European Contest-Game
Math Kangaroo

Online Training
March 19, 2011

Grade 7-8

General Points and Strategy

The Kangaroo math contest consists of 24 multiple-choice questions to be answered in 60 minutes.

They are divided into three parts of 8 questions each:

Part A (easy) - correct answer is worth 3 points

Part B (medium) - correct answer is worth 4 points

Part C (hard) - correct answer is worth 5 points

Questions left blank are worth 0 points.

Wrong answers carry a penalty of -1 point.

Calculators are not permitted.

General Points and Strategy

The Kangaroo math contest consists of 24 multiple-choice questions to be answered in 60 minutes.

That means you have an average of 2.5 minutes for every question.

If you get stuck on a question, skip it, do others and come back to it when you're sure you have time to look at it.

Very few students finish the entire contest in the time allotted and answer every question correctly.

Do not be discouraged if you find you can't do some problems.

Because wrong answers are worth less than no answers, It is important that you don't guess if you don't know the answer.

Part A: Each correct answer is worth 3 points.

1) How many hours are there in half of a third of a quarter of a day?

A) 1/3 B) 1/2 C) 3 D) 2 E) 1

Answering this just involves pulling out the correct fractions and multiplying through:

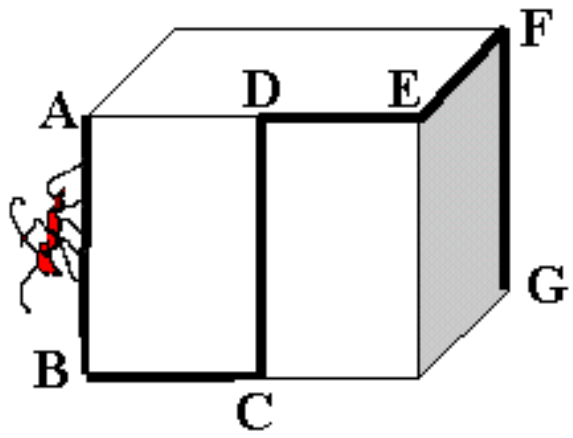
*What is 1/2 * 1/3 * 1/4 of a day?*

*It's 1/(2*3*4) = 1/24, which makes it 1 hour.*

Answer: E)

Part A: Each correct answer is worth 3 points.

2) The diagram shows a cube with sides of length 12 cm. An ant is walking across the cube's surface from A to G using the path shown in bold. How far will it have to walk?



A) 48 cm
B) 40 cm
C) 50 cm
D) 60 cm
E) It is impossible to determine

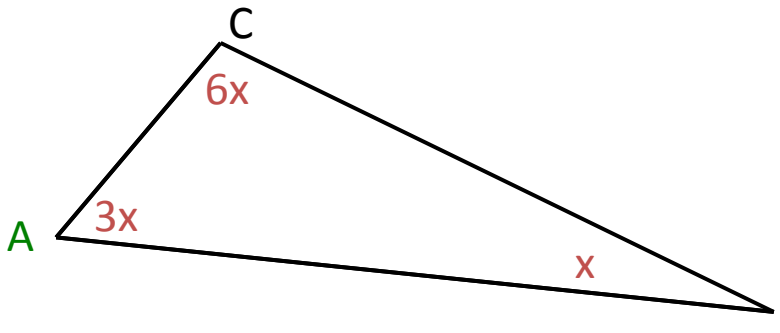
The ant walks AB + BC + CD + DE + EF + FG.
Since the figure is a cube, every side is of length 12cm.
So, AB = CD = EF = FG = 12cm and BC + DE = 12cm.
*The ant walked 12 * 4 + 12 = 60 cm.*

Answer: D)

Part A: Each correct answer is worth 3 points.

3) In triangle ABC, the angle at A is three times greater than the angle at B and half of the angle at C. What is the angle at A?

A) 30°
B) 36°
C) 54°
D) 60°
E) 72°



*If <B is x, then <A is 3*x = 3x and it's half of <C. So, <C is 2*(3x) = 6x*

Now we need to remember what is the total sum of angles in a triangle.

<A + <B + <C = 180 degrees

But <A = 3x, <B = 6x and <C = x

So 3x + 6x + x = 180 degrees

10 x = 180

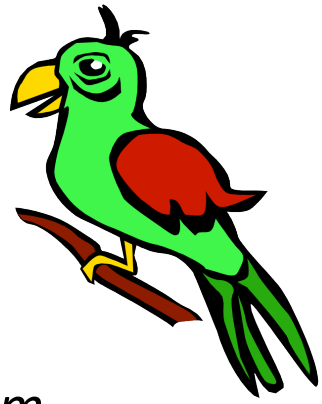
x = 18

*So <A = 3x = 3*18 = 54*

Answer: C)

Part A: Each correct answer is worth 3 points.

4) There were 5 parrots in the cage. Their average price was 6000 dollars. One day during the cleaning of the cage the most beautiful parrot flew away. The average price of the remaining four parrots was 5000 dollars. What was the price of the parrot that escaped?



A) 1000 dollars
B) 2000 dollars
C) 5500 dollars
D) 6000 dollars
E) 10000 dollars

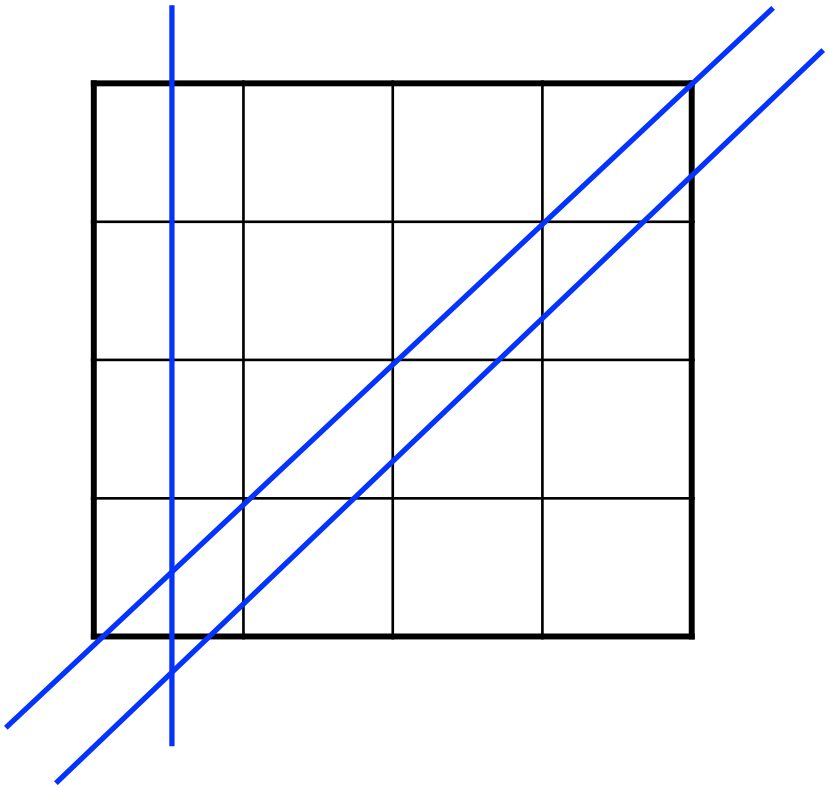
Answer: E)

The average is defined as the sum divided by the number of items.
While there were five parrots, their total price was equal to
*5 * 6000 = 30 000 dollars*
When the most beautiful flew away, the total price of the remaining ones was equal to
*4 * 5000 = 20 000 dollars.*
So, the parrot cost 10 000 dollars.

Part A: Each correct answer is worth 3 points.

5) A straight line is drawn across a 4×4 chessboard. What is the greatest number of 1×1 squares this line can pass through?

A) 3
B) 4
C) 6
D) 7
E) 8



Answer: D)

Part B: Each correct answer is worth 4 points.

6) In a triangle ABC the angle C is three times bigger then the angle A, the angle B is two times bigger then the angle A. Then the triangle ABC

A) is equilateral

B) is isosceles

C) has an obtuse angle

D) has a right angle

E) has only acute angles

Answer: D)

We're back to the total sum of angles in a triangle.

<A + <B + <C = 180 degrees

If <A is x, then

*<C = 3 * <A = 3x*

*<B = 2 * <A = 2x*

This means that since

<A + <B + <C = 180 degrees,

x + 2x + 3x = 180 degrees

6x = 180 degrees

x = 30 degrees

*That makes <C 3*30 = 90 degrees*

Our triangle has a right angle.

Part B: Each correct answer is worth 4 points.

7) At my school, 50% of students swim. Of those, 30% are on the swim team. What percent of students at my school are on the swim team?

A) 15%
B) 20%
C) 25%
D) 40%
E) 80%

Let's remember that percentages are:
- 100th parts of a whole, 30% = 30/100
- the value of a percentage depends on what it is a percentage of.
We have the total student body S.
Out of those, 50% swim.
That's 50/100 = 1/2.
Now, out of that 1/2, 30% are on the team, which makes 30% x 1/2 = 15%.
We can also do this with decimals:
0.3 x 0.5S = 0.15 = 15%

Answer: A)

Part B: Each correct answer is worth 4 points.

8) Which of the following numbers is odd for every integer n?

A) $2011n$

B) $n^2 + 2011$

C) n^3

D) $n + 2012$

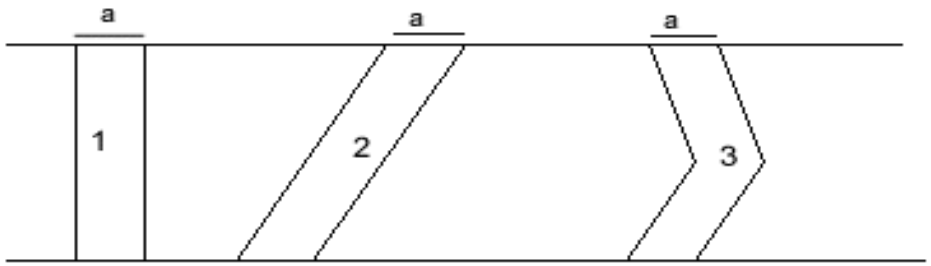
E) $2n^2 + 2011$

What are even/odd numbers?
Even x anything = Even
Odd x Odd = Odd
Odd + Odd = Even
Even + Even = Even
Even + Odd = Odd
If n is even, then A, C and D will be even (even x odd, even cubed, even + even).
If n is odd, then B is even (odd + odd)
In E, $2n^2$ is even, 2011 is odd,
so $2n^2 + 2011$ will be odd for any value of n.

Answer: E)

Part B: Each correct answer is worth 4 points.

9) In the picture, strips 1, 2 and 3 have the same horizontal width a. These strips connect two parallel lines. Which strip has the largest area?



A) All three strips have the same area.

B) Strip 1.

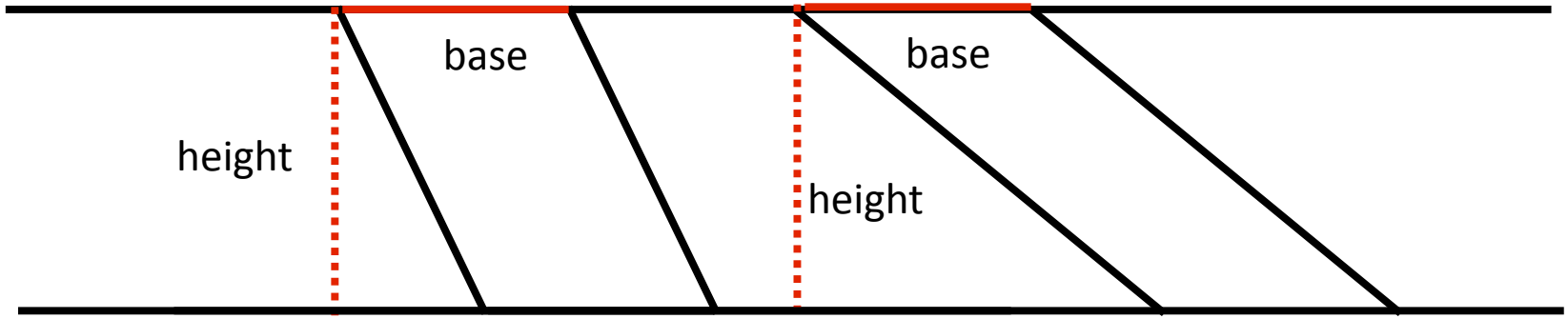
C) Strip 2.

D) Strip 3.

E) It is impossible to answer without knowing a.

Quick Aside

What is the area of a parallelogram?



It's base x height.

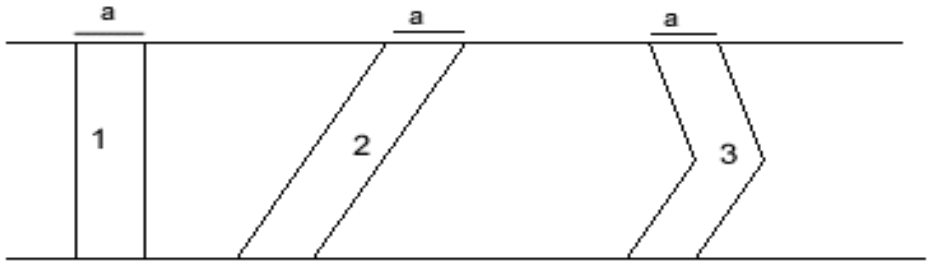
Does it matter how sloped the parallelogram is?

Two parallelograms with the same base and height will have the same area, regardless of their slope.

This applies to rectangles as well, since a rectangle is just a special parallelogram.

Part B: Each correct answer is worth 4 points.

9) In the picture, strips 1, 2 and 3 have the same horizontal width a. These strips connect two parallel lines. Which strip has the largest area?



A) All three strips have the same area.

B) Strip 1.

C) Strip 2.

D) Strip 3.

E) It is impossible to answer without knowing a.

Answer: A)

From our aside, we know that 1 and 2 have the same area.

3 is composed of two smaller parallelograms.

However, the base of either one is still a, and their heights add up to the distance between the parallel lines.

Therefore, the area of 3 is really the same as the areas of 1 and 2.

Part B: Each correct answer is worth 4 points.

10) A is the number 11111...1111 formed by writing 2003 ones in a row. What is the sum of the digits of the product 2003 x A?

A) 10000

B) 10015

C) 10020

D) 10030

E) 2003 x 2003

```
  111111....11111111
                x 2003
  333333....33333333
 0000000....0000000_
0000000....0000000__
2222222....2222222___
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The result of this multiplication will have some 2's, some 3's and some 5's as digits, but overall the sum of its digits would be 2003 2's + 2003 3's, which is 2003 x (2 + 3) = 10015

Answer: B)

Part C: Each correct answer is worth 5 points.

11) When a barrel is 30% empty it contains 30 litres more than when it is 30% full. How many litres does the barrel hold when full?

A) 60

B) 75

C) 90

D) 100

E) 120

If a barrel is 30% empty, how full is it?
It's 100% - 30% = 70% full.
So, a barrel that's 70% full has 30 more litres than a barrel that's 30% full.
Let's say our barrel holds B litres.
We can subtract percent of the same quantity:
70%B - 30%B = 30 litres
This means that 40% of the barrel's capacity is 30 litres.

Just for fun, let's do this with fractions this time:
4/10 B = 2/5 B = 30 l so 1/5 B = 15 l so 5/5 B = B = 15 5 = 75 l*

Answer: B)

Part C: Each correct answer is worth 5 points.

12) In the picture there are four overlapping squares with sides 11, 9, 7 and 5 cm long. How much greater is the sum of the two grey areas than the sum of the two black areas?

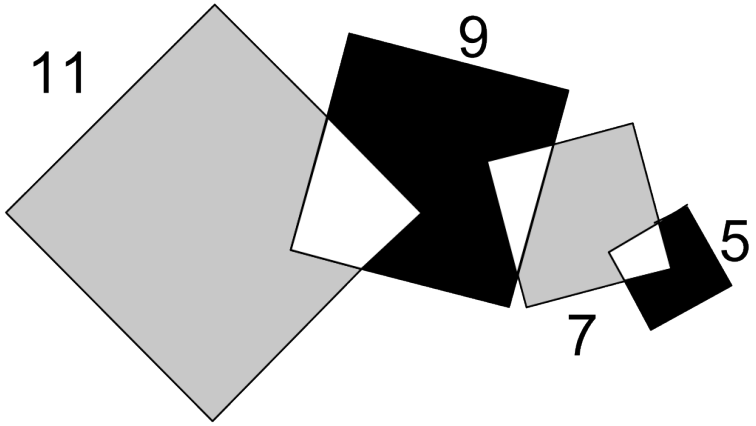
A) 25

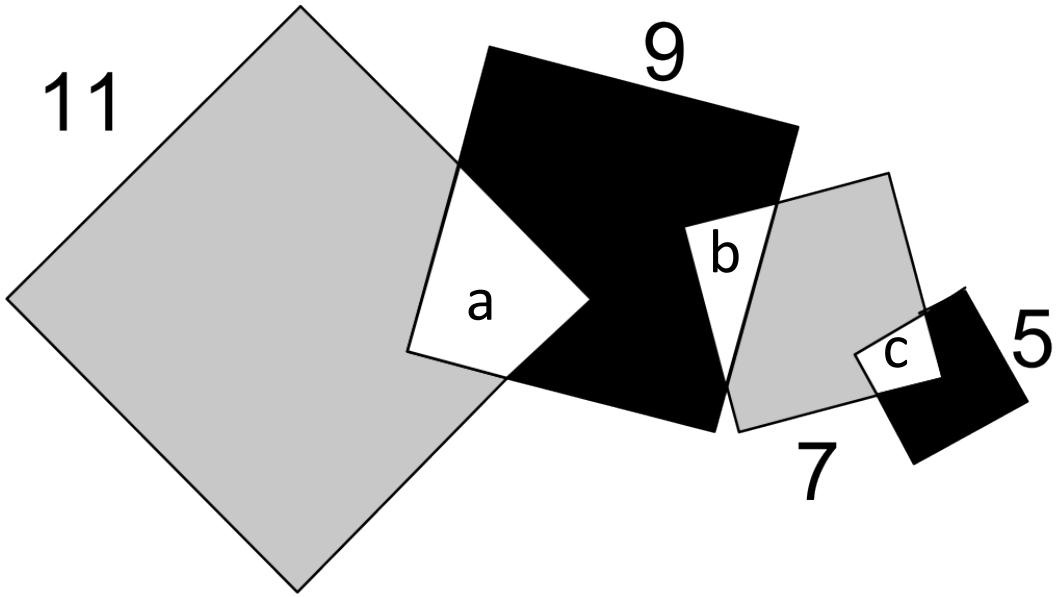
B) 36

C) 49

D) 64

E) 0





We are comparing the grey areas to the black areas.

Let's use letters for the overlapping white areas:

Now let's try to express the grey and black areas in terms of the squares and the overlapping areas:

GREY = 11x11 (big square) - a + 7x7 (third square) - b - c

= 11x11 + 7x7 - a - b - c

BLACK = 9x9 (second square) - a - b + 5x5 (little square) - c

= 9x9 + 5x5 - a - b - c

Without knowing a, b and c we can't know the exact areas, but the question only asks for the difference in areas

Part C: Each correct answer is worth 5 points.

12) In the picture there are four overlapping squares with sides 11, 9, 7 and 5 cm long. How much greater is the sum of the two grey areas than the sum of the two black areas?

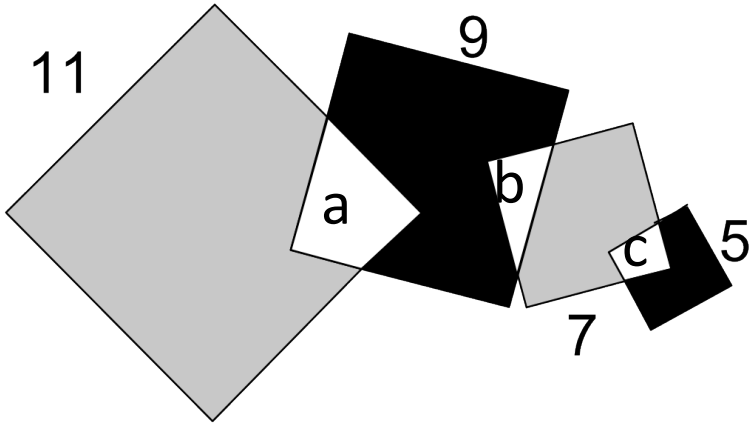
A) 25

B) 36

C) 49

D) 64

E) 0



GREY = 11x11 + 7x7 - a - b - c

BLACK = 9x9 + 5x5 - a - b - c

GREY - BLACK = 11x11 + 7x7 - a - b - c - (9x9 + 5x5 - a - b - c)

= 11x11 + 7x7 - a - b - c - 9x9 - 5x5 + a + b + c

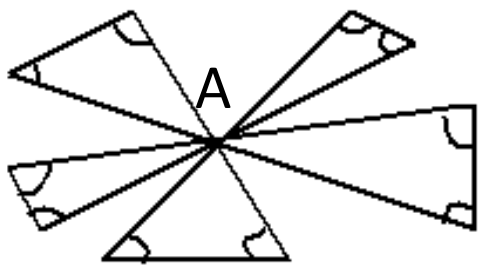
= 121 + 49 - 81 - 25 - a - b - c + a + b + c

= 64

Answer: D)

Part C: Each correct answer is worth 5 points.

13) What is the sum of the 10 angles in the picture?



A) 300^o
B) 450^o
C) 360^o
D) 600°
E) 720^o

The sum of all angles in a triangle is 180^o
The sum of all angles around a point is 360^o

For every angle around the middle point A that belongs to a triangle, there's an equal angle that doesn't.

So, does that mean that the sum of all the angles around A is half of 360^o?

It does indeed. All the angles around A in a triangle add up to 180^o

Now, the sum of the 10 angles we're looking for is equal to the sum of all angles in the 5 triangles minus all the angles at A that belong to a triangle.

$$5 \times 180 - 180 = 4 \times 180 = 720^o$$

Answer: E)

Part C: Each correct answer is worth 5 points.

14) Charles tells the truth every other day; otherwise he lies. Today he stated exactly four of the following statements. Which one could he not have stated today?

A) I have a prime number of friends.
B) I have as many male friends as female.
C) 288 is divisible by 12.
D) I always tell the truth.
E) Three of my friends are older than me.

Either four of these statements are true and one is a lie or
four of these statements are false and one is true.
Since 288 / 12 = 24, C is always TRUE
Since we know that Charles lies half the time, D is always FALSE

A) I have a prime number of friends.

B) I have as many male friends as female.

C) 288 is divisible by 12. TRUE

D) I always tell the truth. FALSE

E) Three of my friends are older than me.

What does B) tell us about the number of Charles' friends?

It tells us that it's even.

But A tells us that the number of Charles' friends is prime. Is this possible?

It is, but only if Charles has exactly 2 friends.

If we assume that Charles is telling the truth today, then since D) is FALSE, every other statement should be TRUE.

Then, from A) and B) it follows that Charles has 2 friends.

But in E) he claims that three of his friends are older, which is not possible if he only has 2 friends.

We reach a contradiction.

Therefore, Charles is lying, and the statement he couldn't have made today was C) 288 is divisible by 12.

Part C: Each correct answer is worth 5 points.

14) Charles tells the truth every other day; otherwise he lies. Today he stated exactly four of the following statements. Which one could he not have stated today?

A) I have a prime number of friends.
B) I have as many male friends as female.
C) 288 is divisible by 12.
D) I always tell the truth.
E) Three of my friends are older than me.

Answer: C)

Part C: Each correct answer is worth 5 points.

15) You have six sticks of lengths 1 cm, 2 cm, 3 cm, 2001 cm, 2002 cm and 2003 cm. You have to choose three of these sticks to form a triangle. How many different choices of three sticks are there that work?

A) 1 B) 3 C) 5

D) 6 E) more than 50

There's a very important property of triangles that deals with the lengths of their sides:

In a triangle, the sum of any two sides has to be greater than the third.

So, to form a triangle with 1, 2, 3, 2001, 2002 and 2003 cm sticks, we have to choose numbers that satisfy that property.

In a triangle, the sum of any two sides has to be greater than the third.
So, to form a triangle with 1, 2, 3, 2001, 2002 and 2003 cm sticks, we have to choose numbers that satisfy that property:

combination	does it work?
1, 2, 3	no, 1 + 2 > 3 is not true
two of (1,2,3) and one of (2001,2002,2003)	no, 2 + 3 > 2001 is not true
2001, 2002, 2003	yes
2001, 2003, 3	yes
2001, 2003 and 2 or 1	no, 2001 + 2 > 2003 is not true
2002, 2003 and 2 or 3	yes
2002, 2003, 1	no, 2002 + 1 > 2003 is not true
2001, 2002, and 2 or 3	yes
2001, 2002, 1	no, 2001 + 1 > 2002 is not true

Part C: Each correct answer is worth 5 points.

15) You have six sticks of lengths 1 cm, 2 cm, 3 cm, 2001 cm, 2002 cm and 2003 cm. You have to choose three of these sticks to form a triangle. How many different choices of three sticks are there that work?

A) 1 B) 3 C) 5

D) 6 E) more than 50

The possibilities are:
2001, 2002, 2
2001, 2002, 3
2002, 2003, 2
2002, 2003, 3
2001, 2003, 3
2001, 2002, 2003

Answer: D)