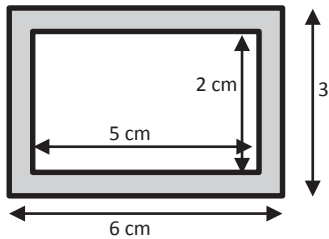# Lesson 26: Volume and Surface Area
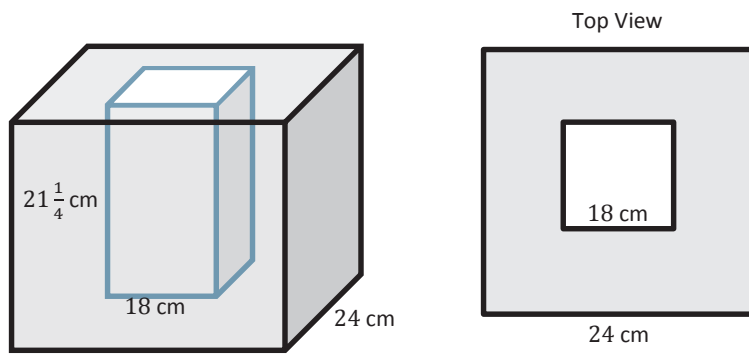
Classwork

Opening Exercise

Explain to your partner how you would calculate the area of the shaded region. Then, calculate the area.

Example 1: Volume of a Shell

Top View

The insulated box shown is made from a large cube with a hollow inside that is a right rectangular prism with a square base. The figure on the right is what the box looks like from above.

　a. Calculate the volume of the outer box.

　b. Calculate the volume of the inner prism.

    c.   Describe in words how you would find the volume of the insulation.

    d.   Calculate the volume of the insulation in cubic centimeters.

    e.   Calculate the amount of water the box can hold in liters.

Exercise 1: Brick Planter Design

You have been asked by your school to design a brick planter that will be used by classes to plant flowers. The planter will be built in the shape of a right rectangular prism with no bottom so water and roots can access the ground beneath. The exterior dimensions are to be 12 ft. $\times$ 9 ft. $\times$ $2\frac{1}{2}$ ft. The bricks used to construct the planter are 6 in. long, $3\frac{1}{2}$ in. wide, and 2 in. high.

    a.   What are the interior dimensions of the planter if the thickness of the planter's walls is equal to the length of the bricks?

    b.   What is the volume of the bricks that form the planter?

c. If you are going to fill the planter $\frac{3}{4}$ full of soil, how much soil will you need to purchase, and what will be the height of the soil?

d. How many bricks are needed to construct the planter?

e. Each brick used in this project costs $0.82 and weighs 4.5 lb. The supply company charges a delivery fee of $15 per whole ton (2,000 lb.) over 4,000 lb. How much will your school pay for the bricks (including delivery) to construct the planter?

    f.   A cubic foot of topsoil weighs between 75 and 100 lb. How much will the soil in the planter weigh?

    g.   If the topsoil costs $0.88 per each cubic foot, calculate the total cost of materials that will be used to construct the planter.

Exercise 2: Design a Feeder

You did such a good job designing the planter that a local farmer has asked you to design a feeder for the animals on his farm. Your feeder must be able to contain at least 100,000 cubic centimeters, but not more than 200,000 cubic centimeters of grain when it is full. The feeder is to be built of stainless steel and must be in the shape of a right prism but not a right rectangular prism. Sketch your design below including dimensions. Calculate the volume of grain that it can hold and the amount of metal needed to construct the feeder.

The farmer needs a cost estimate. Calculate the cost of constructing the feeder if $\frac{1}{2}$ cm thick stainless steel sells for $93.25 per square meter.

Problem Set

1. A child's toy is constructed by cutting a right triangular prism out of a right rectangular prism.

Top View
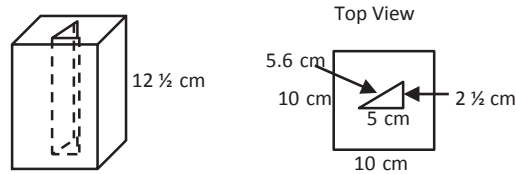
    a. Calculate the volume of the rectangular prism.
    b. Calculate the volume of the triangular prism.
    c. Calculate the volume of the material remaining in the rectangular prism.
    d. What is the largest number of triangular prisms that can be cut from the rectangular prism?
    e. What is the surface area of the triangular prism (assume there is no top or bottom)?

2. A landscape designer is constructing a flower bed in the shape of a right trapezoidal prism. He needs to run three identical square prisms through the bed for drainage.

Side View
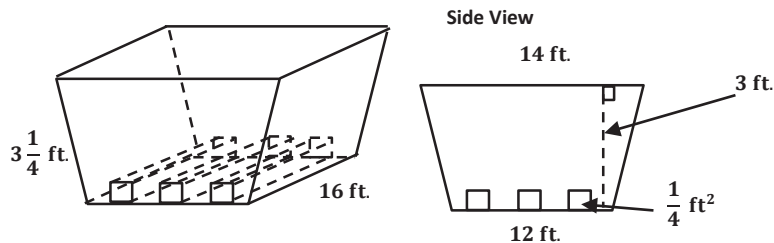
    a. What is the volume of the bed without the drainage pipes?
    b. What is the total volume of the three drainage pipes?
    c. What is the volume of soil if the planter is filled to $\frac{3}{4}$ of its total capacity with the pipes in place?
    d. What is the height of the soil? If necessary, round to the nearest tenth.
    e. If the bed is made of 8 ft. × 4 ft. pieces of plywood, how many pieces of plywood will the landscape designer need to construct the bed without the drainage pipes?
    f. If the plywood needed to construct the bed costs $35 per 8 ft. × 4 ft. piece, the drainage pipes cost $125 each, and the soil costs $1.25/cubic foot, how much does it cost to construct and fill the bed?