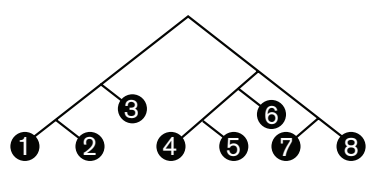




Probability Stretch

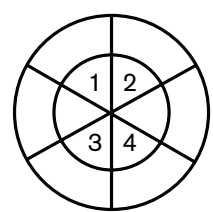
- _____ % Petra randomly selects a card from a standard deck of 52 playing cards. What is the percent probability that the card shows a red number greater than 6? Express your answer to the nearest hundredth.
- _____ Max has eight identical cups. Each cup contains a different combination of nickels, dimes and quarters, each totaling 45 cents. Max randomly selects a cup. What is the probability that the cup he selects contains at least three dimes? Express your answer as a common fraction.
- _____ A bag contains five chips numbered 2 through 6. Danya draws chips from the bag one at a time and sets them aside. After each draw, she totals the numbers on all the chips she has already drawn. What is the probability that at any point in this process her total will equal 10? Express your answer as a decimal to the nearest tenth.
- _____ A drawer contains five socks: two green and three blue. What is the probability that two socks pulled out of the drawer at random will match? Express your answer as a common fraction.
- _____ A penny, a nickel and a dime are flipped. What is the probability that at least two coins land heads up and one of them is the nickel? Express your answer as a common fraction.
- _____ % When the circuit containing blinking lights A and B is turned on, lights A and B blink together. Then A blinks once every 5 seconds and B blinks once every 11 seconds. Lindsey looks at the two lights just in time to see A blink alone. What is the percent probability that the next light to blink will be A blinking alone?
- _____ % What is the percent probability that a randomly selected multiple of 3 less than or equal to 3000 is also a multiple of 5?

- _____ Starting at the top and selecting paths randomly as you move downward, what is the probability of ending at an odd number? Express your answer as a common fraction.



- _____ A five-digit number is made by randomly ordering the digits 1, 2, 3, 4 and 5. What is the probability that this number is divisible by 4? Express your answer as a common fraction.

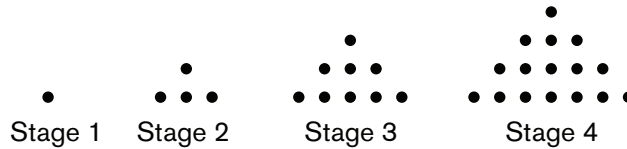
- _____ Pierre throws darts that land randomly in the dartboard shown here. The dartboard is a circle of radius 2 units, with an inner circle of radius 1 unit. Both circles are divided into six congruent sectors. What is the probability that a dart Pierre throws will land in one of the four inner numbered sectors? Express your answer as a decimal to the nearest hundredth.





Patterns Stretch

11. _____ dots The first four stages of a dot pattern are shown. How many more dots are in the figure at Stage 47 than in the figure at Stage 27?



12. _____ The first three terms of a sequence are 1, 2 and 3. Each subsequent term is the sum of the three previous terms. What is the 11th term of this sequence?
13. _____ What is the sum of the terms in the arithmetic series $2 + 5 + 8 + 11 + 14 + \dots + 89 + 92$?
14. _____ Three consecutive terms in an arithmetic sequence are x , $2x + 11$ and $4x - 3$. What is the constant difference between consecutive terms in this sequence?
15. _____ What is the sum of the terms in the geometric series $1 + 4 + 16 + \dots + 1024$?
16. _____ What is the sum of the first 51 consecutive odd positive integers?
17. _____ What is the sum of the terms in the infinite series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots$?
18. _____ What is the sum of the terms in the infinite series $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \dots$? Express your answer as a common fraction.
19. _____ Let $f(x) = 2x + 3$ and $f^2(x) = f(f(x)) = f(2x + 3) = 2(2x + 3) + 3 = 4x + 9$. If $f^5(x) = ax + b$, what is the value of $a + b$?
20. _____ degrees The degree measures of the interior angles of a quadrilateral form a geometric sequence whose terms have integer values and are all integer multiples of the first term. What is the largest possible degree measure of an angle in this quadrilateral?



Travel Stretch

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{distance} = \text{speed} \times \text{time}$$

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

21. _____ mi/h Jack and Jill travel up a hill at a speed of 2 mi/h. They travel back down the hill at a speed of 4 mi/h. What is their average speed for the entire trip? Express your answer as a mixed number.



22. _____ : _____ p.m. At 2:20 p.m., Jack is at the top of the hill and starts walking down at the exact same time that Jill, who is at the bottom of the hill, starts walking up. If they maintain the same uphill and downhill speeds from the previous problem, and the distance from the bottom to the top of the hill is 1.5 miles, at what time will Jack and Jill meet?

23. _____ yards When Jack and Jill meet, as described in the previous problem, how many yards will they be from the bottom of the hill?

24. _____ minutes Alysha's average speed when walking from home to the market is 5 mi/h, and it takes her 21 minutes longer than when she drives to the market. If Alysha drives to the market, along the same route, at an average speed that is eight times her average walking speed, how many minutes does it take her to drive from home to the market?



25. _____ miles Based on problem 24, how many miles does Alysha travel to get from home to the market?

26. _____ minutes

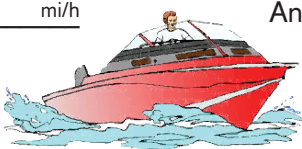


- Jana begins jogging along a path and, 5 minutes later, Zhao begins riding his bicycle along the same path, which has a length of 2 miles. Zhao rides his bicycle at a speed of 10 mi/h, and Jana's jogging speed is 6 mi/h. If they both begin at one end of the path and end at the other, how many minutes after Zhao reaches the end of the path will Jana reach the end of the path?

27. _____ minutes Based on problem 26, how many minutes after Zhao begins riding will he catch up with Jana? Express your answer as a mixed number.

28. _____ miles Again, based on problem 26, how many miles will Jana have traveled when Zhao catches up with her? Express your answer as a mixed number.

29. _____ mi/h



- Ansel left the dock in his motorboat, traveled 10 miles, and then returned to the dock along the same route. On the return trip, Ansel was traveling against the current of the river, and his average speed relative to the water was 20 mi/h. If the round-trip took Ansel 64 minutes, what is the speed of the river's current?

30. _____ Based on problem 29, what fraction of Ansel's total travel time was spent traveling upstream? Express your answer as a common fraction.



Warm-Up 1

31. _____ What is the value of $5 - 5 \times 5 + 5 \div 5$?
32. _____ diagonals How many diagonals are in a convex heptagon?
33. _____ What is the first year after 2018 that is a palindrome?
34. _____ A standard 52-card deck of playing cards includes four aces. What is the probability that two cards selected randomly, without replacement, will both be aces? Express your answer as a common fraction.
35. _____ What is the value of $\sqrt{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 10}$? Express your answer in simplest radical form.
36. _____ °F The temperature dropped from 13°F to -5°F . How many degrees Fahrenheit is the absolute value of the change in temperature?
37. _____ What is the value of $1 \times 2 + 3 \div 6 \times 5 - 4$? Express your answer as a common fraction.
38. _____ If $x \textcircled{R} y$ is defined as $x^2 - y^2$, what is the value of $3 \textcircled{R} (2 \textcircled{R} 1)$?
39. _____ If the digits 7, 8, 2, 3 and 0 are used, each exactly once, to form a three-digit positive integer and a two-digit positive integer that differ by exactly 288, what is the sum of the three-digit integer and the two-digit integer?
40. _____ degrees In rectangle ABCD, point P lies on side BC and point Q lies in the interior of the rectangle so that triangle APQ is equilateral. If the measure of angle PAB is 17 degrees, what is the measure of angle QPC?



Warm-Up 2

41. _____ balls Kim is knitting a baby blanket that requires 750 meters of yarn. There are 180 meters of yarn in each ball. How many balls of yarn must Kim buy to ensure she has enough yarn to complete her blanket?
42. _____ years
old On Chris' birthday in 1992, he was half the age of his brother Joseph. On Chris' birthday in 1998, he was two-thirds the age of Joseph. How old will Chris be on his birthday in 2018?
43. _____ degrees On a standard 12-hour clock, the minute hand moves continuously, at a constant rate, making one full revolution every hour, and the hour hand moves similarly, making one full revolution every 12 hours. What is the measure of the smaller of the two angles between the minute hand and the hour hand, in degrees, when the clock reads 5:42?
44. _____ What is the value of the expression $12 \times 37 + 12 \times 7 + 12 \times 6$?
45. _____ factors How many distinct positive factors does 2018 have?
46. _____ Two fair six-sided dice, with sides numbered 1 through 6, are rolled. What is the probability that the values on the two top faces add to at least 9? Express your answer as a common fraction.
47. _____ If the graph of the equation $y = mx + b$ is a line passing through the points (6, 13) and (10, 31), what is the value of m ? Express your answer as a common fraction.
48. _____ Dewey buys soda in 12-ounce cans that cost \$1.00 each. Peppar buys soda in 20-ounce bottles that cost \$1.25 each. If Dewey and Peppar buy the same volume of soda in one week, then Peppar pays $P\%$ less than Dewey. What is the value of P ?
49. _____ logs Gerald Scheetz is building a log cabin. If each log is 9 inches in diameter, how many logs must be stacked on top of one another to create a wall that has a height of 12 feet?
50. _____ units² A square with area 8 units² is inscribed in a circle. What is the area of the circle? Express your answer in terms of π .



Warm-Up 3

51. _____ If y is a number such that $y^2 = (y + 2018)^2$, what is the value of y ?

52. $\frac{\text{years}}{\text{old}}$ Maura is 5 years younger than her sister Cara. Seven years ago, Maura was half as old as her sister. How old is Maura now?

53. _____ A dartboard consists of three concentric circles with radii 10, 5 and 1, respectively, measured in inches. The area between the largest and middle circles is colored green, the area between the middle and smallest circles is colored yellow, and the area within the smallest circle, the bull's-eye, is colored red. If a thrown dart is guaranteed to hit the board, but its position on the board is uniformly random, what is the probability that it lands in the yellow portion of the board? Express your answer as a common fraction.

54. $\frac{\text{days}}$ The 1990 and 2018 calendars are identical in the number of days in each month and the day of the week on which each day of each month occurs. In fact, the calendar repeats in these ways every 28 years until the year 2100. How many days are there in the 28 years preceding 2018?

55. $\frac{\text{cm}}$ A right triangle has legs with lengths of 5 cm and 10 cm. What is the length of the altitude drawn to the hypotenuse of this triangle? Express your answer in simplest radical form.

56. _____ Min Zhang wrote down all of the two-digit multiples of 5. What is the probability that one of these numbers, chosen at random, has exactly two distinct primes that are factors? Express your answer as a common fraction.

57. _____ Given a set of numbers with median m , the median of all the numbers less than m is called the *lower quartile*. The median of all the numbers greater than m is called the *upper quartile*. The absolute difference between the lower and upper quartiles is called the *interquartile range*. What is the interquartile range for the numbers in the stem-and-leaf plot shown?

0	2	4	6	8	9	
1	1	1	3	8	9	9
2	0	4	6	8		
3	0	3	5	7	7	
4	1	5	7	7		

58. _____

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

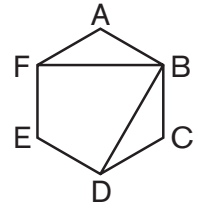
Positive integers 1 to 36 are written in rows in a six-by-six array as shown. Each prime number is crossed off, as well as all the numbers in the diagonal extending up and to the right from that prime. For example, 11 is prime and is crossed off along with the 6 above and to the right. What is the sum of the remaining values after all the primes and associated diagonals have been eliminated?

59. _____ What is the value of $1,000,000! \div 999,999!$?

60. $\frac{\text{inches}}$ A cubic yard of topsoil is to be spread evenly in the garden at Prove It! Math Academy. The garden measures 10 feet by 8 feet. How many inches deep will the topsoil be? Express your answer as a mixed number.



Warm-Up 4



61. _____ Diagonals FB and BD are drawn in regular hexagon ABCDEF. What is the ratio of the sum of the areas of triangles ABF and BCD to the area of quadrilateral BDEF? Express your answer as a common fraction.

62. _____ What is the value of $\frac{11! - (9+1)(9!)}{8(7!)}$?

63. _____ times David's optometrist sold him a bottle of eyeglass cleaner containing 30 mL of glass-cleaning solution. Assuming there are 20 drops per milliliter, and assuming proper cleaning requires 3 drops of glass cleaner on each side of each lens, what is the maximum number of times David can properly clean his glasses before he must buy a new bottle of eyeglass cleaner?

64. _____ combinations The lunch-ordering app for Pete's Pizza Parlor requires you to choose two distinct meats from among pepperoni, Canadian bacon and sausage; or choose two distinct vegetables from among mushrooms, onions, green peppers and black olives; or choose one meat and one vegetable from among the same choices. How many different pizza combinations are possible using the lunch-ordering app?

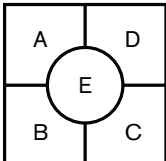
65. _____ pounds Kathy Beckhardt weighs four of her sheep at the fair. She can weigh two of them at a time on the big scale. Sheep A and sheep B have an average weight of 150 pounds, sheep B and sheep C have an average weight of 127 pounds, and sheep C and sheep D have an average weight of 168 pounds. What is the average weight of sheep A and sheep D?

66. _____ meters In circle O, the lengths of chords AB and BC are equal and $m\angle ABC = 90$ degrees. Given that circle O has a radius of 3 meters, what is the length of arc ABC? Express your answer in terms of π .

67. _____ tiles How many 4-inch square tiles are needed to cover a wall that measures 6 feet by 8 feet?

68. _____ What is the units digit of $2^{2017} \times 7^{2017}$?

69. _____ integers How many integers between 100 and 1000 contain no digits other than 3, 4 or 5?

70. _____ paths  The square shown is divided into five cells. How many paths can be drawn that start at any cell, move only to adjacent cells and visit each of the five cells exactly once?



Warm-Up 5

71. _____ paper clips A pencil and 5 paper clips weigh the same as 2 erasers. A pencil weighs the same as 29 paper clips. How many paper clips weigh the same as an eraser?

72. _____ points What is the maximum number of points of intersection of a right triangle with a square, assuming no side of the triangle is collinear with any side of the square?

73. _____ If $p(x) = ax^2 + bx + c$ is a quadratic polynomial satisfying $p(0) = 4$, $p(1) = 15$, $p(2) = 36$, what is the value of the product abc ?

74. _____ units A certain sphere has a volume that is numerically equal to three times its surface area. What is the radius of this sphere?

75. _____ candles A layered candle is made with 5 colors, shown here as candle A. How many different candles can be made using the same 5 colors, with BLUE as the middle layer, shown as candle B, and with no color next to a color that it touched in candle A?

A	B
BLUE	
GREEN	
RED	BLUE
ORANGE	
YELLOW	

76. _____ Suppose Luke spins the pointer on a fair 3-color spinner twice. What is the probability that the pointer lands on the same color twice? Express your answer as a common fraction.

77. _____ shots Kevin is playing basketball and up to now made $\frac{1}{3}$ of his attempted shots. If he makes his next 5 shots, he will improve his shooting percent to 50%. How many shots has Kevin attempted up to now, when he has a $\frac{1}{3}$ success rate?

78. _____ base eight What is 110011_2 when rewritten in base eight?

79. _____ points If the point $(8, 9)$ is the center of a circle of radius 10 units, at how many points does the circle intersect the coordinate axes?

80. _____ If $x + \frac{1}{y} = \frac{1}{5}$ and $y + \frac{1}{x} = 20$, what is the value of the product xy ?



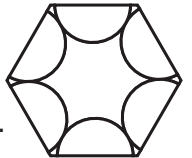
Warm-Up 6

81. _____ If $3x + 5 = 13$, what is the value of the expression $(3x + 2)(3x + 3)(3x + 4)$?
82. _____ units² What is the maximum area of a rectangle with a diagonal of length 16 units?
83. _____ pairs How many pairs of numbers (a, b) satisfy rules I and II shown here?
I. $a = 0$ or $b = -1$ or $b = 1$
II. $a = -1$ or $a = 1$ or $b = 0$
84. _____ If each letter in the sum $A.BC + D.EF$ represents a different nonzero digit, what is the least possible value of the sum? Express your answer as a decimal to the nearest hundredth.
85. _____ ways In the hardware store, Matt goes to the fastener aisle, which has wood screws, sheet metal screws, hex bolts, carriage bolts and lag bolts. How many ways can he choose 10 fasteners if he needs at least one of each kind?
86. _____ A family farm is equally divided among three heirs: Jim, Jan and John. John's share of the farm is then equally divided among his three heirs: Peter, Paul and Patricia. Paul decides to sell his share of the farm, and then later the family decides to sell the remainder of the farm all at once. What portion of the proceeds from the most recent sale should Jim receive? Express your answer as a common fraction.
87. _____ Olivia Justynski earned scores of 82, 86 and 92 on her first three tests. What score does she need on her fourth test to achieve an average score of 90 on the four tests?
88. _____ hours For each child, Kiddie Day Care charges \$330 per month for preschool and \$5.50 per hour for each hour of after-school care. If Cody's cost was \$770 for his son's child care last month, how many hours did his son spend in after-school care?
89. _____ When three consecutive positive integers are multiplied, the product is 16 times the sum of the three integers. What is the difference of the product minus the sum?
90. _____ The lift force on an airplane during flight is directly proportional to the surface area of the wing. Orville builds a model airplane and goes outside to play. Orville's little brother, Wilbur, builds a mini replica of Orville's plane that is half as long in every linear dimension. What is the ratio of the lift force on Wilbur's plane to that on Orville's plane? Express your answer as a common fraction.



Warm-Up 7

91. _____ What whole number n makes $\frac{6}{78} < \frac{1}{n} < \frac{5}{55}$ true?
92. _____ In 2016 the Flying Turtles finished their baseball season with a record of 95 wins and 67 losses. The Dolphins finished the season with 84 wins and 78 losses. The Flying Turtles and Dolphins played each other 19 times during the season. If the Flying Turtles had F wins against teams other than the Dolphins, and the Dolphins had D wins against teams other than the Flying Turtles, what is the value of $F + D$?
93. _____ units A point D is placed on the segment with endpoints $(0, 8)$ and $(8, 0)$, and a point E is placed on the segment with endpoints $(-3, 0)$ and $(0, -2)$. What is the shortest possible distance between D and E ? Express your answer in simplest radical form.
94. _____ terms In the arithmetic sequence $1, 3, 5, 7, 9, 11, \dots$, how many terms appear after the term 315 but before the term 639?
95. _____ Allen Zhang rolls two fair 6-sided dice with faces numbered 1 through 6. What is the probability that the sum of his two rolls has an odd number of positive integer divisors? Express your answer as a common fraction.
96. _____ Six semicircles, each of radius r , are constructed inside a regular hexagon of side length s , one on each side, so that each semicircle is tangent to two others. What is the ratio of r to s ? Express your answer in simplest radical form.



97. _____ Gaylon starts writing down dates from January 1, 2018 onward as follows: 01012018, 01022018, 01032018, etc. What is the 2018th digit Gaylon writes down?
98. _____ lightning bolts Zeus threw, on average, 12 lightning bolts per day in the month of March. During the first week in April, he averaged 15 lightning bolts per day. How many lightning bolts does Zeus need to throw per day on average for the rest of April to maintain a 12-bolt-per-day average over March and April? Express your answer to the nearest integer.
99. _____ For what positive value of x is the equation $9^{2x^2 - 6} = 27^{x^2 - 1}$ true?
100. _____ times The decibel is a unit used to describe the loudness of a sound. For every 20-decibel increase, a sound gets 10 times as loud. Normal conversation is about 60 decibels, and a loud rock concert is about 120 decibels. How many times as loud is a rock concert compared to normal conversation?



Warm-Up 8

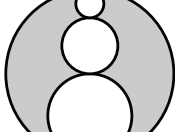
101. _____ Pamela Wickham writes a sequence of four consecutive integers on a sheet of paper. The sum of three of these integers is 206. What is the other integer?

102. _____ seconds Benjamin starts walking up on an escalator that moves down one flight of stairs every 20 seconds. Benjamin takes 10 seconds to walk up a single flight of stairs on the adjacent stationary staircase. Assuming Benjamin walks at the same speed on the escalator and stairs, how many seconds does it take him to walk up two flights on this escalator?

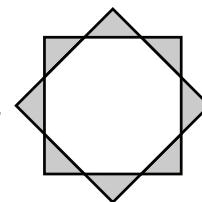
103. _____ divisors Let $K = 168 \times 900 = 151,200$. How many positive integer divisors does K have?

104. _____ Emma Kerwin creates a custom six-sided die by randomly choosing six different integers between 1 and 7, inclusive, to paint on the sides of a blank cube. What is the probability that the faces of her die sum to 24? Express your answer as a common fraction.

105. \$ _____ The owners of two food carts calculate their weekly profits for three weeks. The medians and the highest weekly profit values are the same for the two carts. The mean weekly profit of Cart A is \$27 more than that of Cart B. What is the absolute difference between the lowest weekly profit values of Cart A and Cart B?

106. _____  Each of the circles in the figure is tangent to exactly two others. The centers of all four lie on a line. If the diameters of the three inner circles are in a ratio of 1:2:3, what fraction of the largest circle is shaded? Express your answer as a common fraction.

107. _____ Two congruent squares overlap to form a regular octagon as shown. What is the ratio of the shaded area to the area of the regular octagon? Express your answer in simplest radical form.



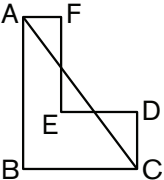
108. _____ hours It takes Avi one half-hour longer to make a basket than it takes Markus. After 28 hours, Markus has made one more basket than Avi has made. How many hours does it take Avi to make one basket?

109. _____ Suppose N is a positive integer such that $N - 1$ is even, $N - 2$ is divisible by 3, $N - 3$ is divisible by 5, and $N - 5$ is divisible by 7. What is the least possible value of N ?

110. _____ What fraction of the positive integer factors of 1000^3 are perfect squares? Express your answer as a common fraction.



Warm-Up 9

111. _____ Sola's lucky numbers are 7 and 11. So he decides his lucky common fraction, f , will be formed by the repeating decimal $f = \overline{0.711}$. What is the value of f as a common fraction?
112. _____ Suppose m is the line given by the equation $6x - 3y = 7$, and suppose n is the line perpendicular to m and passing through the point $(6, 2)$. If k is the line of slope 5 and y -intercept 1, what is the x -coordinate of the intersection of n and k ? Express your answer as a common fraction.
113. _____ In hexagon ABCDEF, shown here, adjacent sides are perpendicular. If $AB = 8$, $BC = 6$, $CD = 3$ and $DE = 4$, what fraction of the segment AC lies inside of the hexagon? Express your answer as a common fraction.
- 
114. _____ $\frac{\text{times}}{\text{stronger}}$ The Richter scale is used to describe the strength of an earthquake. An increase of 1 point on the Richter scale represents a tenfold increase in the strength of an earthquake. How many times stronger is an earthquake rated 7.5 on the Richter scale compared to an earthquake rated 5? Express your answer in simplest radical form.
115. _____ The third term of a geometric sequence of integers is 45. The seventh term of the sequence is 3645. What is the least possible sum of the first five terms of the sequence?
116. _____ words In a new version of Scrabble, a sequence of letters is considered a word if the first and last letters are consonants and every letter in between is a vowel. In this game, how many four-letter words can be formed using each of the letters M, A, T, H, R, U, L, E and S no more than once?
117. _____ mi/h Rebecca and Susan live at opposite ends of a 2-mile-long street. At 8:00 a.m., Rebecca starts jogging from her house toward Susan's end of the street. At 8:06 a.m., Susan starts jogging from her apartment toward Rebecca's end of the street. They pass each other at exactly 8:13 a.m. If Rebecca and Susan jog at the same constant speed, what is this speed, in miles per hour?
118. _____ units^2 We define a *Heronian triangle* to be a triangle with three integer side lengths and integer area. What is the least possible positive area of a Heronian triangle whose longest side has a length of 17 units?
119. _____ For each of the first eight prime numbers, Brian Edwards writes down all the number's positive factors. What is the sum of all the numbers Brian writes down?
120. _____ If $\sqrt{x} - \sqrt{y} = 10$ and $\sqrt{x} + \sqrt{y} = 14$, what is the value of $x + y$?



Warm-Up 10

121. _____ What is the greatest prime factor of $(1!)! \times (2!)! \times (3!)! \times (4!)!$?

122. _____ minutes The table shows how long it takes Anita's fully discharged cell phone battery to fully charge using three methods. When her phone battery fully discharged, Anita charged the phone for half an hour using the wall charger, and now she will continue charging it for 1 hour using her computer. How many minutes are required to fully charge the phone battery using the portable charger, if the phone is not used during or between chargings?

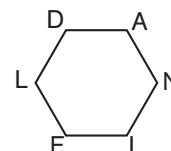
Method	Hours <small>(to fully charge)</small>
Wall Charger	1.5
Computer	3
Portable Charger	5

123. _____ In equilateral triangle ABC , M is the midpoint of side AB . If CMN is also an equilateral triangle, what fraction of the area of triangle $\triangle ABC$ lies inside of $\triangle CMN$? Express your answer as a common fraction.

124. _____ What is the greatest prime factor of $3^7 - 27$?

125. _____ ways In how many ways can eight differently colored balls, including one red, one green and one yellow, be ordered left to right so that the green ball is to the right of the red ball (not necessarily adjacent) and the yellow ball is to the right of the green ball (not necessarily adjacent)?

126. _____ units Sides DL and AN in a regular hexagon $DANIEL$, shown here, are extended until they intersect at a point F . If the sides of the hexagon have length 6 units, what is the length of segment FE ? Express your answer as a radical in simplest form.



127. _____ baskets Annette, Mary and Lynn team up to pick apples. Annette can pick 4 baskets of apples per hour, and Mary can pick 5 baskets of apples per hour. Annette, Mary and Lynn work together to pick 6 baskets of apples in half an hour. How many baskets of apples can Lynn pick by herself in 3 hours?

128. _____ Kayla Straub starts with a pile of 15 stones. She divides the pile into two new piles and finds the product of the numbers of stones in the two new piles. Kayla then divides one of the existing piles into two new piles. She finds the product of the numbers of stones in the two new piles and adds it to the previous product. Kayla continues this process, each time adding the product of the numbers of stones in the two new piles to the previous total, until she has 15 piles with one stone each. What is the greatest possible ending total?

129. _____ A sphere is inscribed in a cube. What is the ratio of the volume of the cube to that of the sphere? Express your answer as a common fraction in terms of π .

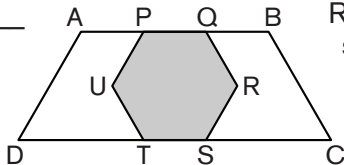
130. _____ Let $\#x$ represent the greatest even integer less than x . If $20 < x < 30$, what is the maximum possible value of $\#(5x) - \#(4x)$?



Warm-Up 11

131. _____ points If two distinct ellipses and a square are drawn, what is the maximum possible number of points at which at least two of the three planar figures intersect?

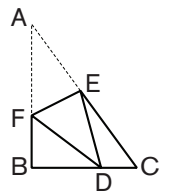
132. _____ Isosceles triangles ABC and DEF have six interior angles altogether, but these six angles have only three different measures among them. If the sum of these three different measures is 156 degrees, and both triangles have at least one angle of measure m degrees, what is the value of m ?

133. _____  Regular hexagon $PQRSTU$ lies inside of trapezoid $ABCD$, as shown, so that vertices P and Q trisect the base AB , S and T lie on the base CD , and sides PU and QR are parallel to sides AD and BC , respectively. The shaded area is what fraction of the area of trapezoid $ABCD$? Express your answer as a common fraction.

134. _____ integers How many positive integers in the set of numbers from 1 to 1000, inclusive, are multiples of 2, 3 and 5 but not 8?

135. _____ In the sum $ABCD + EFGH$, each letter represents a digit selected independently at random from the set $\{1, 2, 3, 4\}$. What is the probability that the sum of the two four-digit numbers contains the digit 5 at least once? Express your answer as a common fraction.

136. _____ cm^2 In right triangle ABC , with $AB = 44$ cm and $BC = 33$ cm, point D lies on side BC so that $BD:DC = 2:1$. If vertex A is folded onto point D to create quadrilateral $BCEF$, as shown, what is the area of triangle CDE ?



137. _____ passes After the first eight games of the football season, Jason Doan had completed 70% of his passes. During the ninth game, he completed 49 of his 50 passes, raising his season pass completion rate to 74%. How many total passes did he throw during the first nine games?

138. _____ The mean of seven distinct positive integers is 20. What is the difference between the greatest and least possible medians of the seven integers?

139. _____ integers How many two-digit positive integers have a units digit that is equal to the product of its two digits?

140. _____ Colleen Kipfstuhl rolls a standard fair six-sided die. If she rolls a number with an odd number of positive integer divisors, she steps 1 meter to her right. Otherwise, she steps 1 meter to her left. After four rolls of the die, what is the probability Colleen ends up right where she started? Express your answer as a common fraction.



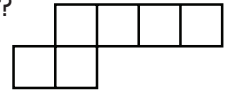
Warm-Up 12

141. _____ What is the absolute difference between the sum of the multiples of 2, from 1 to 100, inclusive, and the sum of the multiples of 3, from 1 to 100, inclusive?
142. _____ If $p(x)$ is a cubic polynomial with $p(0) = 4$, $p(1) = 10$, $p(-1) = 2$ and $p(2) = 26$, what is the value of $p(3)$?
143. _____ inches What is the greatest possible perimeter of an obtuse triangle, each of whose side lengths is a whole number of inches less than or equal to 100?
144. _____ fist bumps After playing a math game, each member of the MATHCOUNTS national office staff gives a fist bump to every coworker. If 25 members of the national office staff participate as described, how many total fist bumps occur?
145. _____ students Several students were trying out for a class play. When asked which roles they were willing to play, 12 of them were willing to play the knight, 15 were willing to play the princess and 6 were willing to play the sorcerer. Of these students, 8 were willing to play either the knight or the princess, 5 were willing to play the knight or the sorcerer, and 4 were willing to play the princess or the sorcerer. Exactly 3 of these students were willing to play any of the roles. How many students were willing to play the sorcerer but no other role?
146. _____ ways Frankie the frog stands at the number 0 on a number line and wants to hop to the number 8. He can hop 1, 2 or 3 units forward in a single jump. How many different ways are there for Frankie to reach the number 8?
147. _____ The median and the mean of the five integers 10, 12, 26, x , x are equal. What is the sum of all possible values of x ?
148. _____ ordered pairs How many ordered pairs of prime numbers (a, b) are there such that $a + b = 100$?
149. _____ cm What is the perimeter of a right triangle with an area of 10 cm^2 and a hypotenuse of length 10 cm? Express your answer in simplest radical form.
150. _____ If $\frac{2}{x+1} + \frac{8}{y-3} = \frac{10}{3}$ and $\frac{4}{x+1} - \frac{2}{y-3} = \frac{2}{3}$, what is the value of $x + y$?

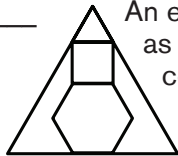


Warm-Up 13

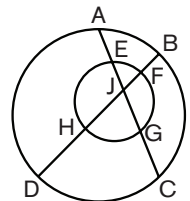
151. ordered triples How many ordered triples of integers (m, n, p) exist such that $mn = p$, $np = m$ and $mp = n$?
152. cm² What is the least possible area of a rectangle that can enclose an equilateral triangle with side length 6 cm? Express your answer in simplest radical form.
153. integers How many of the first 2018 positive integers are either perfect squares or perfect cubes?
154. assortments Alexander Clifton visits Sweet Dreams bakery, which sells three kinds of cookies. How many unique assortments of a dozen cookies can Alexander buy?
155. dimes Gabriel and Isabel each start with a pile of 20 coins consisting of nickels, dimes and quarters. After Gabriel gives Isabel 2 coins, and Isabel gives Gabriel 5 coins, Gabriel's pile is worth twice the value of Isabel's pile. If Gabriel and Isabel have the greatest possible combined value of coins, what is the least number of dimes Isabel could end up with?
156. _____ What is the smallest positive integer multiple of 130 that is divisible by 365?
157. hexominoes A *hexomino* is a planar figure formed by connecting six unit squares so that adjacent squares have a common side. One possible hexomino is shown. How many distinct hexominoes can be drawn that have exactly four squares in a row? Two hexominoes are distinct if one cannot be reflected or rotated to form the other hexomino.



158. _____ An equilateral triangle, a square and a regular hexagon with side length 6 are stacked as pictured. A larger equilateral triangle is then drawn around the stack of polygons, completely enclosing it. The area outside the polygon stack but inside the larger triangle can be expressed in the form $a + b\sqrt{c}$, where a , b and c are integers and $b\sqrt{c}$ is in simplest radical form. What is $a + b + c$?



159. assignments Abhi, Bryan, Meghna and Noreen are each assigned a different integer from 1 to 10, inclusive. Abhi's number is prime and Noreen's number is a perfect square. Bryan's number is half of the value assigned to another person, while Meghna's number is the sum of two other assigned values. The ordered quadruple $(2, 1, 5, 4)$ is one possible assignment. How many such assignments are there?





Warm-Up 14

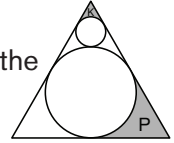
161. _____ If n is the product of three consecutive positive integers and $n = 22 \times 14 \times k$, what is the least possible value of k ?

162. _____

23	12	1		9
4	18	7	21	
10	24	13	2	16
	5	19	8	22
17		25	14	3


A regular 5×5 magic square contains a permutation of the integers from 1 through 25, such that every row, every column, and the two main diagonals sum to the same value. What is the sum of the numbers missing from the magic square shown?

163. _____ Two circles are inscribed in an equilateral triangle as shown. What is the ratio of the areas of the shaded regions K to P? Express your answer as a common fraction.



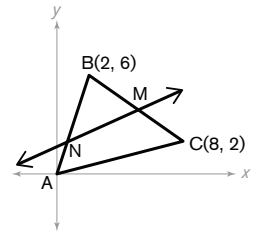
164. _____ What is the sum of the integers strictly between 1 and 100 that are multiples of neither 2 nor 3?

165. _____



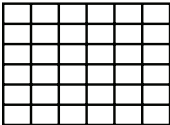
James and John take turns spinning the pointer of a fair spinner that is divided into three congruent sectors. The first player whose spin lands on the WIN sector is the winner of the game. If James goes first, what is the probability that he wins the game? Express your answer as a common fraction.

166. _____ units Triangle ABC has vertices at $(0, 0)$, $(2, 6)$ and $(8, 2)$. The line $x - 3y = -7$ intersects two sides of the triangle at points M and N, as shown. What is the length of segment MN? Express your answer as a common fraction in simplest radical form.



167. _____ bottles Edna enters a room with 1000 bottles lined up in a row left to right. One bottle contains a tasteless magic potion. All bottles to the left of the magic potion contain tasteless water. All bottles to the right of the magic potion contain a bitter poison. Edna can drink from no more than two bottles containing poison before becoming sick and being unable to drink anything else. She can take an unlimited number of drinks from any other bottle. What is the minimum number of bottles from which Edna may need to drink to ensure she can identify the bottle containing the magic potion no matter where it is in the lineup?

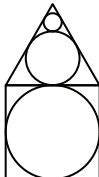
168. _____ silver rectangles



In origami, a *silver rectangle* is any rectangle such that the ratio of the length of the short side to the length of the long side is exactly $1:\sqrt{2}$. Each of the small rectangles in the figure shown is a silver rectangle. How many silver rectangles of any size can be found in the figure?

169. _____ laps Priya, Amanda and Du simultaneously begin jogging in the same direction from the same point on a circular track. Amanda's speed is the average of Priya's and Du's speeds. Du passes Priya for the first time at the moment when Du completes his fourth lap. How many laps has Amanda completed at the moment when she passes Priya for the first time?

170. _____



An equilateral triangle is stacked above a square as shown, with a circle inscribed inside the square and two stacked circles inscribed in the triangle so that they are tangent to each other. What is the ratio of the area of the smallest circle to the area of the largest circle? Express your answer as a common fraction.



Workout 1

171. _____ If three fair coins are simultaneously flipped, what is the probability that exactly two heads will be showing? Express your answer as a fraction in simplest form.
172. _____ m/s^2 An Euler Airline flight is getting ready to take off. Gary McDonald, the pilot, starts from rest at the edge of the runway. He needs to accelerate to a speed of 300 km/h in 30 seconds. Acceleration is defined as the change in speed per unit time. What is Gary's average acceleration, in meters per second per second, which is equivalent to meters per second squared, during takeoff? Express your answer as a decimal to the nearest tenth.
173. _____ pounds An object's weight on a planet is directly proportional to the mass of that planet and inversely proportional to the square of the radius of the planet. Jupiter is 318 times as massive as Earth and has a radius 11 times as large as that of Earth. If Gordon weighs 100 pounds on Earth, how many pounds would he weigh on Jupiter? Express your answer to the nearest whole number.
174. _____ $\frac{\text{million}}{\text{dollars}}$ The 1998 film *Armageddon* had a production budget of \$140 million. The domestic box office gross was about \$200 million, and the international box office gross was about \$350 million. The studio considers a film a financial success if the worldwide (domestic plus international) gross is at least double the sum of the production budget and the advertising budget. In millions of dollars, what was the greatest advertising budget the film could have had to be considered a financial success? Express your answer to the nearest whole number.
175. _____ days What is the mean number of days per month among all months in the year 2018? Express your answer as a decimal to the nearest tenth.
176. _____ If $A = x^2 - 2x + 6$ and $B = \frac{5x^2 - 1}{x + 3}$, what is $A + B$ if $x = -2$?
177. _____ cm What is the height of a cone with a volume of 1187.5 cm^3 and a base of diameter 18 cm? Express your answer to the nearest whole number.
178. _____ degrees What is the degree measure of each interior angle of a regular decagon?
179. \$ _____ Elliott's stock portfolio was valued at \$5000 on January 1. Its value decreased by 20% during January but then increased by 25% during February. What was the value of his stock portfolio at the end of February?
180. _____ hours How many hours are in the decade from January 1, 2011, through December 31, 2020?



Workout 2

181. _____ inches If a television screen with a length-to-height ratio of 16:9 has an area of 576 in^2 , what is its perimeter?
182. _____ miles Alex Zhu bikes between home and school every day. He uses the same route to go to and from school, but it takes him 20 minutes to bike to school and only 15 minutes to bike back. If his average biking pace for the whole round-trip is 7 minutes per mile, how many miles long is the trip from home to school? Express your answer as a decimal to the nearest tenth.
183. _____ m/s Bruce and Lawson are playing ice hockey. Bruce shoots the puck at the goal 40 meters directly in front of him at a speed of 50 m/s. If Lawson is standing exactly 30 meters to the left of Bruce, what is the minimum speed at which he must skate, to reach the goal when the puck does? Express your answer as a decimal to the nearest tenth.
184. \$ _____ In 2015 the average two-adult family in a particular town paid \$619 per month for groceries, excluding sales tax. If groceries in this town were subject to a 9% sales tax, how much sales tax was paid by the average two-adult family in one month?
185. \$ _____ Linda makes 6 cakes per hour. Sara makes 4 cakes per hour. If Linda gets paid \$11 less than Sara for each cake, how much in dollars should Sara be paid for each cake for Linda and Sara to earn the same amount each hour?
186. _____ yards In a golf long-drive competition, Jason Zuback hits his first five drives 394, 401, 387, 414 and 421 yards, respectively. How long must he hit his sixth drive to ensure that the mean of his six drives is at least 400 yards?
187. _____ km A pilot flying due east is forced to make a detour from her original route to avoid turbulent weather. The pilot turns 30 degrees north of east. After traveling some distance, she turns and rejoins her original route and is 1000 km away from where she took the detour. The turn back to her original route put her at a 45 degree angle to that route. How much farther did the pilot travel due to her detour? Express your answer to the nearest whole kilometer.
188. _____ What is 12% of $\frac{3}{4}$ of 1.8? Express your answer to the nearest thousandth.
189. _____ ways In how many different ways can four people be seated around a circular table so that no one ever has the same two neighbors more than once?
190. _____ fluid ounces A cylindrical container holds 20 fluid ounces. It has a radius of 3 inches and a height of 12 inches. How many fluid ounces will a similar container with a radius of 4.5 inches hold? Express your answer as a decimal to the nearest tenth.



Workout 3

191. _____ seats The number of seats per row in an auditorium increases from the front to the back. The first row has 15 seats, the second row has 2 more seats than the first row, the third row has 3 more seats than the second row, the fourth row has 2 more seats than the third row, the fifth row has 3 more seats than the fourth row. This pattern continues, with successive rows alternating between 2 more seats and then 3 more seats than the previous row. How many seats are in the auditorium if there are 30 rows total?
192. _____ degrees Three interior angles of a pentagon measure 110, 120 and 130 degrees, respectively. Of the two remaining interior angles, one is three times the measure of the other. What is the measure of the pentagon's smallest interior angle?
193. _____ Hisham Dimashkieh chooses four distinct positive integers a , b , c and d , each less than or equal to 10. He chooses the numbers so that a is prime, b is composite, c is a perfect square and d is a perfect cube. What is the greatest possible sum of the four numbers?
194. _____ If p , q and r are prime numbers such that $pq + r = 73$, what is the least possible value of $p + q + r$?
195. _____ % If Bella runs 40% as fast as Thomas and 35% as fast as Tam, what percent faster than Thomas is Tam? Express your answer to the nearest percent.
196. _____ % The probability that it will rain today is 50%. The probability that it will rain tomorrow is 40%. Assuming today and tomorrow's precipitation outcomes are independent from one another, what is the percent probability that it will rain on at least one of the two days? Express your answer as a whole number.
197. _____ ft² What is the area of a 60 degree sector of a circle with radius 30 feet? Express your answer in terms of π .
198. _____ kg One year, the U.S. government printed \$700 million worth of paper money every day, for 365 days. Half of the total value came from \$1 bills. If a new \$1 bill weighs exactly 1 gram, what was the weight, in kilograms, of all the \$1 bills printed that year?
199. _____ trees The number of bushels of apples, $B(n)$, that can be harvested from an acre of land is a function of n , the number of trees planted per acre, where $B(n) = 2025n - n^3$. How many trees planted per acre will produce the greatest harvest? Express your answer as a whole number.
200. _____ Three days ago, there were p cupcakes on the counter. Two days ago, exactly 20% of the cupcakes were eaten. Today, there are 30% fewer cupcakes than yesterday and half as many as there were three days ago. If a whole number of cupcakes were eaten every day, what is the least possible value of p ?



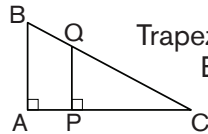
Workout 4

201. _____ The sum of the digits of a two-digit integer is 12. The integer is equal to 15 times its units digit. What is the integer?
202. _____ If $\frac{x^2 + 8x + 15}{x + 5} = 4.01$, then what is the value of x ? Express your answer as a decimal to the nearest hundredth.
203. _____ units² A triangle has three vertices given by coordinates (2, 2), (2, -6) and (-5, -9). What is the area of the triangle?
204. _____ In a regular octagon, the diagonals have three possible lengths —“short,” “medium,” and “long.” What is the ratio of the length of the medium diagonal to the long diagonal? Express your answer as a decimal to the nearest thousandth.
205. _____ % Not realizing that an 18% tip had already been added to the cost of a meal, Emalee added another 15% to the total bill. Given that there is no sales tax, what percent tip did Emalee actually pay? Express your answer as a percent to the nearest tenth.
206. _____ Penner has a deck of 40 cards composed of four suits (red, blue, green, and yellow) and cards numbered 1 through 10 in each suit. Tell secretly chooses a card. Penner then chooses the following 4 cards from the deck: Red-2, Blue-3, Green-5 and Yellow-7. For each card Penner chooses, Tell says “yes” if his secret card is of the same color or shares a common factor greater than 1 with Penner’s card. Otherwise Tell says “no.” Tell says “no,” “yes,” “no,” and “yes,” respectively, in response to Penner’s cards. With this information, what is the best possible probability Penner has of guessing Tell’s secret card? Express your answer as a common fraction.
207. _____ Henry Flannigan chooses a two-digit positive integer at random. What is the probability that the two digits have an absolute difference greater than 1? Express your answer as a common fraction.
208. _____ mi³ Ngorongoro Crater is shaped approximately like a cylinder that is 10 miles across and 2000 feet deep. How many cubic miles of water would it take to fill the crater? Express your answer to the nearest whole number.
209. _____ Mady distributes w candies evenly among 20 bags. The next day, she discovers 5 more empty bags and decides to redistribute the w candies evenly into all of the bags. On the third day, Mady finds 1 more bag and redistributes the w candies evenly again. There are 2 fewer candies on the third day in each of the bags than there were in the bags on the second day. What is the value of w ?
210. _____ If $f(x) = ax^2 + bx + c$, with $f(0) = 4$, $f(2) = 2$ and $f(4) - f(3) = 4$, what is the value of $f(1)$?



Workout 5

211. _____ units²



Trapezoid APQB lies inside of right triangle ABC, as shown. If $AP = 30$, $BQ = 34$ and $AB = 60$, what is the area of triangle ABC?

212. _____

There are five two-digit positive integers arranged in decreasing order. Each digit is unique. What is the absolute difference between the greatest possible range and the least possible range of such a set of integers?

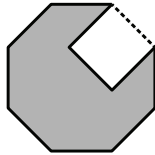
213. _____ ways

Six different donuts are lined up in a box. Six different cookies are lined up in another box. Mackenzie wants to alternate the donuts and cookies in one big long box. How many ways are there to arrange them?

214. _____ workers

For a certain crew of workers, it takes the n th worker exactly n hours to complete a certain job alone. What is the least number of workers needed to complete an identical job in under 20 minutes by working together?

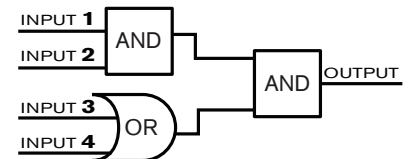
215. _____ cm²



A regular octagon has a perimeter of 64 cm. A square with one side along an edge of the octagon has been cut out of the octagon, as shown. What is the remaining area of the octagon? Express your answer in simplest radical form.

216. _____ inputs

The function machine shown here consists of three logic functions. A Boolean is a member of the set $\{0, 1\}$. For each function, the inputs are on the left and the output is on the right. The output of each function is a Boolean. The input of each function is a pair of Booleans. For the AND function, the output is 1 if and only if both inputs are 1. For the OR function, the output is 0 if and only if both inputs are 0. Among the 16 distinct sets of inputs that can be applied on the far left, how many will produce a 1 as the final output on the far right?



217. \$ _____

Joe, Bob and Randell split a restaurant bill that totaled \$80 before the tip. The group tipped 25%, Joe paid twice as much as Bob, and Randell paid the same amount as Joe. How much did Bob pay?

218. _____ pairs

How many pairs of positive integers a and b exist such that $a^2 - b^2 = 144$?

219. _____ cm³

A rectangular box measures 5 mm \times 10 mm \times 1 m. What is the volume of the box in cubic centimeters?

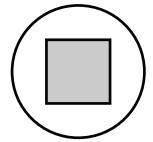
220. _____

A college has been trying to decrease the number of cars on campus and increase the number of bicycles. The price of a parking permit was tripled, and the number of cars on campus decreased 30%. Student tuition was decreased, and the number of bicycles on campus increased by 20%, producing a car to bicycle ratio of 1:3. What was the ratio of cars to bicycles before the changes occurred? Express your answer as a common fraction.



Workout 6

221. _____ in² The gasket shown consists of a circular disk with a square removed. The square and the disk have the same center. If each corner of the square is exactly 1 inch away from the boundary of the disk, and the midpoint of each side of the square is exactly 2 inches away from the boundary of the disk, what is the area of the top face of the gasket? Express your answer as a decimal to the nearest tenth.

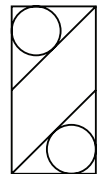


222. _____ % Jeffrey Pribble needs to buy 6 pairs of socks. The Sock Shop is running a limited time promotion: buy 3 pairs of socks and get 3 pairs at half off the regular price. What percent savings does Jeffrey get with the promotion compared to the regular price without the promotion?

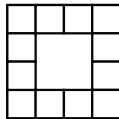
223. _____ What is the greatest possible absolute difference between the median and mean of a list of 10 positive integers that are at most 20? Express your answer as a decimal to the nearest tenth.

224. _____ tourists The tour from Ajim to the Mos Eisley Cantina can take at most 36 tourists. The price for the tour is \$520 per person until at least 15 people have signed up. After that, the price for each person, including the first 15, drops \$5 per additional tourist. If the total amount the tourists paid was \$12,740, how many tourists signed up?

225. _____ inches A rectangle has a width of 1 inch and a height 2 inches. There are two lines drawn, each connecting a vertex to the midpoint of the opposite side, and circles are inscribed in the triangles created, as shown. How far apart are the centers of the circles? Express your answer as a decimal to the nearest hundredth.



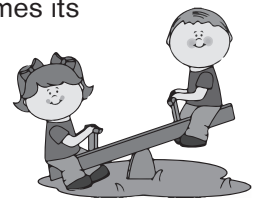
226. _____ rect-angles



How many rectangles of any size are in the figure shown?

227. _____ Let S be the set of all integers N such that both N and the number formed by reversing the digits of N are three-digit perfect squares. What is the sum of the integers in S ?

228. _____ feet A seesaw is in balance when the weight on one side of the fulcrum times its distance from the fulcrum is equal to the weight on the other side of the fulcrum times its distance from the fulcrum. Shandra weighs 96 pounds. Her little sister weighs 72 pounds. The seesaw at their playground has a beam with seats 14 feet apart. The position of the fulcrum can be adjusted as required. If each girl sits in her seat, how far should the fulcrum be from Shandra's seat to achieve perfect balance with her sister?



229. _____ Jennie Weiner has p pennies, n nickels, d dimes and q quarters with a total value of \$1.08. If the numbers p , n , d and q are distinct and positive, and the greatest common divisor of each pair of these numbers is 1, what is the least possible value of $p + n + d + q$?

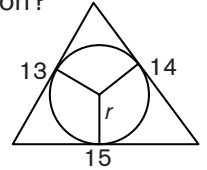
230. _____ ft² What is the total surface area of a right square pyramid with a height of 12 feet and a base with side length 10 feet?



Workout 7

231. \$ _____ Rahul Ilangovan can arrange his dad's collection of quarters as a rectangular array with 10 equal rows, 12 equal rows or 18 equal rows, using all the quarters in each arrangement. What is the least possible monetary value in dollars of the quarter collection?

232. _____ units What is the radius of the largest circle that can be inscribed in an acute triangle with sides 13, 14 and 15 units?



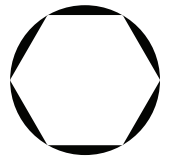
233. _____ Francisco is born at 1:00 a.m. on a Tuesday and gets married exactly 2^{18} hours later. On what day of the week does Francisco get married?

234. _____ Erica drew a 4 of hearts out of a standard 52-card deck, without replacement. If she draws a second card from the deck, what is the probability that her two cards will show consecutive numbers? Express your answer as a common fraction.

235. _____ A set S contains some, but not all, of the positive integers from 3 to 7. Some statements describing S are given below. The statement numbered n is true if the number n is in S and false if n is not in S . What is the product of the numbers that are in S ?

3. The sum of the numbers in S is odd.
4. The sum of the numbers in S is less than 15.
5. S contains exactly one composite number.
6. S contains exactly one prime number.
7. The product of the numbers in S is odd.

236. _____ m^2 The figure shows a regular hexagon of side length 12 meters inscribed in a circle. What is the total area of the shaded regions between the hexagon and the circle? Express your answer to the nearest whole number.



237. _____ Kendra starts at a positive integer k and counts up by 4s until she hits exactly 200. Mason starts at a positive integer m and counts up by 6s until he hits exactly 200. If it takes Kendra half as many steps to reach 200 as it takes Mason, what is the greatest possible value of $k - m$?

238. _____ The graph of the line $3x - 4y = 13$ is translated 2018 units to the right. What is the y -intercept of the translated line? Express your answer as a decimal to the nearest hundredth.

239. \$ _____ Ron works five days a week selling wallets in a booth at the mall. He earns a salary of \$215 per week plus 15% of his weekly sales. If he earned \$383.75 this week, what was the amount of his average daily sales for the week?

240. _____ inches A regular hexagon is inscribed in a circle. If the area of the hexagon is $216\sqrt{3}$ in², what is the circumference of the circle? Express your answer in terms of π .



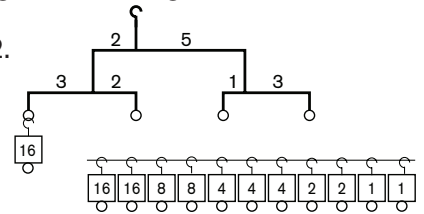
Workout 8

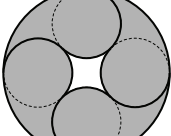
241. _____ Suppose that $A \& B = k \times A^m \times B^n$, where k , m and n are constants. Suppose that $5 \& 3 = 18$, $10 \& 3 = 72$ and $5 \& 6 = 36$. What is the value of $10 \& 6$?

242. combinations How many different combinations of pennies, nickels, dimes and quarters are possible in the cup holder of Terry's car if he counts 15 coins total?

243. in³ If the average length of the edges of a right rectangular prism is 13 inches, and the dimensions of the prism are distinct integers in geometric progression, what is the sum of the volumes of the distinct prisms that meet these criteria?

244. weights In the mobile shown, a beam is *in balance* when the length l_L of the left arm of the beam times the total weight w_L hanging below the left arm of the beam is equal to the length l_R of the right arm of the beam times the total weight w_R hanging below the right arm of the beam. Every beam must be in balance for the mobile to be in balance. Decorative weights are available only in powers of 2. Multiple weights can be hung in a vertical chain, one below another. If the existing weight in the figure is not removed, what is the minimum number of weights that must be added to bring the mobile shown into balance?



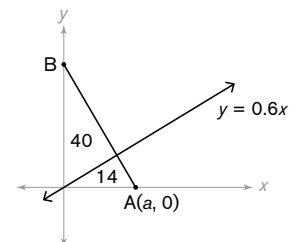
245. %  Four non-overlapping congruent circles are inscribed in a larger circle. Each small circle is shaded. Each region between two adjacent small circles and the enclosing large circle is also shaded. What percent of the figure is not shaded? Express your answer as a decimal to the nearest tenth of a percent.

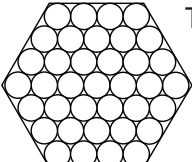
246. _____ Maxine secretly chooses a positive integer between 1 and 2018, inclusive. Martin wants to identify her number with a series of guesses. Each time Martin makes a guess, Maxine tells him whether his number is correct, too high or too low. With an appropriate strategy, Martin can always identify Maxine's number after at most n guesses. What is the least value of n for which Martin can correctly guess Maxine's number?

247. novels Ms. Ault's reading list has novels by only three authors: Mark Twain, Ernest Hemingway and John Steinbeck. For a summer reading assignment, Austen Mazenko must pick one of the authors and read two of that author's novels on the list. If there are exactly 100 ways for Austen to pick two novels that satisfy this requirement, what is the greatest possible total number of novels on the reading list?

248. _____ What is the sum of all prime numbers p less than 60 such that there exists a right triangle whose side lengths are all integers and whose hypotenuse has length p ?

249. _____ A line segment AB from the positive x -axis to the positive y -axis cuts off a triangle of area 54 square units in the first quadrant of the coordinate plane. The line $y = 0.6x$ divides this triangle into two triangles of areas 40 units² and 14 units². If the point A has coordinates $(a, 0)$, what is the value of a ? Express your answer in simplest radical form.



250. %  The Solar Sunflower is made up of super-efficient circular solar panels within a hexagonal frame. Using a two-dimensional diagram of the Solar Sunflower, as shown, with a hexagonal frame with side length 10 meters, what percent of the hexagon is covered in solar discs? Express your answer to the nearest whole number.

ANSWERS

In addition to the answer, we have provided a difficulty rating for each problem. Our scale is 1-7, with 7 being the most difficult. These are only approximations, and how difficult a problem is for a particular student will vary. Below is a general guide to the ratings:

Difficulty 1/2/3 - One concept; one- to two-step solution; appropriate for students just starting the middle school curriculum.

4/5 - One or two concepts; multistep solution; knowledge of some middle school topics is necessary.

6/7 - Multiple and/or advanced concepts; multistep solution; knowledge of advanced middle school topics and/or problem-solving strategies is necessary.

Probability Stretch

Answer	Difficulty	Answer	Difficulty
1. 15.38	(2)	6. 50	(4)
2. 1/4	(3)	7. 20	(2)
3. 0.2	(4)	8. 9/16	(4)
4. 2/5	(3)	9. 1/5	(3)
5. 3/8	(3)	10. 0.17	(4)

Patterns Stretch

Answer	Difficulty	Answer	Difficulty
11. 1480	(4)	16. 2601	(3)
12. 423	(3)	17. 2	(4)
13. 1457	(3)	18. 4/3	(4)
14. 36	(4)	19. 125	(4)
15. 1365	(3)	20. 243	(5)

Travel Stretch

Answer	Difficulty	Answer	Difficulty
21. $2\frac{2}{3}$	(5)	26. 3	(3)
22. 2:35	(4)	27. $7\frac{1}{2}$	(5)
23. 880	(3)	28. $1\frac{1}{4}$	(3)
24. 3	(4)	29. 5	(4)
25. 2	(4)	30. 5/8	(3)

Warm-Up 1

Answer	Difficulty	Answer	Difficulty
31. -19	(1)	36. 18	(2)
32. 14	(2)	37. 1/2	(2)
33. 2112	(2)	38. 0	(3)
34. 1/221	(3)	39. 452	(4)
35. $60\sqrt{(14)}$	(3)	40. 47	(4)

Warm-Up 2

Answer	Difficulty	Answer	Difficulty
41. 5	(2)	46. 5/18	(3)
42. 32	(4)	47. 9/2	(4)
43. 81	(4)	48. 25	(3)
44. 600	(2)	49. 16	(3)
45. 4	(3)	50. 4π	(4)

Warm-Up 3

Answer	Difficulty	Answer	Difficulty
51. -1009	(3)	56. 13/18	(3)
52. 12	(3)	57. 25	(4)
53. 6/25	(5)	58. 348	(3)
54. 10,227	(3)	59. 1,000,000	(3)
55. $2\sqrt{5}$	(4)	60. $4\frac{1}{20}$	(3)

* The plural form of the units is always provided in the answer blank, even if the answer appears to require the singular form of the units.

Warm-Up 4

Answer	Difficulty			
61.	1/2	(3)	66.	3π (4)
62.	900	(4)	67.	432 (3)
63.	50	(3)	68.	4 (4)
64.	21	(4)	69.	27 (3)
65.	191	(4)	70.	48 (5)

Warm-Up 5

Answer	Difficulty			
71.	17	(3)	76.	$1/3$ (2)
72.	6	(2)	77.	15 (4)
73.	120	(5)	78.	63 (3)
74.	9	(3)	79.	4 (3)
75.	4	(3)	80.	1 (5)

Warm-Up 6

Answer	Difficulty			
81.	1320	(2)	86.	$3/8$ (4)
82.	128	(4)	87.	100 (2)
83.	5	(3)	88.	80 (2)
84.	3.81	(2)	89.	315 (4)
85.	126	(5)	90.	$1/4$ (3)

Warm-Up 7

Answer	Difficulty			
91.	12	(2)	96.	$\sqrt{3}/4$ (5)
92.	160	(4)	97.	9 (4)
93.	$5\sqrt{2}$	(5)	98.	11 (4)
94.	161	(3)	99.	3 (4)
95.	$7/36$	(3)	100.	1000 (3)

Warm-Up 8

Answer	Difficulty			
101.	68	(3)	106.	$11/18$ (4)
102.	40	(3)	107.	$\sqrt{2} - 1$ (5)
103.	144	(4)	108.	4 (5)
104.	$1/7$	(3)	109.	173 (5)
105.	81	(3)	110.	$1/4$ (5)

Warm-Up 9

Answer	Difficulty			
111.	$79/111$	(2)	116.	180 (3)
112.	$8/11$	(5)	117.	6 (4)
113.	$17/24$	(5)	118.	36 (7)
114.	$100\sqrt{10}$	(4)	119.	85 (2)
115.	305	(4)	120.	148 (4)

Warm-Up 10

Answer	Difficulty		
121. 23	(3)	126. $6\sqrt{7}$	(6)
122. 100	(3)	127. 9	(4)
123. $3/8$	(5)	128. 105	(5)
124. 5	(3)	129. $6/\pi$	(6)
125. 6720	(5)	130. 30	(4)

Warm-Up 11

Answer	Difficulty		
131. 20	(4)	136. 132	(6)
132. 76	(4)	137. 350	(4)
133. $3/8$	(5)	138. 28	(4)
134. 25	(4)	139. 18	(3)
135. $175/256$	(5)	140. $8/27$	(5)

Warm-Up 12

Answer	Difficulty		
141. 867	(3)	146. 81	(5)
142. 58	(5)	147. 58	(4)
143. 241	(5)	148. 12	(3)
144. 300	(3)	149. $10+2\sqrt{35}$	(6)
145. 0	(3)	or $2\sqrt{35}+10$	
		150. 8	(5)

Warm-Up 13

Answer	Difficulty		
151. 5	(4)	156. 9490	(5)
152. $18\sqrt{3}$	(3)	157. 13	(4)
153. 53	(3)	158. 105	(5)
154. 91	(4)	159. 17	(5)
155. 3	(4)	160. 25	(4)

Warm-Up 14

Answer	Difficulty		
161. 30	(4)	166. $(11\sqrt{10})/8$	(5)
162. 52	(2)	167. 45	(6)
163. $1/9$	(5)	168. 140	(4)
164. 1632	(4)	169. 7	(5)
165. $3/5$	(5)	170. $1/27$	(5)

Workout 1

Answer	Difficulty		
171. $\frac{3}{8}$	(2)	176. 33	(3)
172. 2.8	(4)	177. 14	(3)
173. 263	(5)	178. 144	(4)
174. 135	(3)	179. 5000	(3)
175. 30.4	(2)	or 5000.00	
		180. 87,672	(3)

Workout 2

Answer	Difficulty		
181. 100	(3)	186. 383	(2)
182. 2.5	(5)	187. 250	(5)
183. 62.5	(5)	188. 0.162	(2)
184. 55.71	(2)	189. 3	(4)
185. 33	(4)	190. 67.5	(4)

Workout 3

Answer	Difficulty		
191. 1530	(3)	196. 70	(3)
192. 45	(3)	197. 150π	(3)
193. 34	(3)	198. 127,750,000	(2)
194. 44	(4)	199. 26	(4)
195. 14	(3)	200. 70	(3)

Workout 4

Answer	Difficulty		
201. 75	(2)	206. $\frac{1}{3}$	(4)
202. 1.01	(3)	207. $\frac{32}{45}$	(3)
203. 28	(3)	208. 30	(2)
204. 0.924	(5)	209. 1300	(3)
205. 35.7	(3)	210. 2	(4)

Workout 5

Answer	Difficulty		
211. 3375	(5)	216. 3	(3)
212. 57	(4)	217. 20 or 20.00	(3)
213. 1,036,800	(5)	218. 4	(4)
214. 11	(5)	219. 50	(2)
215. $64 + 128\sqrt{2}$	(5)	220. $\frac{4}{7}$	(4)
or $128\sqrt{2} + 64$			

Workout 6

Answer	Difficulty		
221. 37.9	(5)	226. 52	(5)
222. 25	(2)	227. 2996	(3)
223. 7.6	(4)	228. 6	(3)
224. 28	(3)	229. 11	(4)
225. 1.47	(5)	230. 360	(4)

Workout 7

Answer	Difficulty		
231. 45	(4)	236. 78	(5)
232. 4	(6)	237. 128	(5)
233. Thursday	(4)	238. -1516.75	(4)
234. $\frac{8}{51}$	(3)	239. 225	(3)
235. 72	(5)	or 225.00	
		240. 24π	(5)

Workout 8

Answer	Difficulty		
241. 144	(4)	246. 11	(5)
242. 816	(5)	247. 26	(5)
243. 1729	(6)	248. 195	(6)
244. 5	(4)	249. $3\sqrt{7}$	(6)
245. 4.7	(6)	250. 87	(6)

PROBLEM INDEX

It is difficult to categorize many of the problems in the *MATHCOUNTS School Handbook*. It is very common for a MATHCOUNTS problem to straddle multiple categories and cover several concepts. This index is intended to be a helpful resource, but since each problem has been placed in exactly one category and mapped to exactly one Common Core State Standard (CCSS), the index is not perfect. In this index, the code **9 (3) 7.SP.3** refers to problem 9 with difficulty rating 3 mapped to CCSS 7.SP.3. For an explanation of the difficulty ratings refer to page 49. For an explanation of the CCSS codes refer to page 53.

NUMBER THEORY	45	(3)	4.OA.4	PLANE GEOMETRY	32	(2)	4.G.2	MEASUREMENT	40	(4)	8.G.5		
	56	(3)	4.OA.4		43	(4)	7.G.6		49	(3)	5.MD.1		
	59	(3)	SMP		53	(5)	7.G.4		50	(4)	G-SRT.5		
	68	(4)	N-RN.1		61	(3)	7.G.6		55	(4)	8.G.8		
	69	(3)	S-CP.9		66	(4)	G-C.2		67	(3)	6.G.1		
	78	(3)	SMP		82	(4)	7.G.6		96	(5)	7.G.6		
	84	(2)	SMP		106	(4)	7.G.4		126	(6)	8.G.8		
	109	(5)	SMP		107	(5)	7.G.6		132	(4)	8.G.5		
	110	(5)	8.EE.2		113	(5)	8.G.8		143	(5)	8.G.7		
	111	(2)	7.NS.2		118	(7)	G-SRT.4		155	(4)	7.RP.3		
	119	(2)	4.OA.4		123	(5)	G-SRT.6		172	(4)	6.RP.3		
	121	(3)	6.NS.4		133	(5)	7.G.6		181	(3)	6.EE.7		
	130	(4)	SMP		136	(6)	G-SRT.5		183	(5)	8.G.7		
	134	(4)	6.NS.4		149	(6)	A-REI.4		198	(2)	6.RP.3		
	139	(3)	SMP		152	(3)	G-SRT.6		SOLID GEOMETRY	60	(3)	7.G.6	
	148	(3)	7.NS.3		158	(5)	G-SRT.6			74	(3)	G-GMD.3	
	151	(4)	SMP		160	(4)	G-C.2			129	(6)	G-GMD.3	
	153	(3)	6.EE.2		163	(5)	G-SRT.5			177	(3)	8.G.9	
	156	(5)	6.NS.4		170	(5)	G-SRT.5			190	(4)	8.G.9	
	161	(4)	F-BF.2		178	(4)	8.G.5			208	(2)	8.G.9	
	193	(3)	SMP		187	(5)	G-SRT.6			219	(2)	7.G.6	
	194	(4)	SMP		192	(3)	8.G.5			230	(4)	G-GMD.3	
	201	(2)	7.NS.3		197	(3)	G-C.2			COORDINATE GEOMETRY	79	(3)	G-C.2
	227	(3)	8.EE.2		204	(5)	7.G.1				93	(5)	8.G.8
	229	(4)	6.NS.4		211	(5)	G-SRT.5		112		(5)	8.F.3	
	231	(4)	6.NS.4		215	(5)	G-SRT.6		166		(5)	8.F.3	
233	(4)	SMP	221	(5)	G-SRT.6	203	(3)	6.G.1					
237	(5)	6.NS.4	225	(5)	G-SRT.6	238	(4)	8.G.1					
241	(4)	N-RN.2	232	(6)	G-C.2	249	(6)	8.G.8					
246	(5)	SMP	236	(5)	G-C.2								
LOGIC	83	(3)	S-CP.9	240	(5)	7.G.6							
	92	(4)	SMP	245	(6)	G-SRT.6							
	131	(4)	SMP	250	(6)	G-SRT.6							
	145	(3)	SMP										
	167	(6)	SMP										
	216	(3)	S-CP.9										
	235	(5)	SMP										

ALGEBRAIC EXPRESSIONS & EQUATIONS

38 (3) 6.EE.2
 42 (4) 6.EE.9
 47 (4) 8.F.3
 51 (3) A-REI.4
 52 (3) 6.EE.9
 71 (3) 6.EE.7
 73 (5) A-REI.4
 80 (5) 8.EE.8
 81 (2) 6.EE.3
 89 (4) N-RN.1
 91 (2) 7.EE.4
 99 (4) N-RN.1
 101 (3) A-CED.1
 120 (4) N-RN.1
 142 (5) N-RN.1
 150 (5) 8.EE.8
 176 (3) 6.EE.2
 185 (4) 6.EE.7
 199 (4) A-SSE.3
 202 (3) A-REI.4
 209 (3) A-CED.1
 210 (4) A-SSE.3
 217 (3) 7.EE.4
 218 (4) A-REI.4
 224 (3) A-CED.1

STATISTICS

57 (4) 6.SP.5
 65 (4) 6.SP.5
 87 (2) 6.SP.5
 98 (4) 6.SP.5
 105 (3) 6.SP.5
 138 (4) SMP
 147 (4) 7.EE.4
 175 (2) 6.SP.2
 186 (2) 6.SP.2
 223 (4) 6.SP.5

PROBLEM SOLVING (MISC.)

54 (3) SMP
 58 (3) 4.OA.4
 70 (5) S-CP.9
 72 (2) SMP
 128 (5) SMP
 157 (4) G-CO.3
 162 (2) A-CED.1
 180 (3) 7.NS.3
 212 (4) SMP

GENERAL MATH

31 (1) 3.OA.8
 33 (2) SMP
 35 (3) 6.EE.2
 36 (2) 4.OA.2
 37 (2) 3.OA.8
 39 (4) SMP
 44 (2) 5.OA.1
 62 (4) SMP
 88 (2) 7.NS.3
 100 (3) N-RN.2
 114 (4) N-RN.2
 124 (3) 6.NS.4
 174 (3) 6.NS.3

PROBABILITY, COUNTING & COMBINATORICS

Probability Stretch¹
 34 (3) 7.SP.8
 46 (3) 7.SP.8
 64 (4) S-CP.9
 75 (3) S-CP.9
 76 (2) 7.SP.8
 85 (5) S-CP.9
 95 (3) 7.SP.8
 103 (4) 8.EE.1
 104 (3) 7.SP.7
 116 (3) S-CP.9
 125 (5) S-CP.9
 135 (5) 7.SP.8
 140 (5) 7.SP.7
 144 (3) S-CP.9
 146 (5) S-CP.9
 154 (4) S-CP.9
 159 (5) SMP
 168 (4) S-CP.9
 171 (2) 7.SP.7
 189 (4) S-CP.9
 196 (3) 7.SP.8
 206 (4) 7.SP.8
 207 (3) 7.SP.7
 213 (5) S-CP.9
 226 (5) SMP
 234 (3) 7.SP.8
 242 (5) S-CP.9
 247 (5) S-CP.9
 248 (6) 8.G.7

PERCENTS & FRACTIONS

48 (3) 6.RP.3
 77 (4) 7.RP.3
 86 (4) 7.NS.3
 137 (4) A-CED.1
 179 (3) 7.RP.3
 184 (2) 6.RP.3
 188 (2) 6.NS.1
 195 (3) 6.NS.1
 200 (3) A-CED.1
 205 (3) 7.RP.3
 222 (2) 7.RP.3
 239 (3) 7.NS.3

PROPORTIONAL REASONING

21 (5) 6.RP.3
 22 (4) 6.EE.7
 23 (3) 6.RP.3
 24 (4) 6.EE.7
 25 (4) 6.RP.3
 26 (3) 6.RP.3
 27 (5) 6.RP.3
 28 (3) 6.RP.3
 29 (4) 6.RP.3
 30 (3) 6.RP.3
 41 (2) 5.MD.1
 63 (3) 4.OA.2
 90 (3) 7.RP.2
 102 (3) 6.RP.3
 108 (5) 6.RP.3
 117 (4) 6.RP.3
 122 (3) 7.RP.3
 127 (4) 7.RP.3
 169 (5) SMP
 173 (5) 6.RP.3
 182 (5) 7.RP.2
 220 (4) 7.EE.4
 228 (3) 7.RP.2
 244 (4) SMP

SEQUENCES, SERIES & PATTERNS

Patterns Stretch²
 94 (3) F-BF.2
 97 (4) F-BF.2
 115 (4) F-BF.2
 141 (3) F-LE.1
 164 (4) F-LE.2
 165 (5) F-LE.2
 191 (3) F-LE.2
 214 (5) F-LE.2
 243 (6) F-BF.2

¹ CCSS 7.SP.8 & S-CP.9² CCSS F-BF.2