MATHCOUNTS®

2018
■ State Competition ■
Countdown Round
Problems 1–80

This booklet contains problems to be used in the Countdown Round.

1. _____ Troy has 16 quarters and 13 nickels. Michael has 18 quarters and n nickels. If Troy and Michael have the same amount of money, what is the value of n?

2. _____ (integers) For how many positive four-digit integers is the sum of the digits equal to 3?

3. _____ What is 30% of $\frac{3}{5}$ of 0.85? Express your answer as a decimal to the nearest thousandth.

4. _____ The first five terms of an arithmetic sequence are x, y, $x + 6$, z and 37, in that order, for some numbers x, y and z. What is the value of y?

5. _____ Two spherical scoops of ice cream of radius 1.5 inches sit on top of a cone with base radius 1.5 inches and height h inches. If the cone holds exactly the volume of both scoops together, what is the value of h?

6. _____ (mm) Zola the ant walked 1 meter, then 15 cm and then 3.7 cm. How many millimeters did Zola walk?

7. _____ What is the least integer n such that $\dfrac{10!}{(1!\,2!\,3!\,\cdots n!)}$ is **not** an integer?

8. _____ What is the probability that a randomly chosen positive integer factor of 2018 is prime? Express your answer as a common fraction.

9. _____ What is the mean of all the positive three-digit multiples of 7? Express your answer as a decimal to the nearest tenth.

10. _____ (cases) Bernoulli's Market sells water in cases of 12 bottles each. How many cases of water must Joseph buy to ensure there is at least one water bottle for everyone in his 70-student class?

11. _____ What is the ratio of the number of four-digit positive integers that have four different even digits to the number of four-digit positive integers that have four different odd digits. Express your answer as a common fraction.

12. _____ (right triangles) How many right triangles with integer side lengths can be formed with vertices at A(5, 2), B(5, −1), and a point C(x, y)?

13. _____   If x is a real number such that $\dfrac{x}{1+x} = \dfrac{7}{107}$, what is the value of $\dfrac{x}{3+x}$? Express your answer as a common fraction.

14. _____   The line $2x + 6y = 12$ passes through one point with positive integer coordinates. What is the sum of the coordinates of that point?

15. _____ (dollars)   Abdul wants to buy three shirts whose marked prices are $15, $20 and $25, respectively. He has three coupons, each of which grants a discount on one and only one item. The coupons give discounts of 20%, 30% and 40%, respectively. What is the least amount, in dollars, that he could pay for all three shirts?

16. _____ (ways)   Mr. Thomas wants to divide all seven students in his algebra class into three teams of at least two students each. How many different ways are there for him to divide his students into teams?

17. _____   If $(a^3 b^2)^5 = a^r b^s$ for all positive real numbers a and b, what is the value of $r + s$?

18. _____   If x is an integer, what is the least possible value of $x^2 - 4x + 5$?

19. _____   Jennifer arranges eight tiles, each containing a single letter, in a row to spell the word STRESSED. If Jennifer rearranges the eight tiles in a random order, what is the probability that the new arrangement spells the word DESSERTS? Express your answer as a common fraction.

20. _____   On $\triangle ABC$, point D is one-third of the distance from A to B, point E is one-third of the distance from B to C, and point F is one-third of the distance from C to A. What is the ratio of the area of $\triangle DEF$ to the area of $\triangle ABC$? Express your answer as a common fraction.

21. _____   The diameter of circle A is equal to the radius of circle B. What is the ratio of the area of circle A to the area of circle B. Express your answer as a common fraction.

22. _____   If $x = 0.\overline{45}$, what is the value of x expressed as a common fraction?

23. _____   What is the least positive integer k such that $2^k > 8 - k^2$?

24. _____ If a regular 12-gon has exterior angles measuring m degrees each and has a total of n diagonals, what is the value of $m + n$?

25. _____ (integers) For how many integers n is $2 < n^2 < 20$?

26. _____ (cm) If three interlocking links of a chain have a total length of 23 cm, and each individual link is 9 cm long, what is the total length, in centimeters, of a chain composed of 27 links?
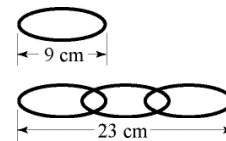
27. _____ (points) The published mean rating of a new computer game was 63 points. When it was discovered that five of the twenty-five ratings used were each 10 points too low, the mean rating was corrected. What is the corrected rating?

28. _____ (triangles) Six points are placed at the vertices of a regular hexagon, and a seventh point is placed at the center of the hexagon. How many distinct triangles can be formed by using three of the seven points?
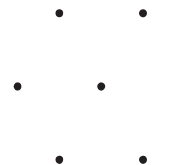
29. _____ If $5x - y = 18$ and $2x + y = -6$, what is the value of $6x - 4y$?

30. _____ (inches) The length of each side of scalene triangle ABC is a whole number of inches. The second-longest side of the triangle is 2018 inches long. What is the absolute difference, in inches, between the greatest possible perimeter and the least possible perimeter of the triangle?

31. _____ If $ab = 10$, what is the value of $(a + b)^2 - (a - b)^2$?

32. _____ For what value of n is $(2019^2 - 2018^2) = (2018^2 - 2017^2) + n$?

33. _____ Yvonne takes a 10-question true and false test. If she randomly guesses on every question, what is the probability she gets at least 75% of the questions correct? Express your answer as a common fraction.

34. _____ (units) What is the perimeter, in units, of a regular hexagon whose longest diagonal has length 8 units?

35. _____ (years) Harry is 10 years older than Caitlin. In 15 years, Caitlin will be twice as old as Harry was 15 years ago. How many years old is Harry now?

36. _____ What is the smallest positive integer n for which $\dfrac{\frac{n^2}{3}}{4}$ is an integer?

37. _____ What is the sum of the number $\dfrac{3}{10}$ of the way from 358 to 383 and the number $\dfrac{3}{8}$ of the way from 496 to 484?

38. _____ Oscar enters the number 2 in his scientific calculator. He presses the square root button, presses the reciprocal button, and presses the square root button again, each time getting a new result. He then raises this last result to the nth power, where n is an integer, and gets an answer of 16. What is the value of n?

39. _____ If $x = -2018$, what is the value of $(x^2 - 2x + 1) - (-1 + x(-2 + x))$?

40. _____ (units) In the figure, ACDH and BCEF are rectangles of areas 50 and 30 units², respectively. The two rectangles intersect in square BCDG. If the total area of concave hexagon ACEFGH is 64 units², what is the perimeter of the hexagon, in units?
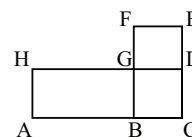
41. _____ Stages 1, 2 and 3 of a regular pattern are shown here. If the upward-pointing triangles are shaded and if U(n) is the number of shaded triangles at Stage n, what is the value of U(26) − U(24)?

Stage 1   Stage 2   Stage 3
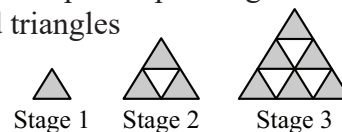
42. _____ If 2^n is a factor of 3384, what is the greatest possible integer value of n?

43. _____ The portion of the number line from 1.6 to 2.85 is divided into five segments of equal length by points A, B, C and D, as shown. What is the number associated with point C? Express your answer as a decimal to the nearest hundredth.

1.6   A   B   C   D   2.85
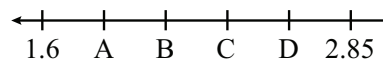
44. _____ (degrees) The ratio of the measures of the angles of a particular triangle is 1:3:5. What is the degree measure of the largest angle?

45. _____ Twice u is 3 less than 5 times u. What is the value of u?

46. _____ If $x^2 + x = 6$, what is the least possible value of $4x + 3$?

47. _____ For what value of x does $5^x = \dfrac{1}{625}$?

48. _____ (cm) The circles of radius 2 cm and 14 cm shown in the figure are concentric. Marcia wants to construct a third circle with the same center that will divide the shaded region into two parts with equal area. In centimeters, what is the radius of the circle she should construct?
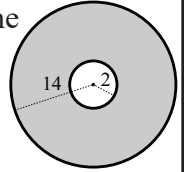
49. _____ Tiger Woods and LeBron James were both born on December 30. In 1987, Tiger turned four times the age LeBron turned. In 1993, Tiger turned twice the age LeBron turned. During what year will Tiger and LeBron turn ages totaling 91?

50. _____ (units²) If $3x^3 - 93x^2 + 933x - 3003 = 0$ can be factored as $3(x - a)(x - b)(x - c) = 0$ for some numbers a, b and c, what is the surface area, in square units, of a rectangular prism with edge lengths a, b and c?

51. _____ If $0.\overline{4} \times n = 12$, what is the value of n?

52. _____ What is the least positive integer with at least five positive integer factors?

53. _____ If $\dfrac{r}{21} = \dfrac{3}{7} = \dfrac{21}{s}$, what is the value of $r + s$?

54. _____ (ft²) The figure shows a circle of radius 4 feet inscribed in a square with a second square inscribed in the circle. What is the absolute difference, in square feet, between the areas of the two squares?
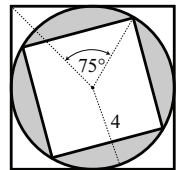
55. _____ (km/h) Allison rides her bike up a steep hill at 4 km/h. She rides down the same hill at 28 km/h. What is Allison's average speed, in kilometers per hour, for the round trip bike ride?

56. _____ What number is 62.5% of 888?

57. _____ (minutes) Greg disolves 10 grams of sugar in 250 mL of water. He then heats the sugar water in a pan to create a more concentrated solution. If the water boils off at a rate of 10 mL per minute while the sugar stays in the pan, for how many minutes will he have to boil the sugar water until it has a concentration of 100 g/L?

58. _____ (diagonals) What is the absolute difference between the number of diagonals in a heptagon and the number of diagonals in an octagon?

59. _____ The second and sixth terms of an arithmetic sequence are 2 and 14, respectively. What is the first term of this sequence?

60. _____ Let $a = 20$ and $b = 25$. If p is the greatest positive integer that divides both a and b, and q is the least positive integer divisible by both a and b, what is $p \times q$?

61. _____ What is the value of $\sqrt{8^2 + 8^2 + 9^2 + 9^2 + 12^2 + 12^2}$? Express your answer in simplest radical form.

62. _____ For what positive integer n does $\dfrac{3^{20} - 3^{18}}{3^2 - 1^8} = n^n$?

63. _____ Billy chose a number he called B. He divided B by $\dfrac{1}{2}$ and then divided that quotient by $\dfrac{1}{3}$. The result was 666. What was the value of B?

64. _____ If x is a real number such that $(x + 2017) + (x + 2018) + (x + 2019) = 114$, what is the value of x?

65. _____ (degrees) If ABCDEFGHI is a 9-sided regular polygon with diagonals AD and CG intersecting at P, what is the degree measure of angle APC?

66. _____ What is the least positive integer with at least three distinct prime factors?

67. _____ (units²) Trapezoid ABCD has parallel bases AB and DC. The area of triangle ABD is 6 units², and the area of triangle ACD is 9 units². What is the area of the trapezoid, in square units?

68. _____ If x and y are positive real numbers such that $\dfrac{x}{3} = \dfrac{5}{y} = \dfrac{x+y}{7}$, what is the value of y? Express your answer in simplest radical form.

69. _____ If you choose any two numbers a and b from the set {3, 5, 7}, what is the least possible value of $\dfrac{a}{b}$? Express your answer as a common fraction.

70. _____ If the cubic equation $x^3 + ax + b = 0$ has three real solutions, $x = 7$, $x = 5$ and $x = m$, what is the value of m?

71. _____ (ostriches) Fiona went to the zoo and saw tigers, lions and elephants, which have four legs each, and ostriches, which have two legs each. In total, she saw 20 animals, which have a combined total of 72 legs. How many ostriches did Fiona see?

72. _____ What common fraction is equal to the ratio of $2\frac{1}{2}$ to $3\frac{1}{3}$?

73. _____ Bernadette selects three numbers at random, without replacement, from the set $\{1, 2, 3, 4, 5, 6\}$. What is the probability that the greatest number she selects is 4? Express your answer as a common fraction.

74. _____ (ways) In how many unique ways can the numbers 1 through 6 be assigned to the faces of a standard cubical die, so that all pairs of opposite faces add to 7, assuming that two assignments are the same if one can be rotated to obtain the other?

75. _____ Equilateral triangle DEF overlays square ABCD, as shown, so that B is the midpoint of side EF. What is the ratio of the area of square ABCD to the area of triangle DEF? Express your answer as a common fraction in simplest radical form.

76. _____ If $2^x \times 2^{30} \times 4^x = 64^x$, what is the value of x?

77. _____ (in³) A rectangular box has integer side lengths each greater than one. Two adjacent faces have a total area of 15 in². In cubic inches, what is the volume of the largest sphere that will fit inside this box? Express your answer as a common fraction in terms of π.

78. _____ What is the value of $\sqrt{40401}$?

79. _____ (degrees) The measure of each of four interior angles of a pentagon is 100 degrees. What is the degree measure of the fifth angle?

80. _____ What is the value of $(36)(34) - (39)(31)$?