## Lesson 16: The Most Famous Ratio of All

## Classwork

## Opening Exercise

a. Using a compass, draw a circle like the picture to the right.

$C$ is the center of the circle.
The distance between $C$ and $B$ is the radius of the circle.
b. Write your own definition for the term circle.
c. Extend segment $C B$ to a segment $A B$ in part (a), where $A$ is also a point on the circle.

The length of the segment $A B$ is called the diameter of the circle.
d. The diameter is $\qquad$ as long as the radius.
e. Measure the radius and diameter of each circle. The center of each circle is labeled $C$.

f. Draw a circle of radius 6 cm .

## Mathematical Modeling Exercise

The ratio of the circumference to its diameter is always the same for any circle. The value of this ratio, $\frac{\text { Circumference }}{\text { Diameter }}$, is called the number $p i$ and is represented by the symbol $\pi$.


Since the circumference is a little greater than 3 times the diameter, $\pi$ is a number that is a little greater than 3 . Use the symbol $\pi$ to represent this special number. Pi is a non-terminating, non-repeating decimal, and mathematicians use the symbol $\pi$ or approximate representations as more convenient ways to represent pi.

- $\pi \approx 3.14$ or $\frac{22}{7}$.
- The ratios of the circumference to the diameter and $\pi: 1$ are equal.
- Circumference of a Circle $=\pi \times$ Diameter.


## Example

a. The following circles are not drawn to scale. Find the circumference of each circle. (Use $\frac{22}{7}$ as an approximation for $\pi$.)

b. The radius of a paper plate is 11.7 cm . Find the circumference to the nearest tenth. (Use 3.14 as an approximation for $\pi$.)
c. The radius of a paper plate is 11.7 cm . Find the circumference to the nearest hundredth. (Use the $\pi$ button on your calculator as an approximation for $\pi$.)
d. A circle has a radius of $r \mathrm{~cm}$ and a circumference of $C \mathrm{~cm}$. Write a formula that expresses the value of $C$ in terms of $r$ and $\pi$.
e. The figure below is in the shape of a semicircle. A semicircle is an arc that is half of a circle. Find the perimeter of the shape. (Use 3.14 for $\pi$.)


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## Relevant Vocabulary

Circle: Given a point $O$ in the plane and a number $r>0$, the circle with center $O$ and radius $r$ is the set of all points in the plane whose distance from the point $O$ is equal to $r$.

Radius of a circle: The radius is the length of any segment whose endpoints are the center of a circle and a point that lies on the circle.

DIAMETER OF A CIRCLE: The diameter of a circle is the length of any segment that passes through the center of a circle whose endpoints lie on the circle. If $r$ is the radius of a circle, then the diameter is $2 r$.

The word diameter can also mean the segment itself. Context determines how the term is being used: The diameter usually refers to the length of the segment, while a diameter usually refers to a segment. Similarly, a radius can refer to a segment from the center of a circle to a point on the circle.


Circle $C$
Radii: $\overline{O A}, \overline{O B}, \overline{O X}$
Diameter: $\overline{A B}$

Circumference


Circumference: The circumference of a circle is the distance around a circle.
PI: The number pi, denoted by $\pi$, is the value of the ratio given by the circumference to the diameter, that is $\pi=\frac{\text { circumference }}{\text { diameter }}$. The most commonly used approximations for $\pi$ is 3.14 or $\frac{22}{7}$.

Semicircle: Let $C$ be a circle with center $O$, and let $A$ and $B$ be the endpoints of a diameter. A semicircle is the set containing $A, B$, and all points that lie in a given half-plane determined by $\overline{A B}$ (diameter) that lie on circle $C$.

## Semicircle



## Problem Set

1. Find the circumference.
a. Give an exact answer in terms of $\pi$.
b. Use $\pi \approx \frac{22}{7}$ and express your answer as a fraction in lowest terms.
c. Use the $\pi$ button on your calculator, and express your answer to the nearest hundredth.


Find the circumference.
d. Give an exact answer in terms of $\pi$.
e. Use $\pi \approx \frac{22}{7}$, and express your answer as a fraction in lowest terms.

2. The figure shows a circle within a square. Find the circumference of the circle. Let $\pi \approx 3.14$.

3. Consider the diagram of a semicircle shown.
a. Explain in words how to determine the perimeter of a semicircle.
b. Using $d$ to represent the diameter of the circle, write an algebraic equation that will result in the perimeter of a semicircle.

c. Write another algebraic equation to represent the perimeter of a semicircle using $r$ to represent the radius of a semicircle.
4. Find the perimeter of the semicircle. Let $\pi \approx 3.14$.

5. Ken's landscape gardening business makes odd-shaped lawns that include semicircles. Find the length of the edging material needed to border the two lawn designs. Use 3.14 for $\pi$.
a. The radius of this flower bed is 2.5 m .

b. The diameter of the semicircular section is 10 m , and the lengths of the sides of the two sides are 6 m .

6. Mary and Margaret are looking at a map of a running path in a local park. Which is the shorter path from $E$ to $F$, along the two semicircles or along the larger semicircle? If one path is shorter, how much shorter is it? Let $\pi \approx 3.14$.

7. Alex the electrician needs 34 yards of electrical wire to complete a job. He has a coil of wiring in his workshop. The coiled wire is 18 inches in diameter and is made up of 21 circles of wire. Will this coil be enough to complete the job? Let $\pi \approx 3.14$.


