MATHCOUNTS ${ }^{\circledR}$

## 2018 <br> School Competition Sprint Round Problems 1-30

Name $\qquad$

## DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.
In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

| Total Correct | Scorer's Initials |
| :---: | :---: |
|  |  |
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1. $\qquad$ The average of Jerrold's vocabulary quizzes is 88.75. What is Jerrold's quiz average expressed to the nearest whole number?
2. $\qquad$ What is the value of $2^{5}-5 \times 6$ ?
3. $\$$ $\qquad$ A particular store sells items for only $\$ 1, \$ 3$ and $\$ 5$ and charges no sales tax. If Georgia buys nine $\$ 1$ items, seven $\$ 3$ items and three $\$ 5$ items from this store, what is the total amount she will be charged?

4. $\qquad$ ${ }^{\circ} \mathrm{F}$ boiling point and freezing point of water?


Boiling Point
$212{ }^{\circ} \mathrm{F}$
5. $\qquad$ If the first five terms of an arithmetic sequence are $26,16,6,-4,-14$, what is the sixth term of this sequence?
6. $\qquad$ If $\frac{2^{12}}{2^{n}}=2^{3}$, what is the value of $n ?$
7. $\qquad$ What is the slope of the line containing the points $(0,0)$ and $(3,-6)$ as shown?

8. $\qquad$ The least common multiple of 15 and 20 is $k$. What is the least common multiple of 12 and $k$ ?
9. $\qquad$ When Allen Iverson and Shaquille O’Neal faced off in the 2001 NBA Finals, O'Neal weighed exactly twice as much as Iverson, and their weights totaled 495 pounds. How many pounds did Iverson weigh?
10. $\qquad$ If $3 x+7=22$, what is the value of $\frac{1}{3 x+11}$. Express your answer as a common fraction.
11. $\qquad$ The table shows the results of Jake's last twenty times at bat. Based on this data, what is the probability that Jake hits a double on his next time at bat? Express your answer as a decimal to the nearest tenth.

Batting Results

| Result | Number |
| :---: | :---: |
| Home Run | 3 |
| Triple | 1 |
| Double | 2 |
| Single | 8 |
| Walk | 3 |
| Out | 3 |

12. $\qquad$ A square pen that is surrounded by a fence is divided into two rectangular regions by adding a new fence that connects the midpoints of two opposite sides. By what percent has the total length of fencing been increased?
13. $\qquad$ What is the sum of the greatest common factor of 4 and 18 and the least common multiple of 4 and 18 ?
14. $\qquad$ years

Claire's three sons, from oldest to youngest, are Evan, Joel and Alex. The difference between the ages of Evan and Joel is the same as the difference between the ages of Joel and Alex. If the sum of the three ages is 39 , how old is Joel?
15. mile
marker

Danica started her trip at mile marker 66 and ended her trip at mile marker 194. She drove at a constant speed the entire trip. Which mile marker had she reached by $75 \%$ of her trip?
16. $\qquad$ What is the value of $(20+12)^{2}-(20-12)^{2}$ ?
17. $\qquad$ visitors

The table shows the number of visitors over a 4 -week period to a new website. What was the average number of visitors to the site per week?

Website Visitors

| Week | Visitors |
| :---: | :---: |
| 1 | 1028 |
| 2 | 1100 |
| 3 | 1060 |
| 4 | 1056 |

18. $\qquad$ percent

The Venn diagram shows the number of students at Ramanujan Middle School who play both volleyball and basketball, the number who play one of these sports but not the other, and the number who play neither of these sports. All students at the school are represented in the diagram. What percent of the students at the school play basketball?

19. $\qquad$ Josie selected a number $n$. She divided $n$ by 2 and then subtracted $\frac{1}{2}$ from the result. She took half of that result and then subtracted $\frac{1}{2}$ to get the final result of 10 . What is the value of $n$ ?
20. $\qquad$ times

Scott repeatedly rolls a pair of standard six-sided dice and keeps track of the sum of the two numbers rolled each time. So far he has not rolled any sum twice. What is the maximum possible number of times he has thrown the dice?
21. $\qquad$ cm

An annulus is a ring bounded by two concentric circles. In the figure, annulus A has an inner circle of radius 2 cm and an outer circle of radius 6 cm . Annulus B has an outer circle of radius 9 cm . If annulus $B$ has area equal to that of annulus A , what is the radius of the inner circle of annulus B ?


Let $\mathrm{Q}=\{1.7,1.1,1.4,2.1,2.3, s\}$. What is the absolute difference between the greatest and least possible values of the median of set Q? Express your answer as a decimal to the nearest hundredth.
23. $\qquad$ tickets

A bag contains 25 tickets, each colored either red or yellow. Red tickets are worth $\$ 0.50$, and yellow tickets are worth $\$ 5.00$. If the expected value of a ticket drawn at random from this bag is $\$ 3.20$, how many of the tickets are red?
24. $\qquad$ If $a$ and $b$ are integers such that $a^{2}-b^{2}=100$, what is the greatest possible value of $a$ ?
25. $\qquad$ cents

In cents, what is the least total amount that cannot be obtained by using a combination of fewer than eight coins from a collection of pennies, nickels, dimes and quarters?
26. $\qquad$ How many of the first one thousand perfect fourth powers have either 1 or 6 as their units digit?
27. $\qquad$ Seven jars hold a total of 250 candies. They are lined up left to right from fullest to emptiest. The left-most jar has 72 candies, and the right-most jar has 13. No two jars hold the same number of candies. What is the positive difference between the greatest number of candies that could be in the second jar from the left, and the least number of candies that could be in that jar?

28. $\qquad$
29. $\qquad$ nets

How many distinct tetrahedron nets, formed from four connected equilateral triangles, can be cut from the figure shown?
30. $\qquad$
What is the greatest integer $k$ such that $2^{k}$ is a factor of $67!?$

30. triangles

The rows and columns of lattice points in a three-by-three
 square array are evenly spaced one unit apart. How many distinct triangles with at least one side of length $\sqrt{2}$ units can be drawn using three lattice points for the vertices?

