## Warm-Up 14

191. $\qquad$
192. $\qquad$ Michael will throw three darts that will hit the dartboard shown. His total score will be the sum of the scores for the three hits. How many different total scores could Michael earn?
193. $\qquad$ For what value of $x$ does $2^{18}=\left(\frac{1}{4}\right)^{x}$ ?

194. $\qquad$ $\mathrm{m}^{3}$
195. $\qquad$
m
196. $\qquad$ $\mathrm{cm}^{2}$

A right triangle is formed by the sides of three squares, as shown. The side length of the square labeled $K$ is 7 cm , and the side length of the square labeled $L$ is 2 cm . What is the area of the square labeled J?

197. $\qquad$ Kathy ate one-eighth of the jelly beans in a jar, and Sue ate one-fifth of the rest. Pat ate twice as many jelly beans as Kathy and Sue combined, and then Drew ate the rest. What is the ratio of the number of jelly beans Drew ate to the number of jelly beans Pat ate? Express your answer as a common fraction.
198. $\qquad$ Randolph's favorite positive integers each have three digits and obey the following rules:

- All three digits are different.
- The sum of the digits is 9 .

How many different favorite positive integers could Randolph have?
199. $\qquad$ 1 What is the $y$-intercept of the line that contains the points $(2,0)$ and $(4,-3)$ ? Express your answer as an ordered pair.
200. $\qquad$ The median of an ordered set of 11 integers is 35 . In this set, the median of the first five integers is 29 , and the median of the last five integers is 39 . If the minimum and maximum values of the set are 12 and 52 , respectively, what is the greatest possible value of the mean of this set?

## Workout 7

201. $\qquad$ cm

A rectangular prism has a volume of $720 \mathrm{~cm}^{3}$ and a surface area of $666 \mathrm{~cm}^{2}$. If the lengths of all its edges are integers, what is the length of the longest edge?
202. $\qquad$ Marco's average score for all 14 quizzes this semester was 81 . If Marco had an average score of 75 for his first 10 quizzes this semester, what was the average score for his last 4 quizzes?
203.


Point $D$ is located in the interior of $\triangle A B C$, as shown. What is the ratio of the area of quadrilateral $A C B D$ to the area of $\triangle A B D$ ? Express your answer as a decimal to the nearest hundredth.
204. $\qquad$ The endpoints of a diameter of a circle are $(-3,-2)$ and $(11,-10)$. What are the coordinates of the center of the circle?
205. $\qquad$ Letters have been substituted for digits in the equation ATE + AT = ROYS. Each distinct letter represents a unique digit. No two distinct letters represent the same digit. If T is odd, S is prime and $E>T$, what is the value of the sum $A+T+E+O$ ?
206. $\qquad$ In a survey of ninth-graders, 8 students reported that they have attempted a marathon. Of those 8 students, 2 reported that they actually completed a marathon. If $2.5 \%$ of the ninth-graders surveyed completed a marathon, how many ninth-graders were surveyed?

207. $\qquad$ The line $y=k x$ intersects the line that passes through points $A(5,0)$ and $B(0,2)$ at a point $P$ such that $\mathrm{AP}: \mathrm{PB}=1: 2$. What is the value of $k$ ? Express your answer as a common fraction.
208.


The shape shown is formed by a circle with radius of length $r$ units and two smaller semicircular regions, each with diameter of length $r$ units. What is the perimeter of the shaded portion of the large circle? Express your answer in terms of $r$ and $\pi$.
209. $\$$ $\qquad$ At the first store Arlene visited, she spent $40 \%$ of her money plus $\$ 4$. At the second store she spent $50 \%$ of her remaining money plus $\$ 5$. At the third store Arlene spent $60 \%$ of her remaining money plus $\$ 6$. When Arlene finished shopping at the three stores, she had $\$ 2$ left over. How many dollars did she have when she started shopping?
210. $\qquad$ Judith received a gift on the first day of the year and every sixth day after that. Mort received a gift on the fourth day of the year and every fifth day after that. Kale received a gift on the ninth day of the year and every fourth day after that. On how many days during the year did all three receive a gift on the same day?

## Warm-Up 15

211. $\qquad$ studs
212. $\qquad$ The first term in a sequence is 5 , and each subsequent term in the sequence is the units digit of 2 more than the square of the preceding term. What is the 100th term in the sequence?
213.___studs

Seventy-eight students participate in one or more of three sports: baseball, tennis and golf. Four students participate in all three sports. Five students play only baseball and golf. Two students play only tennis and golf. Three students play only baseball and tennis. If seven students play only tennis, and one student plays only golf, what is the total number of students who play only baseball?
214. $\qquad$ The lines in the grid shown indicate "sight lines." A soldier standing at one location can see a soldier standing at another location only if the soldiers are on the same sight line. Two soldiers are located at C1 and F2, respectively. What are the locations of two other soldiers if none of the four soldiers are able to see any of the others?

215. $\qquad$ A right triangle has legs of length $\frac{3}{2}$ units and $\frac{20}{3}$ units. If the triangle has a perimeter of $x$ units and an area of $y$ units ${ }^{2}$, what is the value of $x^{2}-y^{2}$ ?

What is the coefficient of $x^{2} y^{2} z^{2}$ in the expansion of $(x+y+z)^{6}$ ?
217. $\qquad$ A two-digit positive integer is randomly selected. What is the probability that the units digit is a multiple of the tens digit? Express your answer as a common fraction.
218. $\qquad$ In Quaternion, the coin with the least value is the qua. Four quas equal one quab, four quabs equal one quac, four quacs equal one quad, four quads equal one quae and four quaes equal one quaf. What is the least number of coins that have a combined value of 2012 quas?

219. $\qquad$ How many different pairs of numbers $(m, n)$ can be formed using numbers from the list of integers $\{1,2,3, \ldots, 99,100\}$ such that $m<n$ and $m+n$ is even?


## Warm-Up 16

221. $\qquad$

How many positive two-digit integers each have the property that the integer's value increases by $75 \%$ when its digits are reversed?
222. $\qquad$ The square pyramid shown has a volume of $64 \mathrm{~cm}^{3}$. A plane parallel to the base of the pyramid bisects its altitude and divides the pyramid into two sections, as shown. What is the volume of the smaller pyramid?

223. $\qquad$ If $x+\frac{4}{x}=y+\frac{4}{y}$ and $x \neq y$, then what is the value of the product $x y$ ?
224.


A large square is divided into four congruent squares. Then those four squares are each divided into four smaller congruent squares, some of which are shaded, as shown. What is the probability that a dart thrown at random that lands in the largest square will also land in a shaded region? Express your answer as a percent to the nearest tenth.
225. $\qquad$ A 4-person dance team composed of 2 boys and 2 girls is to be selected from a group of 8 girls and 11 boys. How many different dance teams are possible?
226. $\qquad$ Mrs. Garcia allowed each student in her class to drop the lowest of their five test scores. When Matt dropped the lowest of his test scores, a 60, his test average increased by 5 points. What is Matt's new test average?
227. $\qquad$ What is the maximum number of $3^{\prime \prime} \times 1^{\prime \prime} \times 1^{\prime \prime}$ blocks that will fit into a box with interior dimensions of $5^{\prime \prime} \times 5^{\prime \prime} \times 10^{\prime \prime}$ ?
228. $\qquad$ Using five identical index cards, the following structure can be created on a flat, level surface. What is the sum of the two marked angles on the side view of this construction?

229. $\qquad$ A cruise ship must average $22 \mathrm{mi} / \mathrm{h}$ for 10 hours to make its next port on schedule. During the first 4 hours, bad weather caused it to reduce its speed to $16 \mathrm{mi} / \mathrm{h}$. What should its speed be for the remainder of the trip to make it to the next port on schedule?
230. $\qquad$


Ms. Robinson gives the following homework assignment: You must write an essay by tomorrow. You may work alone or in boy-girl pairs. As it turned out, $\frac{2}{3}$ of the boys and $\frac{3}{5}$ of the girls worked in pairs. What portion of the class worked alone? Express your answer as a common fraction.

## Workout 8

231. $\qquad$


Beginning January 1st, Marte will put $\$ 100$ in her savings account at the start of every month. The bank pays $4.8 \%$ annual interest compounded monthly ( $0.4 \%$ each month) at the end of each month. Assume the bank rounds the interest amount to the nearest cent. How much money will Marte have in her account at the end of the year? Express your answer to the nearest cent.
232. $\qquad$ The number of units in the length of the line segment with endpoints $(-4,-5)$ and $(4,2)$ is a value between two consecutive positive integers. What is the sum of those two integers?
233. $\qquad$ What is the 2012th digit after the decimal point in the decimal expansion of $\frac{8}{81}$ ?
234. $\qquad$


Tito's bill was $\$ 19.50$ before the $6 \%$ sales tax was added. Tito wants to leave a tip of at least $20 \%$ of the bill amount before tax is added. What is the least amount he should pay for the bill, tax and tip?
235. $\qquad$ The deck used in the card game Krypto contains three each of cards numbered 1 to 10, two each of cards numbered 11 to 17 , and one each of cards numbered 18 to 25 . When five cards are dealt from the deck, what is the probability that all five cards will be 10 or less? Express your answer as a decimal to the nearest thousandth.

236. $\qquad$ If $5 \sqrt{x}-30=2 \sqrt{x}+54$, what is the value of $x$ ?
237. $\qquad$ ft A rectangular prism has dimensions in the ratio of 1:2:3. If the shortest edge is 2 ft , what is the longest distance between any two vertices? Express your answer as a decimal to the nearest hundredth.
238. $\qquad$ $\mathrm{cm}^{2}$


An isosceles triangle has sides of length $13 \mathrm{~cm}, 13 \mathrm{~cm}$ and 10 cm . If a square is inscribed in the triangle, as shown, what is the area of the shaded region? Express your answer as a decimal to the nearest hundredth.
239. $\qquad$ If $\frac{x^{2}+a x+6}{x+1}=x+b$ for all positive values of $x$, what is the value of $6 a-7 b$ ?
240. $\qquad$ ft A 6-ft ladder is leaning against a wall so that the base of the ladder makes a $45^{\circ}$ angle with the floor. The base of the ladder is then pushed in toward the wall so that its distance from the wall is one-third of what it was originally. In feet, how much farther up the wall does the top of the ladder reach? Express your answer as a decimal to the nearest hundredth.


## Warm-Up 17

241. $\qquad$ Each of the letters $A, B, C, D, E, F$ and $G$ in the figure shown will be replaced with a different value from the set $\{1,2,3,4,5,6,7\}$. If the sums of the four numbers in each of the three quadrilaterals are each 15 , what is the sum of all numbers that can replace $A$ ?

242. $\qquad$


Square $A B C D$, shown here, has side length 8 units and is divided into four congruent squares. One of these squares contains an inscribed circle, two other squares contain diagonals and the fourth square has perpendicular line segments drawn from the midpoints of adjacent sides to form a square in the interior. In square units, what is the total area of the shaded regions? Express your answer in terms of $\pi$.
243. $\qquad$ Five integers form an arithmetic sequence with a mean of 18 . If the mean of the squares of the five integers is 374 , what is the largest of the five original integers?
244. $\qquad$ The prime factorization of 75 is written, without exponents, as $3 \times 5 \times 5$. The sum of the prime factors is $3+5+5=13$. For how many whole numbers is the sum of each number's prime factors, without exponents, equal to 13 ?
245. $\qquad$ Xavier gave Yvonne and Zeena the same number of marbles as each already had. Then Yvonne gave Xavier and Zeena the same number of marbles as each already had. Then Zeena gave Xavier and Yvonne the same number of marbles as each already had. At that point, each person had 48 marbles. How many fewer marbles did Xavier have at the end than he had at the start?
246. \$ $\qquad$


The ratio of Barbara's cell phone bill to Tina's cell phone bill was 7:5. Barbara's bill was $\$ 14$ more than Tina's bill. How much was Barbara's bill?
247. $\qquad$ In how many ways can four different positive integers be placed, one per box, so the sum of the integers is 13 ?

248. $\qquad$ Three numbers have a sum of 5 and the sum of their squares is 29 . If the product of the three numbers is -10 , what is the least of the three numbers? Express your answer in simplest radical form.
249. $\qquad$ A box contains only quarters and dimes. If there were $10 \%$ more quarters, the total value of the money in the box would increase by $7.5 \%$. What is the ratio of the number of quarters to the number of dimes in the box? Express your answer as a common fraction.

250. $\qquad$ In trapezoid $A B C D$, angles $A$ and $D$ are right angles, and $A B=4$ units, $D C=16$ units and $B C=15$ units. The trapezoid is revolved $360^{\circ}$ around side $A D$ to form the frustum of a cone. What is the volume of this frustum? Express your answer in terms of $\pi$.

## Warm-Up 18

251. $\qquad$ There are 240 pairs of numbers such as $\{7,5318\}$ or $\{17,358\}$ that can be formed using each of the digits $1,3,5,7$ and 8 exactly once. What is the largest possible product of two such numbers?
252. $\qquad$ ft If a rectangle with an area of $400 \mathrm{ft}^{2}$ is divided into two smaller rectangles with areas in the ratio of $3: 1$, what is the least possible value of the perimeter of the smaller rectangle?
253. $\qquad$ A set of seven different positive integers has a mean of 13 . What is the positive difference between the largest and smallest possible values of its median?
254. $\qquad$ When written in a certain positive base $b, 363$ (base 10) is 123 (base $b$ ). What is the value of $b$ ?
255. $\qquad$ units ${ }^{2}$

Rhombus $E F G H$ is inscribed in rhombus $A B C D$ with point $E$ on $\overline{A B}$, point $F$ on $\overline{B C}$, point $G$ on $\overline{C D}$ and point $H$ on $\overline{A D}$. If $A E: E B=B F: F C=C G: G D=D H: H A=1: 2$, and if the area of rhombus $A B C D$ is 180 units $^{2}$, what is the area of rhombus EFGH?

256. $\qquad$ The solutions $x=u$ and $x=v$ of the quadratic equation $r x^{2}+s x+t=0$ are reciprocals of the solutions of the quadratic equation $(2+a) x^{2}+5 x+(2-a)=0$ for some integer $a$. If the GCF of $r, s$ and $t$ is 1 , what is the value of $r+s+t$ ?
257. $\qquad$ A club with 22 students is forming a committee of either 4 or 5 students. What is the ratio of the number of different 4-person committees possible to the number of different 5 -person committees possible? Express your answer as a common fraction.
258. $\qquad$ Quadrilateral ABCD has vertices at points $\mathrm{A}(-9,4), \mathrm{B}(-7,8), \mathrm{C}(-3,6)$ and $\mathrm{D}(-6,2)$. Quadrilateral WXYZ is congruent to quadrilateral ABCD and has vertices $W(2,-3), X(4,1)$ and $Y(8,-1)$ and a fourth vertex, $Z$. What is the sum of the coordinates of vertex $Z$ ?

259. $\qquad$ A right triangle has integer side lengths $a, b$ and $c$ with $a<b<c$. If $a+c=49$, what is the area of the triangle?


The cube with vertices A, B, C, D, E, F, G, H has edges of length 2 units. Point $M$ is the midpoint of $\overline{E H}$ and $N$ is the midpoint of $\overline{E F}$. What is the volume of the tetrahedron with vertices A, C, M, N? Express your answer as a common fraction.

## Workout 9

261. $\qquad$ units ${ }^{3}$ A 6-8-10 triangle is revolved about the side of length 10 units. What is the volume of the resulting solid? Express your answer as a decimal to the nearest tenth.
262. $\qquad$ Some students in a gym class are wearing blue jerseys, and the rest are wearing red jerseys. There are exactly 25 ways to pick a team of 3 players that includes at least 1 player wearing each color. How many students are in the class?

263. $\qquad$ \% Suppose that the price of game tickets has gone up 4\% every year for the past 25 years. What percent of today's game ticket price is the price of a game ticket 13 years ago? Express your answer to the nearest whole number.
264. $\qquad$ The three sides of a right triangle form a geometric progression. What is the ratio of the length of the shortest side to the length of the hypotenuse? Express your answer as a decimal to the nearest hundredth.
265. $\qquad$ The radii of circles $C$ and $D$ are each 3 units long. The two circles are tangent at point $E$. Side $A B$ of rectangle $A B C D$ is tangent to circle $D$ at point $A$ and tangent to circle $C$ at point $B$. What is the area of the shaded region? Express your answer as a decimal to the nearest tenth.

266. $\qquad$ What is the smallest positive integer that is greater than 100 and leaves a remainder of 1 when divided by 3 , a remainder of 2 when divided by 5 and a remainder of 3 when divided by 7 ?
267. $\qquad$


Trapezoid MPRS has $\overline{\mathrm{MP}}$ and $\overline{\mathrm{RS}}$ as bases and $\overline{\mathrm{MS}} \perp \overline{\mathrm{SR}}$. Additionally, $\mathrm{MP}=\mathrm{MS}=2$ units, and $\mathrm{RS}=3$ units. If point Q lies on $\overline{\mathrm{SR}}$ such that $\overline{\mathrm{PQ}}$ bisects the area of trapezoid MPRS, what is the length of $\overline{\mathrm{PQ}}$ ? Express your answer as a decimal to the nearest tenth.
268. $\qquad$ The mean of a set of five different positive integers is 21 . How many values are possible for the median of this set of positive integers?
269. $\qquad$
hours
One candle burns entirely in 6 hours, and another burns entirely in 9 hours. The candles were lit at the same time, and they were the same height when lit. After how many hours will one candle be half the height of the other? Express your answer as a decimal to the nearest tenth.

270. $\qquad$ For years, Mrs. Meany has had a tradition of giving her students a list of books from which they had to choose two to read during winter break. This year, Mrs. Meany added $n$ book(s) to the list. If there are now 99 more ways that two books can be selected, what is the sum of all possible values of $n$ ?

For problems 271-276, the Pythagorean Theorem or a knowledge of Pythagorean triples can be used to determine the value of $x$ in each figure. For each figure, provide the exact value of $x$ (as an integer or in simplest radical form). If $x$ is not an integer value, also provide the value of $x$ expressed as a decimal to the nearest tenth.
271. $\qquad$ units
72. $\qquad$
273. $\qquad$
271.

274. isosceles triangle


275. right rectangular prism

273.

276.


For problems 277-280, a knowledge of 30-60-90 or 45-45-90 triangles can be used to determine the value of $x$ in each figure. For each figure, provide the exact value of $x$ (as an integer or in simplest radical form). If $x$ is not an integer value, also provide the value of $x$ expressed as a decimal to the nearest tenth.
277.
278. $\qquad$
279. $\qquad$
280. $\qquad$
277. square

279. regular hexagon inscribed in a circle

278. equilateral triangle



For problems 281-282, a knowledge of the properties of similar triangles and right triangles can be used to determine the value of $x$ and $y$ in each figure. If $x$ and/or $y$ is not an integer value, provide the value expressed as a decimal to the nearest tenth.

## 281. $x=$ <br> units

$\qquad$
282.
$x=$ $\qquad$
$y=\quad$ units
281.

282.


## Sequences Stretch

283. $\qquad$ What is the 100th term of the arithmetic sequence $3,11,19,27, \ldots$ ?
284. $\qquad$ What is the sum of the first 100 terms of the arithmetic sequence $3,11,19,27, \ldots$ ?
285. $\qquad$ What is the 10th term of the geometric sequence $729,243,81,27, \ldots$ Express your answer as a common fraction.
286. $\qquad$ The 1st and 18th terms of an arithmetic sequence are 4 and 8.25 , respectively. What is the 35th term of the sequence? Express your answer as a decimal to the nearest tenth.
287. $\qquad$ The first three terms of an arithmetic sequence are $p, 2 p+6$ and $5 p-12$. What is the 4th term of this sequence?
288. $\qquad$ All terms in a geometric sequence are positive integers, and the first three terms are $n$, $n+3$ and $2 n+6$. What is the 4 th term of this sequence?
289. $\qquad$ The 3 rd term of an arithmetic sequence is 17 , and the 9 th term is 83 . What is the 1 st term?
290. $\qquad$ The 2 nd term of a geometric sequence is 24 , and the 5 th term is 81 . What is the 1 st term?
291. $\qquad$ The 6th term of an arithmetic sequence is 24. What is the sum of the 5 th and 7 th terms?
292. $\qquad$ cells The number of bacterial cells within a Petri dish doubles every hour. If there are 8 cells in the dish at the end of the 2nd hour, how many cells will be in the dish at the end of the 8 th hour?

## Similarity Stretch

Two geometric figures are similar if all of their corresponding angles are congruent and all of their corresponding sides are proportional. This means that the figures have the exact same shape but not necessarily the same size. For two triangles to be similar, it is sufficient to know that two pairs of corresponding angles are congruent.
293. $\qquad$
294. $\qquad$ units

The two pentagons shown here are similar, with the side of length 4 in the smaller pentagon corresponding to the side of length 8 in the larger pentagon, and with the indicated lengths given. What is the perimeter of the smaller pentagon?


In the figure below, $\overline{\mathrm{RS}}, \overline{\mathrm{VW}}$ and $\overline{\mathrm{UT}}$ are parallel. If $\mathrm{RS}=3, \mathrm{VW}=12, \mathrm{UT}=48$ and $\mathrm{XW}=10$, what is the length of $\overline{Y T}$ ?

296. $\qquad$


Chords KM and NL of the circle shown intersect at point P. If $K N=8, P M=6$ and $L M=10$, what is the length of PN? Express your answer as a decimal to the nearest tenth.
297. $\qquad$ Given that $\overline{\mathrm{UV}}$ and $\overline{\mathrm{XZ}}$ are parallel and that $\mathrm{YV}=3$ and $\mathrm{YZ}=5$, what is the ratio of the area of $\Delta U Y V$ to the area of trapezoid UVZX? Express your answer as a common fraction.

298. $\qquad$ units ${ }^{2}$

Trapezoid ABCD has right angles at A and D, and diagonals AC and BD intersect at point E. The area of $\triangle A B E$ is 25 units $^{2}$, and the area of $\triangle D E C$ is 49 units $^{2}$. If $A D=6$, what is the area of trapezoid ABCD?
299. $\qquad$ In the figure shown, the largest cone has been divided into a smaller cone and two frustums by two planes that trisect the altitude of the original cone. What is the ratio of the volume of the smaller frustum to the volume of the larger frustum? Express your answer as a common fraction.

300. $\qquad$ $\mathrm{cm}^{2}$

The two toy houses shown here are similar. If the volume of the larger one is $1000 \mathrm{~cm}^{3}$ and the volume of the smaller one is $216 \mathrm{~cm}^{3}$, what is the surface area of the smaller house if the larger one has a surface area of $400 \mathrm{~cm}^{2}$ ?

301. $\qquad$ ft解 strung from the top of each pole to the base of the other pole. How high above the ground do the two wires cross?

302. $\qquad$ ft

Two vertical poles are $a \mathrm{ft}$ and $b \mathrm{ft}$ tall, respectively. A wire is strung from the top of each pole to the base of the other pole. How high above the ground do the two wires cross? Express your answer as a common fraction in terms of $a$ and $b$.


# What About Math? American Meteorological Society 

Lightning is the electrical discharge in a thunderstorm. It results in both a visible flash and the heating of air, causing a sonic boom commonly referred to as thunder. The speed of light is so fast that one may consider seeing the flash as instantaneous with the lightning. The speed of sound is slower, about 340 meters per second, or 1100 feet per second, so counting the seconds from the lightning flash to the lightning bang (thunder) gives the distance to the thunderstorm, based
 on 3 seconds per kilometer, or 5 seconds per mile.

1. $\qquad$ km Using the flash-to-bang method based on $3 \mathrm{sec} / \mathrm{km}$, what is the distance, in kilometers, to a thunderstorm if the time elapsed between the flash and the bang is 5 sec? Express your answer as a decimal to the nearest tenth.
$\qquad$
2. mi thunderstorm if the time elapsed between the flash and the bang is 12 sec? Express your answer as a decimal to the nearest tenth.

Rotating atmospheric storm systems, called cyclones in general, have centers of lower pressure about which the air swirls. In the tropics, these are tropical cyclones. At middle latitudes, they are extra-tropical cyclones and are associated with fronts - boundaries of changing temperatures. Cyclones, tropical cyclones and extra-tropical cyclones are all three-dimensional flows of air with stronger horizontal winds but also upward motions that result in their cloud patterns.

## 3. <br> $\qquad$

In a typical extra-tropical cyclone, the vertical motion of air is typically only $1 \%$ to $10 \%$ of the horizontal wind speed. If the horizontal wind in a cyclone is blowing at a rate of $20 \mathrm{~km} / \mathrm{h}$, what is the positive difference between the greatest and least typical values for the vertical air motion? Express your answer as a decimal to the nearest tenth.


Storms in the tropical regions do not have fronts and temperature changes. Instead they are organized thunderstorm masses about a center of lower air pressure. As these thunderstorm systems develop, they grow and are categorized by their associated wind speeds. When maximum sustained winds reach $37 \mathrm{~km} / \mathrm{h}$, the system is termed a tropical depression. Sustained winds of $63 \mathrm{~km} / \mathrm{h}$ denote a tropical storm. A storm with maximum sustained winds reaching $119 \mathrm{~km} / \mathrm{h}$, by definition, is a hurricane (referred to as a typhoon in the western Pacific Ocean, or cyclone in the Indian Ocean and southern Pacific Ocean). Wind pressure is the force per unit area caused by air in motion. During a storm the factor by which the wind pressure increases is equal to the square of the factor by which the wind speed increases.
4. $\qquad$ Suppose that a tropical depression with a maximum sustained wind speed of $50 \mathrm{~km} / \mathrm{h}$ intensifies to hurricane strength with a maximum sustained wind speed of $150 \mathrm{~km} / \mathrm{h}$. By what factor does the wind pressure increase?

Density is the mass of material in a unit volume. For example, 1 g of fresh water fills $1 \mathrm{~cm}^{3}$, so the density of fresh water is $1 \mathrm{~g} / \mathrm{cm}^{3}$. For comparison, the density of cork is $0.25 \mathrm{~g} / \mathrm{cm}^{3}$ and iron has a density of $7.9 \mathrm{~g} / \mathrm{cm}^{3}$. The general rule is that 10 cm of fresh snow melts to become 1 cm of liquid water. Hence, the average density of fresh snow is considered to be $0.1 \mathrm{~g} / \mathrm{cm}^{3}$.
5. $\qquad$ What is the density of snow if the ratio of snowfall depth to melted water depth is 3 to 1 ? Express your answer as a decimal to the nearest hundredth.
$\qquad$ If 10 in of freshly fallen snow melts down to 1 in of liquid water, what is the liquid water equivalent depth of 30 cm of fresh snow?

The atmosphere can be arbitrarily divided into layers based on the vertical temperature pattern. The lowest layer is called the troposphere, where the temperature generally decreases with altitude from Earth's surface up to about 11 km .
7. $\qquad$ ${ }^{\circ} \mathrm{F}$ If the air temperature within the troposphere drops $3.5^{\circ} \mathrm{F}$ for every $1000-\mathrm{ft}$ increase in altitude, what is the air temperature outside an airplane cruising at $18,000 \mathrm{ft}$ if the temperature at ground level (sea level) is $70^{\circ} \mathrm{F}$ ?

Earth's atmosphere consists mainly of nitrogen and oxygen. These gases do not react with most radiation, thus transmitting almost 100\%. Some gases transmit some types of radiation, such as visible light, but absorb other portions, particularly infrared radiation. Gases that absorb infrared (heat) radiation from Earth outward to space, of which the atmosphere in turn can emit part back down to the surface, help warm the planet. This is called the greenhouse effect. Such gases are called greenhouse gases or GHG. The major GHG (although minor components of the atmosphere) are water vapor and carbon dioxide ( $\mathrm{CO}_{2}$ ), as well as methane, nitrous oxide, chlorofluorocarbons and others. Rising concentrations of $\mathrm{CO}_{2}$ pose a problem in an enhanced greenhouse effect.
$\qquad$ In 2004, the average annual concentration of atmospheric carbon dioxide (a GHG) was 377.6 parts per million by volume (ppmv), and the annual rate of increase was about 1.5 ppmv . At this rate of increase, how many years would it take the 2004 level of $\mathrm{CO}_{2}$ to double? Express your answer as a decimal to the nearest tenth.
$\qquad$ Suppose the rate of annual increase were to average 3.0 ppmv. How many years would it take the 2004 level of $\mathrm{CO}_{2}$ to double? Express your answer as a decimal to the nearest tenth.

All objects including the Sun and Earth emit radiation. Depending on their temperature, the radiation has varying characteristics and differing names. Earth emits mainly infrared radiation (heat) outward to space. The Sun emits mostly visible light into space but also some ultraviolet and infrared, a small portion of which strikes Earth. Radiation that goes to Earth and encounters matter may be absorbed to become heat, may be reflected or
 scattered away, or may be transmitted to the Earth's surface without change.
10. $\qquad$ \%

Consider what happens to a solar beam traveling through the atmosphere on a particular day. If $15 \%$ of the radiation is absorbed and $20 \%$ is scattered, or reflected back to space mostly by clouds, what percent of the solar radiation is actually transmitted to Earth's surface?

# ANSWERS TO HANDBOOK AND WHAT ABOUT MATH? PROBLEMS 

In addition to the answer, we have provided a difficulty rating for each problem. Our scale is 1-7, with 7 being the most difficult. These are only approximations, and how difficult a problem is for a particular student will vary. Below is a general guide to the ratings:

Difficulty 1/2/3-One concept; one- to two-step solution; appropriate for students just starting the middle school curriculum.
4/5 - One or two concepts; multistep solution; knowledge of some middle school topics is necessary. 6/7-Multiple and/or advanced concepts; multistep solution; knowledge of advanced middle school topics and/or problem-solving strategies is necessary.

| Answer | MarmaUp |  |  |
| :---: | :---: | :---: | :---: |
|  | Difficulty |  |  |
| 1. 420 | (1) | 6. 64 | (3) |
| 2. 3 | (2) | 7. 480 | (4) |
| 3. 69 | (2) | 8. $5 / 8$ | (3) |
| 4. 5 | (2) | 9. 6 | (3) |
| 5. 40 | (3) | 10. 3 | (2) |
|  | YM |  |  |
| Answer | Difficulty |  |  |
| 11. 4 | (1) | 16. 35 | (3) |
| 12. 120 | (3) | 17. 60 | (3) |
| 13. 300 | (3) | 18. 152 | (3) |
| 14. 36 | (3) | 19. 48 | (3) |
| 15. 1500 or | (2) | 20. 12 | (4) |
| 1500.00 |  |  |  |

## Workout 1

Answer
21. 35
22. 2.80
23. 27
24. 20
25. 54

## Difficulty

(2)
26. 812
(3)
(3)
(3)
(4)
. 0.51
(2)
(4)
(2)
(4)
(3)
27. 12.65
28. 12
29. 6

## Warm-Up 3

## Answer

31. 36
32. 4
33. 1067
34. 400
35. $3 / 4$
(3)
(1)
(3)
(3)
(3)
)
36. 8

## Warm-Up 4

## Answer Difficulty

| 41. triangle or $\triangle$ | $(2)$ | 46. 108 |
| :--- | :--- | :--- | :--- |
| 42. 104 | (3) | 47. 15 |
| 43. 2 | $(2)$ | 48. 18 |
| 44. $1 / 5$ | $(2)$ | 49. 13 |
| 45. $14 / 95$ | $(4)$ | 50. 23 or 23.00 |

## Workout 2

## Answer

51. 16.39
52. 1320
53. 3
54. $343 / 4096$
55. 13
.

Difficulty
(2) 56.20
(3)
(2)
(4)
(2)

57. 38
58. 13
59. $2 / 25$
60. 7.75

| Answer |  |  |  |
| :--- | :---: | :--- | :--- |
| 61. 66 | Difficulty |  |  |
| 62. $4 \sqrt{ } 3$ | $(2)$ | 66.5 | $(3)$ |
| 63. -256 | $(2)$ | 67.6 | $(4)$ |
| 64. $6 \pi$ | $(4)$ | 69.75 | (3) |
| 65. 24 | $(3)$ | 70.40 | (4) |

## Warm-Up 6

## Answer

71. 20
72. $16 \pi$
73. $15 / 56$
74. 6
75. 0

Difficulty
(4)
(3)
(2)
(4)
(5)
76. 64
77. 82
78. $-2 \frac{1}{2}$
79. $1^{*}$ or 1.00
80. 8

## Workout 3

## Answer

81. 380
82. 30
83. 288
84. 12.50
85. 14.072

Difficulty
(2) 86. 66
(2)
(3)
(4)
(4)
4)

89. 60
90. 9
(5)
(4)
(3)
(3)
(5)
(4)
(3)
(3)
(3)
(4)

## Answer

## Warm-Up 7

Difficulty
91. 960 or 960.00 (1)
92. 2
93. 12
94. $7 / 15$
95. 22
(3) 100. $s^{2}-2 s b$
96. 25 or 25.00
(2)
(3) $97 . \quad 16$
(1) $\quad 98.18$
(3) $\quad 99.1 / 4$
or $s^{2}-2 b s$

## Warm-Up 8

## Answer

101. 400
102. 35
103. 4
104. 45
105. 6

Difficulty
(3)
(2)
(4)
(4)
110. 52

[^0]| Answer | Mifficulty <br> 121. | 126. 44 | (4) |
| :--- | :--- | :--- | :--- |
| 122. 24 | $(3)$ | 127. $1 / 16$ | (3) |
| 123. 390 | $(3)$ | 128. $V(89)$ | (4) |
| 124. 216 | $(3)$ | 129. 72 | (2) |
| 125. 115 | $(5)$ | 130. $(-3,6)$ | (4) |

121. H
122. 24
123. 390
124. 216
125. 115

Answer
131. 24
132. 1.25
133. 48
134. 14
135. 90

Answer
141. 6

Difficulty
(2)
(4)
(5)
(5)
(4)
150. 35
146. 2771.3
(5)
(4)
(5)
(5)
(5)
147. 5
148. 39
149. 512

Answer
151. 13
152. $5 / 18$
153. $1 / 4$
154. 162
155. 80

## Warm-Up 10 <br> Difficulty

(2)
(2)
(3)
(3)
(3)
140. 42

## Workout 5

## Answer

 161. 100 162. $3: 45$163. 36
164. 141
165. 20

## Answer

171. 78
172. 30
173. 18.50
174. 52.6
175. 17

## Warm-Up 11

Difficulty
(4)
156. $1 / 3$
(4)
(4)
157. 17
(5)
158. 3
(5)
(4)
(5)

## Warm-Up 12

Difficulty
(2)
(4)
(3)
(4)
170. V(73)

## Workout 6

Difficulty
(3)
176. 11.3
(5)
(3)
(4)
(4)
177. 2
178. 33/50
179. 12
180. 80

| Marmivo 3 |  |  |
| :---: | :---: | :---: |
| Answer | Difficulty |  |
| 181. 7/18 | (4) | 186. 4536 |
| 182. 60 | (4) | 187. $256 \pi$ |
| 183. $25 / 24$ | (2) | 188. 40 |
| 184. $2 a+b$ | (5) | 189. $1 / 126$ |
| 185. 24 | (3) | 190. 100 |

## Warm-Up 14

Answer
191. 4
192. 10
193. -9
194. 40
195. 18

## Difficulty

(4)
(3)
(4)
(5)
(6)
200. 36
(4)
(6)
(4)
(5)
(4)

|  |  |  |
| :--- | :---: | :--- |
| Answer |  |  |
| 191. 4 | Difficulty <br> $(4)$ | 196. 45 |
| 192. 10 | $(3)$ | 197. $1 / 6$ |
| 193. -9 | $(4)$ | 198. 34 |
| 194. 40 | $(5)$ | 199. 10,3$)$ |
| 195. 18 | $(6)$ | 200. 36 |

(4)
(4)
(4)
(4)
(5)

## Workout 7

## Answer

201. 16
202. 96
203. 0.73
204. (4, -6)
205. 16

## Difficulty

(6)
(4)
(6)
(3)
(5)

210. 6
206. 80
207. $1 / 5$
208. $2 \pi r$
209. 90 or 90.00
(3)
(5)
(5)
(5)
(5)
)
)
)
)
)

| 211. | 325 |
| :--- | :--- |
| 212. | 3 |
| 213. | 56 |
| 214. | E5, B4 |
| or B4, E5 |  |
| 215. | 200 |

## Warm-Up 15

## Answer

Difficulty
(3)
216. 90
(5)
(4) 217. 16/45
(4) 218. 11
(4)
(3)
219. 2450
(4)
220. 7
(6)
(5)

## Warm-Up 16

## Answer

221. 4
222. 8
223. 4
224. 37.5
225. 1540

Difficulty
(5)
(5)
(6)
(3)
(5)
)
230. $7 / 19$

## Workout 8

## Answer

| 231. | 1231.66 |
| :--- | :--- |
| 232. | 21 |
| 233. | 6 |
| 234. | 24.57 |
| 235. | 0.055 |

231. 1231.66
(4)
(5)
(4)
(3)
(5)

Difficulty
239. 0
240. 1.59
(4)
(4)
(6)
(6)
(4)

| Answer |  |  |  |
| :---: | :---: | :---: | :---: |
|  | Difficulty |  |  |
| 241. 4 | (6) | 246. 49 or 49.00 | (4) |
| 242. $44-4 \pi$ | (4) | 247. 72 | (5) |
| 243. 28 | (5) | 248. - V 2 | (7) |
| 244. 9 | (4) | 249. 6/5 | (6) |
| 245. 30 | (4) | 250. $1008 \pi$ | (6) |
|  | Merツinvore |  |  |
| Answer | Difficulty |  |  |
| 251. 62,333 | (5) | 256. 9 | (7) |
| 252. 40 | (4) | 257. 5/18 | (4) |
| 253. 15 | (5) | 258. 0 | (5) |
| 254. 18 | (5) | 259. 210 | (6) |
| 255. 100 | (5) | 260. $4 / 3$ | (7) |

## Answer

261. 241.3
262. 
263. 60
264. 0.62
265. 3.9

Workout 9

Difficulty
(5)
(6)
(4)
(6)
(4)
(4)
(6)
(4)
(5)
(6)

## Right Triangles Stretch

271. V97 $\approx 9.8$ (2)
272. 12
273. $\sqrt{ } 39 \approx 6.2$
274. 12
275. $30 \sqrt{ } 2 \approx 42.4$
276. 9

9
(4)
277. $5 \mathrm{~V} 2 \approx 7.1$
278. 8
279. $4 \sqrt{ } 3 \approx 6.9$
280. $3 \sqrt{ } 2 \approx 4.2$
281. $x=102$
$y=16$

$$
\text { 282. } x=3.6
$$

$$
y=4.8
$$

## Sequences Stretch

## Answer

283. 795
284. 39,900
(5)
285. -5
286. 16
287. 12.5
(4)
288. 48
289. 66
(5)
290. 512

## Answer

293. 9
294. 13

## Similarity Stretch

## Difficulty

(4)
(3)
(4)
(5)
(5)
266. 157
267. 2.5
268. 31
269. 4.5
270. 32

Difficulty

| 1. | 1.7 | $(2)$ | 6. | 3 | (3) |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 2. | 2.4 | $(2)$ | 7. | 7 | (3) |
| 3. | 1.8 | $(3)$ | 8. | 251.7 | (3) |
| 4. | 9 | $(4)$ | 9. | 125.9 | (3) |
| 5. | 0.33 | $(4)$ | 10. | 65 | $(2)$ |

## PROBLEM INDEX

It is difficult to categorize many of the problems in the MATHCOUNTS School Handbook. It is very common for a MATHCOUNTS problem to straddle multiple categories and cover several concepts. This index is intended to be a helpful resource, but since each problem has been placed in exactly one category and mapped to exactly one Common Core State Standard (CCSS), the index is not perfect. In this index, the code 8 (3) 7.SP. 5 refers to problem 8 with difficulty rating 3 mapped to CCSS 7.SP.5. For an explanation of the difficulty ratings refer to page 65. For an explanation of the CCCS codes refer to page 91.

|  | 1 | (1) | 4.OA. 2 |
| :---: | :---: | :---: | :---: |
|  | 11 | (1) | 6.G.3 |
|  | 17 | (3) | 5.OA. 1 |
|  | 21 | (2) | SMP |
|  | 37 | (3) | 6.EE. 2 |
|  | 43 | (2) | 7.NS. 2 |
|  | 53 | (2) | 4.MD. 2 |
|  | 102 | (3) | SMP |
|  | 164 | (3) | SMP |
|  | 191 | (4) | SMP |
|  | 3 | (2) | 4.OA. 3 |
|  | 10 | (2) | 4.OA. 5 |
|  | 22 | (3) | 7.NS. 3 |
|  | 33 | (3) | 4.OA. 5 |
|  | 40 | (4) | 7.NS. 2 |
|  | 49 | (3) | 8.EE. 2 |
|  | 69 | (3) | 6.EE. 6 |
|  | 84 | (4) | 7.RP. 3 |
|  | 95 | (3) | SMP |
|  | 98 | (3) | 4.OA. 4 |
|  | 112 | (3) | 6.NS. 4 |
|  | 123 | (3) | 3.NBT. 2 |
|  | 134 | (3) | SMP |
|  | 145 | (4) | SMP |
|  | 149 | (5) | S-CP. 9 |
|  | 151 | (4) | SMP |
|  | 154 | (3) | 7.NS. 3 |
|  | 167 | (4) | 7.EE. 4 |
|  | 169 | (3) | SMP |
|  | 171 | (3) | 4.OA. 3 |
|  | 175 | (4) | SMP |
|  | 179 | (2) | SMP |
|  | 183 | (2) | 7.NS. 1 |
|  | 198 | (4) | SMP |
|  | 205 | (5) | SMP |
|  | 218 | (4) | 6.RP. 3 |
|  | 219 | (4) | 7.SP. 8 |
|  | 241 | (6) | SMP |
|  | 244 | (4) | 7.NS. 3 |
|  | 248 | (7) | 7.NS. 3 |
|  | 251 | (5) | SMP |
|  | 254 | (5) | SMP |
|  | 266 | (4) | 6.NS. 4 |



[ 5 (3) 6.RP. 3
15 (2) 6.RP. 1
39 (4) 7.RP. 3
44 (3) 6.NS. 1
51 (2) 8.NS. 1
(2) 6.NS. 1
(3) 7.RP. 3
(2) $6 . R P .3$
(4) 6.G. 1
(2) $7 . R P .3$
(5) 7.RP. 3
(3) 7.RP. 3
(4) 7.NS. 3
(5) $6 . R P .3$
(3) 7.NS. 3
(5) 7.NS. 3
(4) A-CED. 1
(3) 7.RP. 3
(6) A-CED. 2
(4) 6.EE. 1

(4) 7.EE. 4
(3) $6 . R P .3$
(2) $6 . R P .3$
(2) $6 . R P .3$
(3) $6 . R P .3$
(3) 6.EE. 7
(3) 6.RP. 3
(2) 7.SP. 6
(4) G-CO. 10
(1) 4.OA. 2
(4) $6 . \mathrm{RP} .3$
(4) 7.RP. 2
(2) 6.EE. 9
(4) 6.EE. 9
(3) 7.RP. 2
(3) $6 . E E .7$
(3) $6 . R P .3$

163 (4) 6.RP. 3
190 (4) 5.MD. 5
194 (5) 8.G.9
206 (3) 6.RP. 3
246 (4) 7.RP. 2
255 (5) 6.G. 1

|  | 27 | (4) | 6.G.1 |
| :---: | :---: | :---: | :---: |
|  | 31 | (1) | 6.EE. 3 |
|  | 42 | (3) | 7.G. 5 |
|  | 46 | (4) | G-CO. 10 |
|  | 55 | (2) | 6.EE. 7 |
|  | 64 | (4) | 7.G. 4 |
|  | 76 | (4) | 4.MD. 3 |
|  | 92 | (3) | 7.G. 4 |
|  | 97 | (3) | 6.RP. 3 |
|  | 116 | (4) | 4.MD. 3 |
|  | 120 | (5) | 7.G. 4 |
|  | 128 | (4) | 8.G.7 |
|  | 133 | (3) | 7.G. 6 |
|  | 146 | (5) | G-SRT. 6 |
|  | 161 | (3) | 7.G.6 |
|  | 196 | (4) | 8.G.7 |
|  | 208 | (5) | 7.G. 4 |
|  | 242 | (4) | 7.G. 4 |
|  | 252 | (4) | 7.G.6 |
|  | 267 | (6) | 8.G.8 |


|  | 80 | (4) | 6.G.3 |
| :---: | :---: | :---: | :---: |
|  | 89 | (3) | 6.G.3 |
|  | 106 | (4) | G-GPE. 6 |
|  | 130 | (4) | 8.G. 1 |
|  | 138 | (5) | 8.F. 4 |
|  | 180 | (4) | 6.G.1 |
|  | 199 | (4) | 8.F. 3 |
|  | 204 | (3) | G-GPE. 6 |
|  | 207 | (5) | 8.F. 4 |
|  | 232 | (5) | 8.G.8 |
|  | 258 | (5) | G-CO. 5 |

## 

$\begin{array}{rll}9 & (3) & 6 . E E .9 \\ 23 & (3) & 7 . G .6 \\ 29 & (4) & 8 . E E .7 \\ 72 & (3) & 8 . G .9 \\ 83 & (3) & 6 . G .2 \\ 111 & (5) & 8 . G .9 \\ 137 & (4) & 7 . G .6 \\ 156 & (4) & 8 . G .9 \\ 187 & (6) & 8 . G .9 \\ 201 & (6) & 6 . G .2 \\ 222 & (5) & \text { G-GMD. } 3 \\ 227 & (4) & 7 . G .6 \\ 237 & (4) & 8 . G .7 \\ 250 & (6) & \text { G-GMD. } 4 \\ 260 & (7) & 7 . G .6 \\ 261 & (5) & \text { G-GMD. } 4\end{array}$


[^0]:    * The plural form of the units is always provided in the answer blank, even if the answer appears to require the singular form of the units.

