –	8 + (–77)	+ *or* –	–3 – (–4)	+ *or* –	–44 + 73	+ *or* –
6 – (–11)	+ *or* –	–65 + 23	+ *or* –	–9 + 640	+ *or* –	3 + (–52)	+ *or* –
98 – 44	+ *or* –	–1 – (–7)	+ *or* –	217 + 84	+ *or* –	–6 – (–1)	+ *or* –
8 + (–77)	+ *or* –	101 – 7	+ *or* –	6 – (–10)	+ *or* –	9 – 900	+ *or* –
–5 – (–2)	+ *or* –	–77 + 0	+ *or* –	12 + (–4)	+ *or* –	–34 + 44	+ *or* –
–617 + 3	+ *or* –	4,753 + 9	+ *or* –	–117 + 8	+ *or* –	0 – (–9)	+ *or* –
74 – 101	+ *or* –	4 – (–52)	+ *or* –	7 – 3	+ *or* –	4 + 0	+ *or* –

Number Lines

These are a useful way to determine which direction your second number should move. Again, it is helpful to eliminate the extra signs so that each number has only one sign. Begin by making a crude number line on your paper. You don't need to make tick marks for every number and label them. You only need to start with an arrow and a zero, like this.

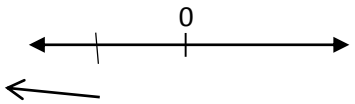
To solve a problem, such as – 4 + 7, draw a tick mark on the proper side of the zero and label it with the first number.

Draw an arrow to the left to subtract the second number, or an arrow to the right for to add. Will the second number pass the zero? This will give you an idea of where the answer will be.



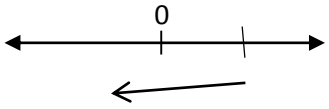
Use the number lines to solve these equations.

–25 – 40 = _____



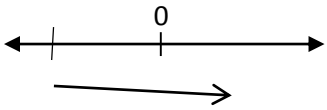
add the numbers, keep the sign

14 – 20 = _____



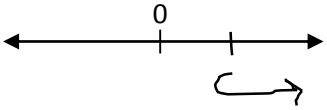
subtract small number from large, answer is on the negative side

–100 + 150 = _____



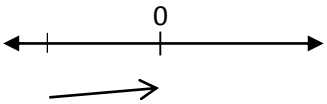
subtract, small number from large, answer is on the positive side

15 – (– 10) = _____



"minus minus equals plus plus," as we move the opposite direction of subtraction

–31 + 31 = _____



opposites add up to zero

The Scoreboard

Let's imagine that the numbers given to us are scores in a soccer or basketball game. As long as we are imagining, let's give the teams names – the New York Negatives vs. the Pittsburgh Positives.

Game One: –4 – 5 Low score. Must be a soccer game. The scoreboard shows that the negatives scored 4 goals in the first half and 5 in the second. Final score: New York 9, Pittsburgh 0. Final answer: – 9

Game Two: 80 – 96 High score. Probably basketball. The scoreboard shows that both teams scored this time. Which team won? New York. By how much? 16. Final answer: –16

Game Three: –700 + 850 Very big numbers. Perhaps a card game. Both teams scored. Final score: Pittsburgh 850, New York 700. Final answer: +150

With the scoreboard method, you look to see if the signs are the same or the opposite. If they are the same, you add up the numbers and keep the sign. If the signs are opposite, you determine which team won and by how much. The scoreboard method is particularly useful to solve a string of numbers:

$$5 - 4 + 12 + 3 - 10 - 11 + 4 - 8 - 2 + 1 + 14 - 6 = \_\_\_\_\_$$

Pittsburgh +'s	5	12	3	4	1	14	= +39
New York –'s	4	10	11	8	2	6	= –41
						Final Score	–2

Fill in these scoreboards to determine a winner. Use zeroes to fill in the extra spaces.

Equation	Teams	1st Half	2nd Half	Winner
–7 + 12	Poland Positives			+ *or* –
	Norway Negatives			
108 – 104	Philadelphia Positives			+ *or* –
	New England Negatives			
–2 + (–5)	Phoenix Positives			+ *or* –
	Nebraska Negatives			
3 – (– 4)	Portland Positives			+ *or* –
	Nashville Negatives			
2 + 3	Pennsylvania Positives			+ *or* –
	New Jersey Negatives			
9 – (–2)	Pawtucket Positives			+ *or* –
	New Haven Negatives			
93 + (–88)	Petersburg Positives			+ *or* –
	Norfolk Negatives			
–119 + 107	Panama Positives			+ *or* –
	Nicaragua Negatives			
–355 – 685	Pensacola Positives			+ *or* –
	Naples Negatives			
8 – 9	Peru Positives			+ *or* –
	Nepal Negatives			

Basic ideas to remember:

- The extra signs can be reduced down to a single sign for each number
  - 2 + (–3) can be simplified to 2 – 3
  - 4 – (– 3) can be changed to 4 +(+3), then simplified to 4 + 3
- Minus means the same thing as negative and plus means the same thing as positive

Word problems related to adding and subtracting integers include:

- Submarines rising up and diving
- Hot air balloons rising and descending
- Money being deposited into or withdrawn form an account
- Temperature rising or falling
- A subway travelling north, then south

To solve these problems, assign a positive to numbers going up (or into a bank account) and a negative to numbers going down (or being withdrawn).

Assign an integer and its sign to each situation:

The submarine dove 120 feet on the Captain's command: _____

Taylor received a bill from the electric company for $140: _____

Rene took a taxi north 15 blocks after the meeting: _____

On the third day, the climber's ascended 540 feet on the mountain: _____

To avoid turbulence, the pilot climbed 1,800 feet, then leveled off: _____

Jimmy dropped a water balloon on his cousin 35 feet below: _____

Even though the sun was out, the temperature plummeted 14 degrees: _____

When it was the little girl's turn, she skipped back eight sidewalk squares: _____

The golfer used a 4-iron to reach the green located 74 feet above the tee box: _____

Carrie tossed a penny into the ocean for luck from the deck 23 feet above the waterline (hint: describe how far the penny moved before it hit the water): _____

A dozen cupcakes were missing when Trevor returned: _____

Gonzales accounted for 83 RBI's in his rookie year: _____

Mike's grandfather said that he lost $100 by betting on "slow horses": _____

Multiplying and Dividing Integers

Rules: If the signs are the same, the answer is positive
If the signs are opposite, the answer is negative

Multiplication signs include: 2 x 4 $2 \cdot 3$ 2(3) (2)(3)

Division signs include: 10 ÷ 2 $\frac{10}{2}$ 10/2

Indicate the sign that goes with the answer:

(–327)(468) = + or –

64(7,108) = + or –

6,015 · –874 = + or –

2,492/–356 = + or –

–485 x (–948) = + or –

108 ÷ 2 = + or –

–630 ÷ –5 = + or –

$\frac{-28}{14}$ = + or –