MATHCOUNTS®
COMPETITION SERIES
EST. 1983

2014 ☆ 2015
SCHOOL HANDBOOK

Warm-Up 1

1. _____ What is the sum of the two-digit multiples of 11?

2. _____ If $a \# b = a + 2b$, for integers a and b, what is the value of $3 \# 4$?

3. _____ times  Kimba chewed a piece of gum 42 times in one minute. If she continued to chew at the same rate, how many times would she chew her gum in 100 seconds?

4. _____ miles Carver jogs 4 times a week. Last week, his distances were $4\frac{1}{3}$, $3\frac{1}{2}$, $3\frac{5}{6}$ and $4\frac{1}{6}$, all measured in miles. What was Carver's average distance for the 4 days? Express your answer as a mixed number.

5. _____ socks Rob has 10 white, 8 red and 6 blue socks in his drawer. If he selects socks from the drawer randomly, without looking, what is the least number of socks Rob must select to guarantee that he has removed a pair of white socks?

6. _____ hamburgers At a particular restaurant, hamburgers are priced $3 each, 2 for $5 and 5 for $9. What is the maximum number of hamburgers that can be purchased for $48?

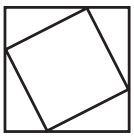
7. _____ Statement

_____ Statement

Which two of the following four statements, labeled A through D, are true statements?

A: Statement B is false, but statement C is true.
B: Statement C is true, but statement D is false.
C: Statement D is false, and statement A is false.
D: Statement A is true, and statement B is true.

8. _____ seconds How many seconds are in 3.14 hours?

9. _____  The vertices of the smaller square in the figure are at trisection points of the sides of the larger square. What is the ratio of the area of the smaller square to the area of the larger square? Express your answer as a common fraction.

10. _____ students Students at Central School were surveyed regarding lunch choices. Of the students that responded, exactly $\frac{1}{3}$ wanted more fresh fruits and vegetables as choices. Of those students not wanting more fresh fruits and vegetables, exactly $\frac{1}{8}$ wanted more seafood. What is the minimum number of students that responded to the survey?

Warm-Up 2

11. _____ Two cylinders are equal in volume. The radius of one is doubled, and the height of the other cylinder is increased to k times its original height. If the two new cylinders are equal in volume, what is the value of k?

12. _____ What positive integer must be included in the set {1, 2, 4, 8} so that the new set of five integers has a median that is equal to its mean?

13. _____ coins

Debbie has equal numbers of dimes and quarters with a total value of $1.40. How many coins does she have altogether?

14. _____ What is the average of the first 99 counting numbers?

15. _____ minutes How many fewer minutes does it take to drive 35 miles at 30 mi/h than to drive the same distance at 25 mi/h?

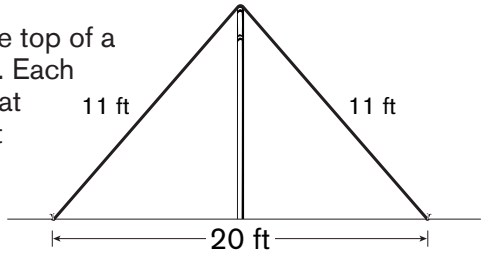
16. $ _____ After a 20% reduction, the price of a computer was $800. What was the price of the computer before the reduction?

17. _____ ounces Koka-Kola is sold in packages of eight 12-ounce bottles. Pepsy-Kola is sold in packages of six 16-ounce bottles. In ounces, what is the absolute difference in the total number of ounces in a package of Koka-Kola and a package of Pepsy-Kola?

18. (_____ , _____) Point A lies at the intersection of $y = x$ and $y = -\frac{2}{3}x + 5$. What are the coordinates of A? Express your answer as an ordered pair.

19. _____ When rolling two fair, standard dice, what is the probability that the sum of the numbers rolled is a multiple of 3 or 4? Express your answer as a common fraction.

20. _____ feet Two wires are connected by stakes from the ground to the top of a vertical pole that is anchored halfway between the stakes. Each wire is 11 feet long, and the stakes are 20 feet apart. What is the height of the pole? Express your answer in simplest radical form.

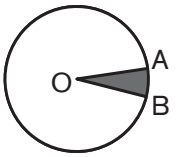


Workout 1

21. _____ What is the sum of the prime factors of 2015?

22. _____ What is $(3.5 \times 10^4)^2$ when written in scientific notation with four significant digits?

23. _____ meters The area of the shaded region of circle O is 9π m^2, and the measure of $\angle AOB$ is 22.5 degrees. What is the length of the radius of circle O?



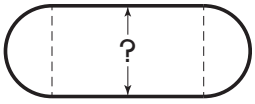
24. $ _____ In Fuelville, the cost of gas averaged $3.50 per gallon at the start of April, then rose 6% during April and dropped 10% during May. What was the cost of a gallon of gas at the end of May?

25. _____ ft^2 A square residential lot is measured to be 100 feet on each side, with a possible measurement error of 1% in each of the length and width. What is the absolute difference between the largest and smallest possible measures of the area given this possible error?

26. _____ years old Currently, the sum of the ages of Yumi, Rana and Victoria is 42 years. Four years ago, the sum of the ages of Rana and Victoria was equal to the current age of Yumi. What is Yumi's current age?

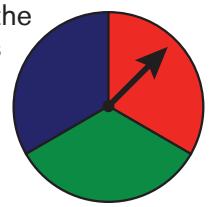
27. _____ students Of 600 students at Goodnight Middle School in Texas, 85% are not native Texans. Of those non-native students, 60% have lived in Texas more than 10 years, and 30 students have lived in Texas less than a year. How many non-native students have lived in Texas for at least 1 year but not more than 10 years?

28. _____ miles The track at Dividend Middle School, depicted here, has two semicircular ends joined by two parallel sides. The total distance around the track is 1/4 mile, and each of the semicircular ends is 1/4 of the total distance. What is the distance between the two parallel sides of the track? Express your answer as a decimal to the nearest hundredth.



29. _____ What is the sum of the coordinates of the point at which $y = x - 3$ and $y = -2x + 9$ intersect?

30. _____ A spinner is divided into three congruent sections colored red, blue and green. The numbers 1 through 6 appear on the faces of a fair number cube. When the pointer on the spinner is spun and the number cube is rolled, what is the probability that the pointer lands within the blue section and an even number is rolled? Express your answer as a common fraction.



Warm-Up 3

31. _____ What is the value of $\frac{6!}{5!+4!}$?

32. _____ The same digit A occupies both the thousands and tens places in the five-digit number 1A,2A2. For what value of A will 1A,2A2 be divisible by 9?

33. _____ pounds In Lewis Carroll's *Through the Looking-Glass*, this conversation takes place between Tweedledee and Tweedledum. Tweedledum says, "The sum of your weight and twice mine is 361 pounds." Tweedledee answers, "The sum of your weight and twice mine is 362 pounds." What is the absolute difference in the weights of Tweedledee and Tweedledum?

34. _____ pieces A 63-inch long string is cut into pieces so that their lengths form a sequence. First, a 1-inch piece is cut from the string, and each successive piece that is cut is twice as long as the previous piece cut. Into how many pieces can the original length of string be cut in this way?

35. _____ Two fair, six-sided dice are rolled. They are marked so one die has the numbers 1, 3, 5, 7, 9, 11 and the other has the numbers 2, 4, 6, 8, 10, 12. What is the probability that the sum of the numbers rolled is divisible by 5? Express your answer as a common fraction.

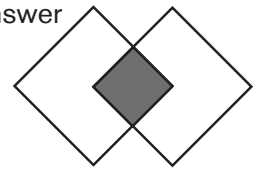
36. $_____ A shirt company charges a one-time setup fee plus a certain price per shirt. An order of 10 shirts costs $84. An order of 20 shirts costs $159. How much does an order of 30 shirts cost?

37. _____ On a number line, what is the nearest integer to π^2?

38. _____ minutes Cara needs to drive to Greenville. She can drive 50 mi/h along a 200-mile highway, or she can take a different route, which requires her to drive 150 miles at 60 mi/h and then 50 miles at 40 mi/h. How many minutes would Cara save by taking the faster route?

39. $_____ Square tiles with 4-inch sides are 20¢ each, and square tiles with 6-inch sides are 40¢ each. How much will Jerry save tiling a 4-foot by 6-foot floor with the 6-inch tiles laid side by side instead of the 4-inch tiles laid side by side?

40. _____ units^2 Two unit squares intersect at the midpoints of two adjacent sides, as shown. What is the area of the shaded intersection of the two square regions? Express your answer as a common fraction.



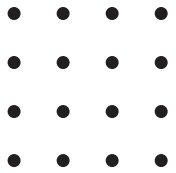
Warm-Up 4

41. _____ If $a - b = 0$, what is the value of $a \times b$? Express your answer in terms of a.

42. _____ When rolling two fair, eight-sided dice, each with faces numbered 1 through 8, what is the probability that the two numbers rolled have a sum of 9? Express your answer as a common fraction.

43. _____ If $2015 = 101a + 19b$, for positive integers a and b, what is the value of $a + b$?

44. _____ squares How many squares can be drawn using only dots in this grid of 16 evenly spaced dots as vertices?



45. _____ ounces A cup contains 6 ounces of milk. Two ounces of chocolate syrup are added to the cup and thoroughly mixed. Then 2 ounces of that mixture are poured out. How many ounces of chocolate syrup are in the remaining mixture? Express your answer as a mixed number.

46. _____ If $f(x) = \sqrt{x+4}$, for what value of x does $f(x) = 3$?

47. _____ What is the product of the greatest common factor and least common multiple of 48 and 72?

48. _____ trips There are 13 stations along the Cheshire Railroad, which runs in a straight line from east to west. A "trip" is defined by its starting and ending stations (regardless of intermediate stops) and must always go westward. How many different trips are possible along the Cheshire Railroad?

49. _____ units^2 What is the area of $\triangle ABC$ with vertices A(2, 3), B(17, 11) and C(17, 3) ?

50. _____ If x is positive, what is the result when $8x$ is doubled and then divided by one half of $8x$?

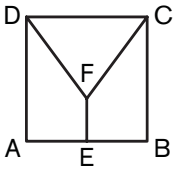
Workout 2

51. _____ points In a basketball game, Aisha and Britney scored 23 points in all; Aisha and Courtney scored 21 points in all; and Britney and Courtney scored 20 points in all. How many points did the three girls score altogether?

52. _____ % The peak of volcano Mauna Kea is 13,803 feet above sea level. When measured from its oceanic base, it measures 33,100 feet vertically to its peak. What percent of Mauna Kea's altitude is below sea level? Express your answer to the nearest whole number.

53. _____ inches A pizza of diameter 16 inches has an area large enough to serve 4 people. What is the diameter of a pizza with an area large enough to serve 12 people? Express your answer as a decimal to the nearest tenth.

54. _____ cm



Square ABCD has sides of length 12 cm. The three interior segments divide the square, as shown, into two congruent trapezoids and an isosceles triangle, all with equal areas. What is the length of segment CF?

55. _____ Given the following facts about the integers a, b, c, d, e and f, what is the value of a if $0 \le a \le 60$?

a is odd. $\quad c = \frac{b}{2}$ is even. $\quad e = \frac{d-1}{2}$ is odd.

$b = \frac{a-1}{2}$ is even. $\quad d = \frac{c}{2}$ is odd. $\quad f = \frac{e-1}{2}$ is even.

56. $ _____ The box office staff at a theater sets the ticket price so that total ticket sales will be $7200 if tickets are sold for all 240 seats. By what amount should they increase ticket prices if they want to keep total ticket sales at $7200 but they sell 15 fewer tickets?

57. _____ minutes Lindsey's e-mail account was bombarded with 34,000 spam messages. The web interface allows Lindsey to delete batches of 50 messages at one time, a process that takes 0.5 second to complete. What is the fewest number of minutes will it take Lindsey to delete all 34,000 messages if she deletes batches of 50 messages at a time? Express your answer to the nearest whole number.

58. _____ Let n represent the smallest positive integer such that $2015 + n$ is a perfect square. Let m represent the smallest positive integer such that $2015 - m$ is a perfect square. What is the value of $n + m$?

59. _____ Emilio started a job on July 1, 2000 at a salary of $40,000. If he got an increase of 10% each year, in what year did his salary first exceed $80,000?

60. _____ inches Anne is trying out for the track team, and she had three triple jump attempts. The distance of each triple jump attempt is measured to the nearest inch, and her first two attempts measured 36' 4" and 38' 4", respectively. If Anne's third attempt measured no less than her first attempt and no more than her second attempt, what is the difference between the greatest and least possible averages for the three triple jump attempts?

Warm-Up 5

61. $_____ Rachel, Siriana, Tanya and Ursula each contributed to a lottery pool. Rachel contributed $12, Siriana $15, Tanya $20 and Ursula $8. One of their tickets was a winner, paying $1100. If the winnings were distributed in proportion to each person's contribution, what was Siriana's share of the winnings?

62. _____ ways The Statesville Middle School basketball team has 8 players. If a player can play any position, in how many different ways 5 starting players be selected?

63. _____ What is 5 times the sum of all the distinct positive factors of 144?

64. _____ The denominator of a fraction is 2 more than its numerator. The reciprocal of this fraction is equal to the fraction itself. What is the sum of its numerator and denominator?

65. _____ Julius has some spare change consisting of quarters, dimes and nickels. If the ratio of quarters to dimes is 3:4 and the ratio of quarters to nickels is 4:5, what is the ratio of dimes to nickels? Express your answer as a common fraction.

66. _____ miles When Mr. Tesla drives 60 mi/h, the commute from home to his office takes 30 minutes less than it does when he drives 40 mi/h. What is the distance of Mr. Tesla's commute from home to his office?

67. _____ hours Jack Frost makes 18 snowballs every hour, but 2 snowballs melt each 15 minutes. How many hours will it take Jack to accumulate 2 dozen snowballs? Express your answer as a decimal to the nearest tenth.

68. _____ years old A mother was 26 years old when her daughter was born. The mother is now 6 years less than 3 times as old as her daughter. How old is the daughter now?

69. _____ red marbles A bag contains a total of 36 marbles colored either red or blue. Twice the number of red marbles is 6 less than the number of blue marbles. How many red marbles are in the bag?

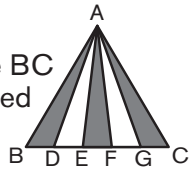
70. _____ When $a > 5$ and $b \leq 5$, $a \mathbin{@} b = (a + b)(a - b)$, and when $a \leq 5$ and $b > 5$, $a \mathbin{@} b = (b + a)(b - a)$. What is the value of $2 \times ((2 \mathbin{@} 6) \mathbin{@} -4) - 1$?

Warm-Up 6

71. _____ If $\frac{1}{n} + \frac{1}{2n} + \frac{1}{3n} = k$, what is the value of nk? Express your answer as a common fraction.

72. _____ minutes Ellie can paint a room in 2 hours, and Diedre can paint the same room in 4 hours. How many minutes will it take Ellie and Diedre, working together, to paint the room?

73. _____ cm^2 Equilateral triangle ABC has sides of length 10 cm. If D, E, F and G divide base BC into five congruent segments, as shown, what is the total area of the three shaded regions? Express your answer in simplest radical form.



74. _____ degrees Given that the Earth is turning at a constant rate around its axis and it makes exactly one complete rotation every 24 hours, how many degrees does the center of home plate at Fenway Park rotate around Earth's axis from 6 a.m. on May 8th to noon on May 10th of the same year?

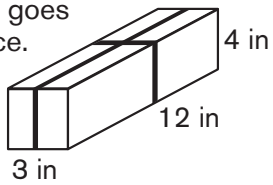
75. _____ sides If the difference in the degree measure of an interior and exterior angle of a regular polygon is 100°, how many sides does the polygon have?

76. _____ What positive four-digit integer has its thousands and hundreds digits add up to the tens digit, its hundreds and tens digits add up to its ones digit and its tens and ones digits add up to the two-digit number formed by the thousands and hundreds digits?

77. _____ meters The surface area of a sphere, in square meters, and its volume, in cubic meters, are numerically equal. What is the length of the radius of the sphere?

78. _____ slips Ben won a writing contest at school. To determine his prize, he draws slips of paper from a bag containing 16 slips, with the names of 4 different prizes written on 4 slips each. He continues to draw slips without replacement until he has selected 4 slips for the same prize. What is the maximum number of slips that Ben can draw without knowing his prize?

79. _____ inches The wrapped present shown has two loops of ribbon around it. Each loop goes completely around the box once and always runs down the middle of a face. The two loops overlap each other at a right angle, but the ends of the ribbon do not overlap when making a loop. How many inches of ribbon were used?



80. _____ times On a 12-hour digital clock, at how many times during a 24-hour day will all of the digits showing the time be the same?

Workout 3

81. _____ If $(x + y)^2 = x^2 + y^2$, what is the value of xy?

82. _____ % Kiera and Aubrey were the only candidates for SGA President. Kiera received 44% of the 7th graders' votes, and Aubrey received 42% of the 8th graders' votes. The student body consists of 325 7th graders and 350 8th graders, and every student voted for one of these two candidates. What percent of the students' votes did the winner receive? Express your answer to the nearest whole number.

83. _____ inches Enoch has a photo that is 8 inches high and 6 inches wide. He wants to enlarge it so that it can fit into a frame that is 10 inches by 7 inches. How much will he need to crop from the width to exactly fill the entire frame? Express your answer as a decimal to the nearest tenth.

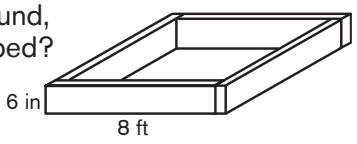
84. _____ miles At 1:00 p.m. an airplane left the local airport and flew due east at 300 mi/h. At 3:00 p.m. a second plane left the same airport and flew due north at 400 mi/h. Assuming the curvature of the earth is negligible, how many miles apart are the planes at 8:00 p.m.?

85. _____ If the mean of the integers 7, 3, 11, 13, 5 and x is 4 more than the mode of the six integers, what is the value of x?

86. _____ students After 6 new students entered the Happy Hearts Childcare Center and 2 students exited, there were 3 times as many students as before. How many students were present before students entered and exited the center?

87. _____ The product of three consecutive prime numbers is 2431. What is their sum?

88. _____ ft^3 Boards that are 8 feet long, 2 inches thick and 6 inches wide are used to make the raised garden bed shown here. If the boards are placed on level ground, how many cubic feet of soil are needed to completely fill the bed? Express your answer to the nearest whole number.



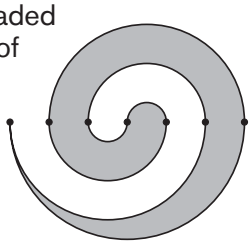
89. _____ dollars Bennie is ordering a new computer. A 6.25% sales tax will be added to the price of the computer, and then an $11 delivery charge will be added to that total. If Bennie has $500 to spend, what is the maximum price of a computer that he can afford? Express your answer to the nearest whole number.

90. _____ teams Each team in the softball league plays each of the other teams exactly once. If 21 games are played, how many teams are in the league?

Warm-Up 7

91. _____ % Mrs. Smith's 1st period class of 28 students averaged 84% on the last test. Her 2nd period class of 24 students averaged 86% on the same test. What must her 3rd period class of 32 students average so that all of the students in her three classes have a combined average of 80%?

92. _____ units^2 In the figure the collinear dots are equally spaced 2 units apart, and the shaded region is formed from two semicircles of diameter 2 units, two semicircles of diameter 6 units, two semicircles of diameter 10 units and one semicircle of diameter 12 units. What is the area of this shaded region? Express your answer in terms of π.



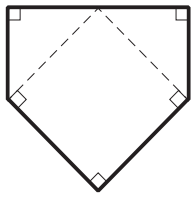
93. _____ If $5a - b - c = 36$ and $b = c = \frac{1}{2}a$, what is the value of a?

94. _____ seats A bus stopped at Main Street and boarded passengers, after which half of its seats were filled. At its next stop, at Oak Street, 2 passengers got off and 7 got on, and then 60% of the seats were filled. How many seats are there on the bus?

95. _____ Three fractions are inserted between $\frac{1}{4}$ and $\frac{1}{2}$ so that the five fractions form an arithmetic sequence. What is the sum of these three new fractions? Express your answer as a common fraction.

96. _____ In the equation $123 \times 4A6 = 5B548$, what is the value of $A \times B$, the product of the two missing digits?

97. _____ ft^2 Home plate at a school's baseball field was constructed by adding two right isosceles triangles to a 1-foot by 1-foot square, as shown. What is the area of this home plate? Express your answer as a common fraction.



98. _____ goldfish Elisa had a new fishbowl but no fish, so Kendell gave her half of his goldfish. Elisa's fishbowl was not very large, so she gave half of her new goldfish to Rocky. Rocky kept 8 of the goldfish he was given and gave the remaining 10 goldfish to Aster. What is the difference between Kendell's starting number of goldfish and the number of goldfish Elisa kept for her new fishbowl?

99. _____ teachers In a school that has 20 teachers, 10 teach mathematics, 8 teach social studies and 6 teach science. Two teach both mathematics and social studies, but none teach both social studies and science. How many teach both mathematics and science?

100. _____ A line segment has endpoints $(-5, 10)$ and (a, b). If the midpoint of the segment is $(13, -2)$, what is the absolute difference between a and b?

Warm-Up 8

101. ____ integers For how many positive integers n is it possible to have a triangle with side lengths 5, 12 and n?

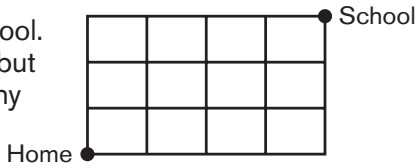
102. ____ laps By the time Shana had completed $\frac{3}{8}$ of her first lap in a race, she had also completed $\frac{1}{32}$ of the entire race. How many laps were there in the race?

103. ____

1	2	3	4	5	6
12	11	10	9	8	7
13	14	15	16	17	18
24	23	22	21	20	19
25	…	…	…	…	…

The counting numbers are written in a table with six columns so that in each successive row the numbers alternate between increasing from left to right and increasing from right to left, as shown. What is the first number in the 15th row?

104. ____ days Alvin lives 4 blocks west and 3 blocks south of his school. He wants to take a different route to school each day, but each route must be exactly 7 blocks long. For how many days can he do this without repeating any route?



105. ____ If x and y are positive integers, and the mean of 4, 20 and x is equal to the mean of y and 16, what is the smallest possible value of $x + y$?

106. ____ cm^2 Five rectangles are arranged in a row. Each rectangle is half as tall as the previous one. Also, each rectangle's width is half its height. The first rectangle is 32 cm tall. What is the sum of the areas of all five rectangles?

107. ____ If one-half of a number is 8 less than two-thirds of the number, what is the number?

108. ____ coins Colton's soda cost $1.95. He paid for it with nickels, dimes and quarters only. He used 2 more dimes than twice the number of nickels. The number of quarters was 1 more than the number of nickels and dimes combined. How many coins did he use to pay for the soda?

109. $ ____ Hiro started a new job with a salary of $50,000 per year. He received increases of 10%, 20% and 30% at the end of his first, second and third years of employment, respectively. How much did Hiro's salary increase after working three years at his new job?

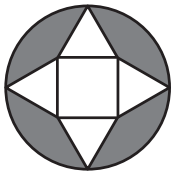
110. ____ A positive integer plus 4 times its reciprocal is equal to the product of the integer and 4 times its reciprocal. What is the integer?

Workout 4

111. _____ cm^2 What is the area of a circle that circumscribes a 6-cm by 8-cm rectangle? Express your answer to the nearest whole number.

112. _____ For positive integer n, $n? = n! \cdot (n-1)! \cdot \ldots \cdot 1!$ and $n\# = n? \cdot (n-1)? \cdot \ldots \cdot 1?$. What is the value of $4\# \cdot 3\# \cdot 2\# \cdot 1\#$?

113. _____ % An equilateral triangle is placed on the outside of each side of a square of length 6 cm, and a circle is then drawn through four vertices of the triangles as shown. What percent of the area of the circle is the unshaded region? Express your answer to the nearest whole number.



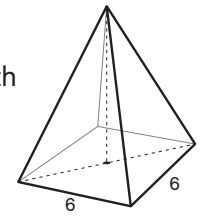
114. _____ If 21 is written as a sum of n consecutive positive integers, what is the greatest possible value of n?

115. _____ minutes Normally, the hose in Elena's garden will fill her small pool in 15 minutes. However, a leak in the hose allows $\frac{1}{3}$ of the water flowing through the hose to spill into the flower bed. How many minutes will it take to fill the pool? Express your answer as a decimal to the nearest tenth.

116. _____ people How many people must be in a group to guarantee that at least 3 of them share the same first initial and last initial?

117. _____ inches The ratio of width to height for Carl's TV screen is 16:9. Carl's TV screen has a 37-inch diagonal. What is the width of his TV screen? Express your answer as a decimal to the nearest tenth.

118. _____ m^3 A pyramid with a square base of side length 6 m has a height equal to the length of the diagonal of the base. What is the volume of the pyramid? Express your answer in simplest radical form.



119. _____ in^3 Boynton's sheet cake measures 18 × 24 inches and has a height of 4 inches. However, these measurements include a $\frac{3}{4}$-inch thick layer of frosting on the top and sides. What is the volume of Boynton's cake excluding the frosting? Express your answer to the nearest whole number.

120. _____ What is the smallest positive integer value of x for which $54x$ is a perfect square?

Warm-Up 9

121. ____ degrees The ratio of the angles of a quadrilateral is 3:4:5:6. What is the number of degrees in the largest of the angles?

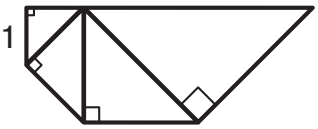
122. _____ Six positive integers have a mean of 6. If the median of these six integers is 8, what is the largest possible value of one of these six integers?

123. $ _____ The prize money for the Math County Science Fair was divided among the top three projects so that the 1st place winner got as much as the 2nd and 3rd place winners combined, and the 2nd place winner got twice as much as the 3rd place winner. If the total prize money awarded was $2400, how much did the 3rd place winner receive?

124. _____ For the function defined as $f(x) = \begin{cases} x + 4 \text{ when } x < -1 \\ x^2 - 6 \text{ when } x \geq -1 \end{cases}$, what is the value of $f(f(2))$?

125. _____ What is the value of $\frac{20^2 - 15^2}{18^2 - 17^2}$?

126. ____ units Starting with an isosceles right triangle with legs of length 1 unit, a second isosceles right triangle is built using the hypotenuse of the first triangle as a leg. A third isosceles right triangle is then built using the second triangle's hypotenuse as a leg, and so on, as demonstrated in the figure. If this pattern continues, what will be the number of units in the length of the hypotenuse of the 20th isosceles right triangle?



127. ____ miles Tony's Towing Service charges $30.00 to hook a vehicle to the tow truck and $1.75 for each mile the vehicle is towed. Mr. Alman's car broke down at school and was towed to his house. If the total amount charged by Tony's Towing was $59.75, what is the distance Mr. Alman's car was towed from the school to his house?

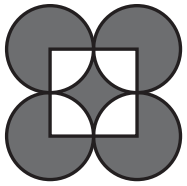
128. $ _____ A car's present value is $20,000, and its value decreases by the same percentage every year. At the end of one year, it will be worth $18,000. What will it be worth at the end of 3 years?

129. _____ What is the least possible sum of two positive integers whose product is 182?

130. ____ colors Chin-Chin is constructing a tetrahedron and a cube using gumdrops for vertices and toothpicks for edges. She wants to have different-colored gumdrops on the two ends of each toothpick, and no gumdrop used for the cube should be the same color as any gumdrop used for the tetrahedron. What is the least number of colors Chin-Chin needs?

Warm-Up 10

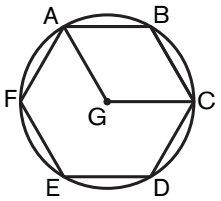
131. _____ mm^2



In the figure shown, four circles of radius 4 mm are centered at the corners of a square of side length 8 mm. What is the total area of the shaded regions? Express your answer in terms of π.

132. _____ Consider any positive three-digit integer that has all of its digits distinct and none equal to zero. What is the largest possible difference between such an integer and any integer that results from rearranging its digits?

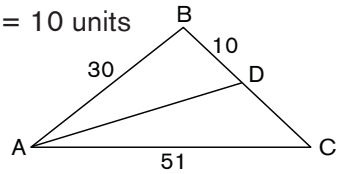
133. _____ $units^2$ A regular hexagon ABCDEF is inscribed in a unit circle with center G, as shown. What is the area of quadrilateral ABCG? Express your answer as a common fraction in simplest radical form.



134. _____ inches The amount of paint required to cover a surface is directly proportional to the area of that surface. The amount of paint needed to cover five spheres of radius 10 inches is the same as the amount of paint required to cover a solid right circular cylinder with radius 20 inches. What is the height of the cylinder?

135. _____ If $x + 2y + 3z = 6$, $2x + 3y + z = 8$ and $3x + y + 2z = 10$, what is the value of $x + y + z$?

136. _____ units In triangle ABC, segment AD bisects angle A. If AB = 30 units, BD = 10 units and AC = 51 units, what is the length of segment BC?



137. _____

1
2
3,4
5,6,7,8,9,10,11

The positive integers are written in order in rows of various lengths. The first row contains the number 1. For every row after the first, the number of entries in the row is the sum of the numbers in the previous row. The first four completed rows are shown. What is the last number in the sixth row?

138. _____ If x and y are negative integers and $x - y = 1$, what is the least possible value for xy?

139. _____ $units^2$ ABCD is a square with side length 4 units, and AEFC is a rectangle with point B on side EF. What is the area of AEFC?

140. _____ liters One hundred liters of a salt and water solution contains 1% salt. After some of the water has evaporated, the solution contains 5% salt. How many liters of water evaporated?

Workout 5

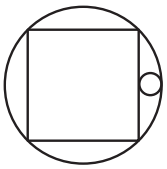
141. _____ The mean of a set of n numbers is 12, and the mean of a set of $3n$ numbers is 6. What is the mean when the two sets are combined? Express your answer as a decimal to the nearest tenth.

142. _____ units^2 The diagonals of convex quadrilateral ABCD intersect at X. If the areas, in square units, of $\triangle AXB$, $\triangle BXC$ and $\triangle CXD$ are 8, 15 and 5, respectively, what is the area of $\triangle DXA$? Express your answer as a decimal to the nearest tenth.

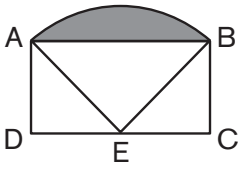
143. _____ What is the sum of all of the four-digit numbers whose digits are permutations of 1, 2, 3 and 4?

144. _____ Duncan picks a number and will subtract 6 each second. Taz picks a different number and will add 8 each second. They begin at the same time, and after 14 seconds, both arrive at the number 25. What is the sum of Duncan's and Taz's starting numbers?

145. _____ units A square is inscribed in a circle of radius 2 units, and then the largest possible circle is inscribed between the square and the original circle, as shown. What is the radius of the inscribed circle? Express your answer as a decimal to the nearest tenth.



146. _____ cm^2



In rectangle ABCD, shown here, AB = 4 cm and BC = 2 cm. If E is the midpoint of side DC and also is the center of a circle that contains points A and B, what is the area of the shaded segment of the circle determined by chord AB? Express your answer as a decimal to the nearest tenth.

147. _____ : _____ a.m. Ellie and Emma live 1.04 miles from each other. They decide to meet by walking toward each other, Ellie at 2.4 mi/h and Emma at 2.8 mi/h. If they both leave at 8:00 a.m., at what time in the morning will they meet?

148. _____ What is the smallest absolute difference between the squares of two distinct positive four-digit integers?

149. _____ bowls At the school fair, MATHCOUNTS parents sold chocolate and vanilla ice cream as a fund-raiser. Forty bowls of chocolate ice cream were sold for $2.15 per bowl. Bowls of vanilla ice cream sold for $1.90 each. How many bowls of ice cream were sold if the total amount of money collected was $158.20?

150. _____ Driving at an average speed of 66 mi/h, Dusty traveled m miles in n minutes. What is the least possible whole-number value of n such that the value of m also is a whole number?

Warm-Up 11

151. _____ times The indicator lights on two different pieces of machinery blink at different intervals, one every 4 seconds and the other every 7 seconds. If they blink together at 10:00 p.m., how many more times will they blink together before 10:15 p.m.?

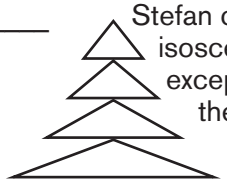
152. _____ base 10 What is the absolute difference, expressed in base 10, between the largest three-digit base 5 number and the smallest four-digit base 4 number?

153. _____ The Chug-A-Long Train Company boxes toy trains with either 5 cars or 7 cars per box. The trains in stock have a total of 53 cars. If Charles selects a box at random, what is the probability that the box contains 7 cars? Express your answer as a common fraction.

154. _____ Given the following facts about the numbers a, b, c and d, what is the value of $a + b$? Express your answer as a mixed number.

$ab = 1$
$bc = -9$
$b + c + d = 0$
$b = -c$
$c < -a$

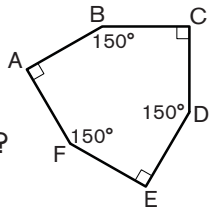
155. _____ units^2 Twenty-seven unit cubes are arranged to form a $3 \times 3 \times 3$ cube. The center unit cube from each face is then removed. What is the surface area of the resulting solid?

156. _____  Stefan created this tree design in his computer drawing class. It consists of four isosceles triangles of the same height arranged vertically as shown. With the exception of the top triangle, the apex of each triangle is the midpoint of the base of the triangle above it, and the base of each triangle is 50% larger than the base of the triangle above it. What is the ratio of the area of the smallest triangle to the area of the largest? Express your answer as a common fraction.

157. _____ (,) What are the coordinates of the point at which the segment with endpoints (2, 6) and (5, 9) intersects the segment with endpoints (−1, −1) and (5, −7)? Express your answer as an ordered pair.

158. _____ If a and b are positive integers such that $ab = 48$ and $a - b = 8$, what is the value of $a + b$?

159. _____ As shown, convex hexagon ABCDEF has right angles at A, C and E, and 150-degree angles at B, D and F. If each side is $\underline{2}$ inches long, its area can be expressed in simplest radical form as $p + q\sqrt{3}$. What is the value of pq?



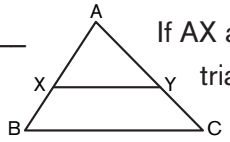
160. _____ integers How many integers from 100 to 999 have three different digits?

Warm-Up 12

161. _____ What is the value of $\frac{2 \times 4 \times 6 \times 8 \times 10 \cdots \times 20}{10!}$?

162. _____ What is the sum of all positive integers less than 20 that cannot be written as the sum of two prime numbers?

163. _____ cm A circle has a diameter of 8 cm. A chord perpendicular to its diameter divides the diameter into segments of lengths 1 cm and 7 cm. What is the length of the chord? Express your answer in simplest radical form.

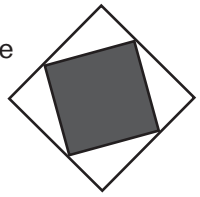
164. _____ 

If AX and AY are $\frac{2}{3}$ of AB and AC, respectively, what is the ratio of the area of triangle AXY to trapezoid XYCB? Express your answer as a common fraction.

165. _____ integers The *digital sum* of a number is the sum of its digits. How many positive three-digit integers have a digital sum of 5?

166. _____ mi/h The Zed Zee courier drove to a location 180 miles away to deliver an urgent package. The courier's average speed driving from the delivery location back to her starting location was 20 mi/h less than her average speed driving to the delivery location. If the entire trip took 7.5 hours, what was her average speed driving to the delivery location?

167. _____ One square is inscribed in another so that the sides of the inner square make 30-degree and 60-degree angles with the sides of the outer square. Each side of the inner square is 4 units, and the area of the outer square, in simplest radical form, is $a + b\sqrt{3}$. What is $a + b$?



168. _____ A set is said to be *closed* under multiplication if the product of elements in the set also is an element in the set. For what number k is the set $\{0, -1, k\}$ closed under multiplication?

169. _____ units^3 A certain box of width w has a length that is twice its width, and its height is three times its width. What is the total volume of 24 of these boxes? Express your answer in terms of w.

170. _____ If $a_1 = 13$ and $a_n = 77$, for an arithmetic sequence of integers, $a_1, a_2, a_3, \ldots, a_n$, with n terms, what is the median of all possible values of n?

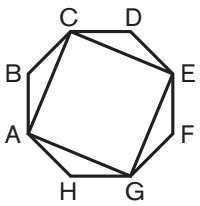
Workout 6

171. _____ 1s Becky tries an experiment. She writes some numbers on the blackboard and then applies the following rule: she picks any number on the board that is greater than 1, erases it and replaces it with the list of its proper divisors. For example, if the number 6 was on the board, she would apply the rule by erasing the 6 and replacing it with the numbers 1, 2 and 3. The experiment ends when there are only 1s left on the board. If Becky begins with just the number 72 on the board, how many 1s will be on the board when she is finished?

172. _____ mm A bug is walking on the ticking second hand of a clock, starting from the center and walking outward. Every second, the bug walks 1 mm along the stationary second hand, and then the hand ticks while the bug stands still. If the bug starts at the very center of the clock and proceeds for exactly 60 seconds, what is the total distance that the bug will travel? Express your answer as a decimal to the nearest tenth.

173. _____ seconds A Ferris wheel has the same height as a building with 60 floors of identical height. After boarding at the bottom of the Ferris wheel, Courtney used a stopwatch to find that it took 8 minutes 26 seconds to rise to the top of the 45th floor of the building. How many seconds will it take from there for the Ferris wheel to bring her back around to where she started, assuming the wheel rotates at a constant rate?

174. _____ units^2 Regular octagon ABCDEFGH has side-length 1 unit. What is the area of square ACEG? Express your answer as a decimal to the nearest tenth.



175. _____ strings How many distinct three-letter strings can be formed using three of the five letters in the word *SILLY*?

176. _____ bunnies Sammie took his little sister to the petting zoo. His sister really liked the area with chicks and bunnies. Altogether, in the group of 27 chicks and bunnies, Sammie counted 78 legs. Assuming every chick had two legs and every bunny had four legs, how many bunnies were in the group?

177. _____ % A path crosses a rectangular field on a diagonal. If someone travels across the field on the diagonal, instead of walking along the sides, what is the greatest possible percent reduction in total distance traveled? Express your answer to the nearest whole number.

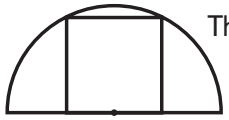
178. _____ cm^3 For a particular rectangular solid with integer dimensions, the sum of its length, width and height is 50 cm. What is the absolute difference between the greatest possible volume and the least possible volume of the solid?

179. _____ A jar contains 28 red jelly beans, 14 black jelly beans and 6 green jelly beans. What is the probability that two jelly beans selected at random, and without replacement, from this jar are the same color? Express your answer as a common fraction.

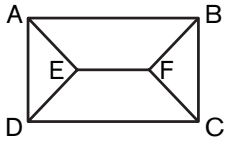
180. _____ What is the units digit of the product $2^{2015} \times 7^{2015}$?

Warm-Up 13

181. _____ perfect squares How many perfect squares are factors of 12!?

182. _____  The figure shows a square inscribed in a semicircle. What is the ratio of the radius of the semicircle to the side length of the square? Express your answer as a common fraction in simplest radical form.

183. _____ units^2 In the figure shown, AE = ED = EF = BF = CF = 1 unit, and $m\angle AED = m\angle BFC = 90$ degrees. What is the area of rectangle ABCD? Express your answer in simplest radical form.



184. _____ What is the decimal difference between 1111_3 and 1111_2?

185. _____ inches A candle that burns at a uniform rate was 11 inches tall after burning for 4 hours and 8 inches tall after burning for a total of 6 hours. How many inches tall was the candle before it was lit?

186. _____ minutes The express train from Addington to Summit travels the 18-mile route at an average speed of 72 mi/h, stopping only in Summit. The local train stops for 1.5 minutes at each of 6 stops between these two locations, and it averages 54 mi/h while it is in motion. How many minutes more does the local train take for this trip than the express train?

187. _____ Jack randomly chooses one of the positive integer divisors of 20, and Jill randomly picks one of the positive integer divisors of 30. What is the probability that Jack and Jill pick the same number? Express your answer as a common fraction.

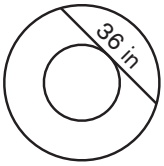
188. _____ miles One dragonfly flew in a straight path at a rate of 36 mi/h for 45 minutes. Meanwhile, a second dragonfly rode for 45 minutes on the windshield of a car that was driving in a straight path at 60 mi/h. How many more miles did the second dragonfly travel than the first dragonfly?

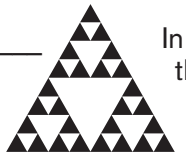
189. _____ The sum of two numbers is 1, and the absolute difference of the two numbers is 2. What is the product of the two numbers? Express your answer as a common fraction.

190. _____ If $(2a - 3b)^{\sqrt{x}} = 16a^4 - 96a^3b + 216a^2b^2 - 216ab^3 + 81b^4$, what is the value of x?

Warm-Up 14

191. _____ in^2 Two concentric circles and a line segment tangent to the smaller circle are shown. If the length of the line segment is 36 inches, what is the area of the region between the circles? Express your answer in terms of π.



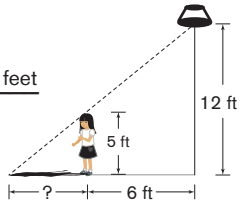
192. _____  In the equilateral triangle, each downward facing white triangle has its vertices at the midpoints of the sides of the larger upward facing triangle that just contains it. What fraction of the entire figure is white? Express your answer as a common fraction.

193. _____ For non-negative integers m and n, $\frac{m+n}{m-n} = \frac{25}{4}\left(\frac{m-n}{m+n}\right)$ and $m > n$. What is the value of $\frac{m}{n}$? Express your answer as a common fraction.

194. _____ cm^3 How many cubic centimeters of silver are needed to cover just the faces of a cube with a layer of silver that is 1 mm thick if the cube has edges of length 6 cm? Express your answer to the nearest whole number.

195. _____ Stage 1 of a pattern is a black square of side length 2015 units. At stage 2, a white square of side length 2014 units is placed on top of the square in stage 1 and positioned so that the upper left vertices of the squares coincide. The pattern continues with alternating white and black squares placed and positioned in this manner at each stage. Each new square placed has sides 1 unit shorter than the square placed at the previous stage. The first five stages are shown. At stage 2015, what portion of the figure will be black? Express your answer as a common fraction.

196. _____ integers The domain of $f(x) = x^2 - 3$ is $\{-4, -3, ..., 3, 4\}$. How many integers are in both the range and the domain of f?

197. _____ feet 

The lamp of a streetlight is 12 feet above the street below. A girl who is 5 feet tall stands at a point 6 feet from the spot directly below the light. How long is her shadow? Express your answer as a mixed number.

198. _____ integers How many positive integers have the same digits in the same order when written in base 7 and in base 13?

199. _____ Maggie writes the numbers 1, 2, 3, …, 10 on separate slips of paper and tosses the 10 slips into a hat. She then randomly pulls three slips from the hat at the same time. What is the probability that the arithmetic mean of the three selected values is in fact written on one of those same three slips? Express your answer as a common fraction.

200. _____ If $2015 + a = b$ for positive integers a and b, both of which are palindromes, what is the smallest possible value of a?

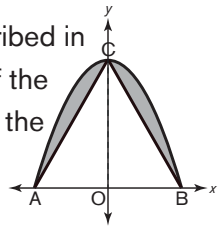
Workout 7

201. _____ If a, b, c and d are positive integers such that $a^b c^d = 2^{10} \times 7^9$, what is the least possible value of $a + b + c + d$?

202. _____ What common fraction is equivalent to $0.2\overline{36}$?

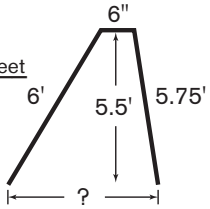
203. _____ units A 3-4-5 right triangle made of paper is cut along the altitude from the right angle, resulting in 2 smaller right triangles. These 2 triangles are then cut along the altitudes from their right angles, then the 4 resulting right triangles are cut the same way and finally, 8 triangles are cut the same way, resulting in 16 smaller right triangles. What is the sum of the perimeters of these 16 triangles? Express your answer as a decimal to the nearest tenth.

204. _____ units^2 The figure shows $\triangle ABC$, with base length $b = AB$ and height $h = CO$, inscribed in the region bounded by the curve $y = -\frac{1}{4}x^2 + 16$ and the x-axis. If the area of the entire region bounded by the curve and the x-axis is $\frac{2}{3}bh$, what is the area of the shaded region? Express your answer as a decimal to the nearest tenth.



205. _____ What is the value of $1^2 - 2^2 + 3^2 - 4^2 + \ldots + 99^2 - 100^2$?

206. _____ feet A ladder is 5.5 feet high when opened. The platform at the top is 6 inches wide and parallel to the floor, and the two supports are 6 feet and 5.75 feet, as shown. How many feet long apart are the two supports on the floor? Express your answer as a decimal to the nearest hundredth.



207. _____ The product of three consecutive odd integers is between 64,000,000 and 65,000,000. What is the greatest of the three integers?

208. _____ What is the 2015th digit after the decimal point in the decimal representation of $\frac{1}{13}$?

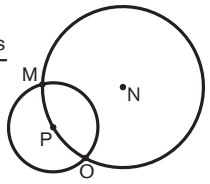
209. _____ sprinklers Jason has a square garden with an area of $\frac{1}{10}$ acre. He is installing sprinklers that each water a circular area with a radius of 11 feet. Jason does not want water from the sprinklers to extend beyond the perimeter of the garden. He also does not want any portion of the garden to receive water from more than one sprinkler, though he realizes that by doing so some portions of the garden will not be watered. Given that an acre is equivalent to 43,560 ft^2, what is the maximum number of sprinklers Jason can install?

210. _____ years A certain computer program will take 2000 years to run using current technology. Every year, advances in technology make it possible to run the program in half the time it would have taken starting in the previous year. However, once the program is started it cannot be interrupted to apply newer technology. Including the years spent waiting to start, what is the least number of years it will take to finish running the program? Express your answer to the nearest whole number.

Warm-Up 15

211. _____ ways In how many ways can the vertices of an equilateral triangle be colored with four available colors? Two colorings are considered the same if they can be obtained from each other by any combination of rotations and reflections.

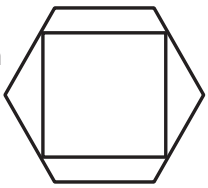
212. _____ degrees



Circle P has its center on circle N. The central angle formed by the radii of circle N that intersect M and O, the points of intersection of the circles, has measure 64 degrees. What is the measure of the central angle formed by the radii of circle P that intersect points M and O?

213. _____ Given that $f(x) = 3x - 7$ and $g(x) = x^2 - 4$, what is the value of $f(g(f(3)))$?

214. _____ cm A square has its four vertices on the sides of a regular hexagon with side length 1 cm. What is the side length of the square? Express your answer in simplest radical form.

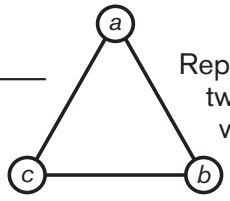


215. _____ % The original price for a pair of shoes was increased by 150%, and then this new price was decreased by 75%. By what percent must the current price be increased to return to the original price?

216. _____ Let P(n) denote the probability that a randomly selected n-digit number contains the digits 42, adjacent and in that order, among its digits. Two 4-digit examples are 3422 and 4205. What is the absolute difference between P(2) and P(3)? Express your answer as a common fraction.

217. _____ % Each year for the first five years of life, a baby elephant's weight increases by 10%. By what percent of its birth weight does an elephant's weight increase during these five years? Express your answer to the nearest whole number.

218. _____



Replace a, b and c with three different positive integers so that the sum of the two numbers along each side is a perfect square. What is the smallest possible value of the sum $a + b + c$?

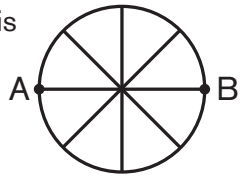
219. $ _____ Ronny had 9 oranges, and Donny had 15 oranges. They met up with Lonny, who had no oranges. Lonny gave $8 to Ronny and Donny, and the three of them shared the oranges equally. If Ronny and Donny split the $8 in proportion to the number of oranges each contributed, how much of the $8 should Ronny receive?

220. _____ The point (8, k) in the first quadrant is the same distance from the point (0, 4) as it is from the x-axis. What is the value of k?

Warm-Up 16

221. _____ If the counting numbers are written in order, what is the value of the 2015th digit written?

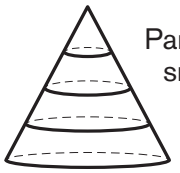
222. _____ cm The circle shown has four equally-spaced diameters of length 2 cm. What is the length of the longest path that can be drawn in one continuous pen stroke from A to B without retracing and without having the path cross itself? (The path may meet itself only at the center but may not cross over itself.) Express your answer in terms of π.



223. _____ cm Two chords of a circle intersect. The point of intersection divides the first chord into two segments of length 5 cm and 8 cm and divides the second chord into two segments, one which has length 4 cm. How long is the second chord?

224. _____ vertices A soccer ball is a polyhedron comprised of 12 pentagons and 20 hexagons. How many vertices does a soccer ball have?

225. _____



Parallel planes divide a cone into a smaller cone and three frustums so that the smaller cone and the three frustums have equal heights, as shown. What is the ratio of the volume of the smallest frustum to the volume of the largest frustum? Express your answer as a common fraction.

226. _____ All points with coordinates (x, y) that are equidistant from the points (1, 3) and (7, 11) lie along a single line. When the equation of the line is written in the form $y = mx + b$, what is the value of b?

227. _____ A sequence begins 1, 2, …, and each term after the second term is the sum of all preceding terms. What is the 15th term of this sequence?

228. _____ Abe chooses a number from Group A, Bob chooses a number from Group B and Charlie chooses a number from Group C.

Group A:	741	345	624	813
Group B:	519	825	717	456
Group C:	134	260	503	152

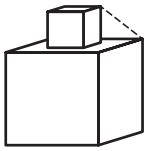
Then Abe chooses a digit X from his number, Bob chooses a digit Y from his number and Charlie chooses a digit Z from his number. The digits are arranged to form the three-digit number XYZ. Abe, Bob and Charlie, in that order, then choose different digits from their selected numbers to form a second three-digit number. Finally, in the same order, they use the remaining digits to form a third three-digit number. What is the sum of the three numbers that are formed?

229. _____ If $6^{12} = 6(6^n + 6^n + 6^n + 6^n + 6^n + 6^n)$, what is the value of n?

230. _____ ways Donald has nine one-day passes to Dizzyworld. He can go alone and use one, or he can take a friend and use two. If he visits every day until he uses all the passes, in how many different ways can he use them? Using two a day for four days and then going alone at the end (2, 2, 2, 2, 1) is different from reversing the order (1, 2, 2, 2, 2).

Workout 8

231. _____ inches



A cube with 2-inch edges is centered on a cube with 4-inch edges so that corresponding edges are parallel, as shown. What is the distance from a top vertex of the smaller cube to the nearest vertex of the larger one? Express your answer in simplest radical form.

232. $_____ Writing down the four-digit dollar amount of a deposit, a bank teller accidentally transposed two adjacent digits. What is the greatest possible value of the resulting difference between the correct amount and the incorrect amount?

233. _____ Two equilateral triangles are drawn in a square. Two opposite sides of the square coincide with one side of each of the triangles, as shown. What is the ratio of the area of the center rhombus to the area of the entire square? Express your answer as a decimal to the nearest thousandth.

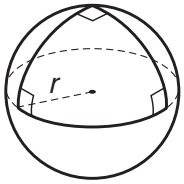
234. _____ paths

B	A	N
A	N	A
N	A	N

How many paths through this grid spell the word *BANANA*? A path consists of a series of moves from one cell to any cell with which it shares a side and may include multiple moves to a single cell.

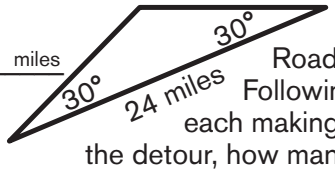
235. _____ On average, Josiah allows one hit for every five pitches. If he were to throw nine pitches, what is the probability that he would allow exactly three hits? Express your answer as a decimal to the nearest hundredth.

236. _____ The figure shows a triangular region bounded by three congruent arcs drawn on the surface of a sphere so that the intersection of any two arcs is a right angle. The triangular region's perimeter and surface area have numerical values A and B, respectively. If $\frac{A^2}{B} = \pi k$, what is the value of k? Express your answer as a decimal to the nearest tenth.



237. _____ The two solutions of the equation $x^2 + 7x + 5 = 0$ each can be expressed as a common fraction in simplest radical form as $\frac{l + u\sqrt{m}}{p}$. What is the least value of the product *plum*?

238. _____ miles



Road repairs prevented Anita from taking the 24-mile direct route home. Following a detour, she traveled on two different roads of equal distance, each making a 30-degree angle with the main road, as illustrated. As a result of the detour, how many more miles did Anita travel? Express your answer as a decimal to the nearest tenth.

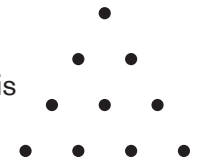
239. _____ units^2 Points A(−1, 2), B(−2, −5) and C(7, −2) are on a circle. What is the area of the circle? Express your answer as a decimal to the nearest tenth.

240. _____ The sum of the pairwise products of three consecutive natural numbers is 8111. What is the largest of the three numbers?

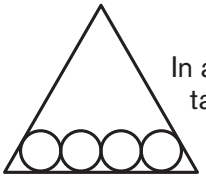
Warm-Up 17

241. _____ In the sum $4 + 2\sqrt{2} + 2 + \sqrt{2} + \ldots$ each term is obtained by dividing the previous term by $\sqrt{2}$. If the sum of the series, in simplest radical form, is $m + n\sqrt{2}$, what is the value of $m + n$?

242. _____ circles How many different circles can be drawn that intersect exactly four points in this triangular grid, made up of 10 points equally spaced?



243. _____ cm



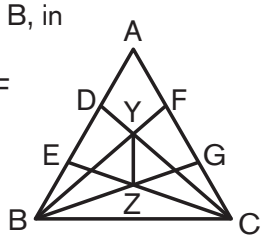
In an equilateral triangle with edge length 12 cm, four congruent circles are tangent to each other and at least one side of the triangle as shown. What is the radius of each circle? Express your answer in simplest radical form.

244. _____ Isosceles triangle ACB has a right angle at C and shares a leg with equilateral triangle BCD of side length 2 in. The triangles, otherwise, do not intersect. Segments BC and AD intersect at E. What is the value of BE/EC? Express your answer in simplest radical form.

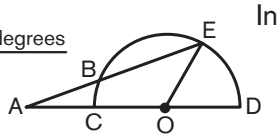
245. _____ What is the value of $a^2(b + c) + b^2(a + c) + c^2(a + b)$ if $a + b + c = 6$, $a^2 + b^2 + c^2 = 40$ and $a^3 + b^3 + c^3 = 200$?

246. _____ days Kevin and Devin each make one hat per day for charity, but they started on different days. Today, Kevin made his 520th hat, and Devin made his 50th. A celebration is planned for the next day that Kevin's hat count is evenly divisible by Devin's hat count. In how many days from today will they celebrate?

247. _____ In equilateral triangle ABC with side length 6 inches, points A, D, E and B, in that order, are equally spaced along side AB, and points A, F, G and C, in that order, are equally spaced along side AC as shown. Segments BF and CD intersect at Y, and segments BG and CE intersect at Z. When expressed as a common fraction in simplest radical form, the length of segment YZ is $\frac{r\sqrt{3}}{s}$ inches. What is the value of $r + s$?



248. _____ degrees



In the figure, $\overset{\frown}{CBD}$ is a semicircle with center O and diameter CD. If AB = OD and the measure of angle EOD is 60 degrees, what is the measure of angle A?

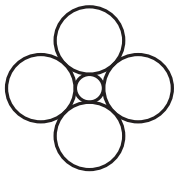
249. _____ times During a game of paintball, ten friends were positioned in a field so that no two of them stood the same distance apart. Each person aimed at his or her closest opponent, and at the signal everyone fired. What is the maximum number of times one player could have been hit?

250. _____ members In a certain state legislature, a proposed bill was defeated with 6 fewer votes for the bill than against it. After the bill was amended, 9 members who had previously voted against the bill were now for it. This resulted in 60% of the legislature now being in favor of the bill. If all those who previously voted for the bill remained in favor of it, how many members are in this state legislature, assuming every member voted each time?

Warm-Up 18

251. _____ units^2 An equiangular hexagon has side lengths 3, 4, 5, 3, 4 and 5 units in that order. What is its area? Express your answer as a common fraction in simplest radical form.

252. _____ mm Each of the four large circles shown here is tangent to two other circles of equal size and is tangent to the center circle, which has radius 1mm. What is the radius of each of the large circles? Express your answer in simplest radical form.



253. _____ units^2 If point Q lies on side AB of square ABCD such that QC = $\sqrt{10}$ units and QD = $\sqrt{13}$ units, what is the area of square ABCD?

254. _____ If $45_a = 54_b$ for positive integers a and b, what is the smallest possible value of $a + b$?

255. _____ If $\sqrt{2\sqrt[3]{2\sqrt[4]{2}}} = 2^c$, what is the value of c? Express your answer as a common fraction.

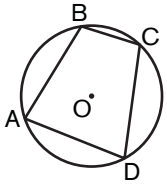
256. _____ Connecting the centers of the four faces of a regular tetrahedron creates a smaller regular tetrahedron. What is the ratio of the volume of the smaller tetrahedron to the volume of the original one? Express your answer as a common fraction.

257. _____ Daniel began painting a room at 9:00 a.m. Yeong, who can paint twice as fast as Daniel, started helping Daniel at 9:20 a.m., and they worked together until the room was fully painted at 10:00 a.m. What fraction of the room had been painted by 9:30 a.m.? Express your answer as a common fraction.

258. _____  Three different dots are randomly chosen from the 16 equally spaced dots in the grid shown. What is the probability that the three dots are collinear? Express your answer as a common fraction.

259. _____ pounds To weigh an object by using a balance scale, Brady places the object on one side of the scale and places enough weights on each side to make the two sides of the scale balanced. Brady's set of weights contains the minimum number necessary to measure the whole-number weight of any object from 1 to 40 pounds, inclusive. What is the greatest weight, in pounds, of a weight in Brady's set?

260. _____ degrees Quadrilateral ABCD is inscribed in circle O as shown. Arc AB = 100 degrees, and arc BC = 50 degrees. What is the measure of angle ADC?

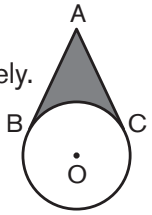


Workout 9

261. _____ If a, b, c and d represent four different positive integers such that $a^2 + d^2 = b^2 + c^2$, what is the least possible value of $|d - a|$?

262. _____ units² The line with equation $ax + by = c$, where a, b and c are positive, forms a right triangle with legs on the x- and y-axes. What is the area of the triangle? Express your answer as a common fraction in terms of a, b and c.

263. _____ in² In the figure, segments AB and AC are tangent to circle O at B and C, respectively. If AB = 12 inches, and minor arc BC measures 120 degrees, what is the area of the shaded region? Express your answer as a decimal to the nearest tenth.



264. _____ Each morning, Khalid and Jack arrive at the dining hall separately, at a random time from 8:00 to 9:00 a.m., and each remains there for exactly 15 minutes while eating. What is the probability they will see each other in the dining hall tomorrow morning? Express your answer as a common fraction.

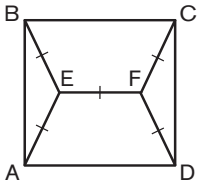
265. _____ Colby is practicing his basketball shots. He knows that he has a 0.7 chance of making each 2-point shot and a 0.4 chance of making each 3-point shot. If he takes five 2-point and five 3-point shots, what is his probability of earning 20 points or more? Express your answer as a decimal to the nearest thousandth.

266. _____ in² One circular base of a cylinder with radius 2 inches and height 4 inches is glued flush to the center of each face of a 4-inch cube. What is the surface area of the resulting solid? Express your answer as a decimal to the nearest tenth.

267. _____ fish Sam, Taylor and Pat counted the number of fish in each of their fish tanks. They noticed that Sam's tank had exactly 25% more fish than Taylor's tank, and Pat's tank had exactly 24% more fish than Sam's tank. If each tank had at least one fish, what is the minimum combined number of fish that could have been in the three tanks?

268. _____ degrees  The lateral surface of a right circular cone of radius 48 feet and height 14 feet is cut along a line drawn from the apex perpendicular to the base and unfurled to form the two-dimensional shape shown. What is the central angle measure of the sector that is missing from the circle? Express your answer as a decimal to the nearest tenth.

269. _____ cm² Inside square ABCD, shown here, points E and F are chosen in such a way that AE = BE = CF = DF = EF = 4 cm. What is the area of triangle ABE?



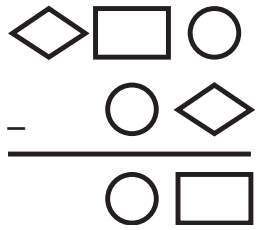
270. _____ Each digit 0 through 9 is used exactly once to create two five-digit numbers. What is the sum of the digits of the greatest product of two such numbers?

Logic Stretch

271._____ Celia, Desi and Everett are each wearing a hat that displays a different whole number from 1 to 9, inclusive. Each number cannot be seen by the person wearing it, but that number is visible to the other two individuals. Everett says, "The sum of the numbers I see is 6." Celia says, "The product of the numbers I see is 10." What is the sum of the numbers that Everett could possibly have on his hat?

272._____ people In a survey, 30 people reported that they enjoy some combination of walking, hiking and jogging. The number who enjoy only walking, the number who enjoy only hiking and the number who enjoy only jogging are all equal. Likewise, the number who enjoy only walking and hiking, the number who enjoy only walking and jogging and the number who enjoy only hiking and jogging are equal. In addition, the survey showed that half as many people enjoy exactly two of these activities as those who enjoy only one activity. If three people enjoy all three activities, how many people enjoy jogging?

273._____



In the subtraction problem shown, the shapes ◇, □, and ○ each represent a different digit. What is the value of □ ◇ ÷ ○?

274. Box_____ Three identical boxes contain tennis balls, baseballs or both. A label is affixed to each box. The three labels correctly describe the three boxes, but none of the labels is on the correct box. Box 1 is labeled "Tennis Balls." Box 2 is labeled "Baseballs." Box 3 is labeled "Tennis Balls & Baseballs." Devon reaches into Box 3 and pulls out a baseball. Which box contains only tennis balls?

275. Page_____ Drew purchased a used 50-page book at the book fair. Drew later realized that the book, in which left-hand pages contained even page numbers and right-hand pages contained odd page numbers, did not contain all 50 pages. The sum of the page numbers on the pages that Drew's book did contain was 1242. What is the greatest page number that could be on a page missing from Drew's book?

276._____ In the addition problem shown, each letter stands for a different digit. If T = 3, what is the value of the four-digit number MATH?

$$\begin{array}{cccc} & G & E & T \\ + & T & H & E \\ \hline M & A & T & H \end{array}$$

277._____ seconds Starting at the lower landing of a staircase, Porscha goes up the steps by repeating a three-step sequence: moving two steps up and then moving one step down. Starting at the upper landing of the same staircase, Micah goes down the steps by repeating a different three-step sequence: moving two steps down and then moving one step up. After simultaneously moving to their first steps, Porscha and Micah both move to another step every 3 seconds. To go from the upper landing to the lower landing of the staircase involves a net movement of 12 steps. How many seconds after moving to the starting steps will Porscha and Micah reach the same step?

278._____ If the six-digit number 3D6,D92 is divisible by 11, what is the value of D?

279._____ A special deck of cards contains cards numbered 1 through 4 for each of four suits. Each of the 16 cards has a club, diamond, heart or spade on one side and the number 1, 2, 3 or 4 on the other side. After a dealer mixed up the cards, three were selected at random. What is the probability that of these three randomly selected cards, displayed here, one of the cards showing the number 2 has a heart printed on the other side? Express your answer as a common fraction.

280._____ The units digit of a three-digit number, ABC, is moved to the left of the remaining two digits to make a new three-digit number, CAB. If CAB – ABC = 162, what is the sum of the least and greatest possible values of ABC?

Solving Inequalities Stretch

Quick Review of Inequality Properties

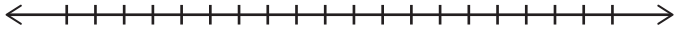
For any numbers a, b and c,

- if $a > b$, then $a + c > b + c$ and $a - c > b - c$. (applies to $>$, $\geq$, $<$ and $\leq$)
- if $a > b$ and $c > 0$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$. (applies to $>$, $\geq$, $<$ and $\leq$)
- if $a > b$ and $c < 0$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$. (applies to $>$, $\geq$, $<$ and $\leq$)
- if $|a| < b$, then $a < b$ and $a > -b$. (applies to $<$ and $\leq$)
- if $|a| > b$, then $a > b$ or $a < -b$. (applies to $>$ and $\geq$)

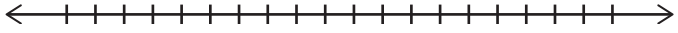
Solve each inequality, and graph the solution on the number line provided.

281. _____ $3 - \frac{x}{3} \leq 5$

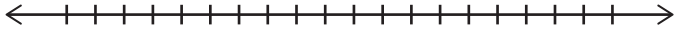
282. _____ $3 - \frac{x}{3} \geq -5$



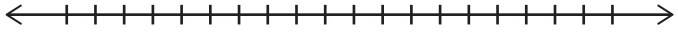
283. _____ $\left|3 - \frac{x}{3}\right| \leq 5$



284. _____ $3 - \frac{x}{3} < x - 5$



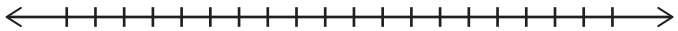
285. _____ $3 - \frac{x}{3} > 5 - x$



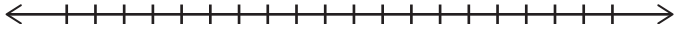
286. _____ $\left|3 - \frac{x}{3}\right| < x - 5$

287. _____ $x^2 \leq 25$

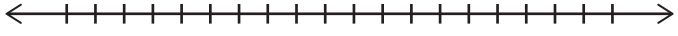
288. _____ $x^2 \geq 25$



289. _____ $x^2 + 4x - 4 > -8$

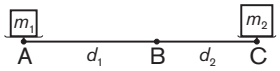


290. _____ $x^2 + 4x - 4 > -7$

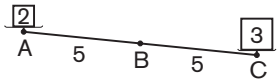


Mass Point Geometry Stretch

Center of Mass



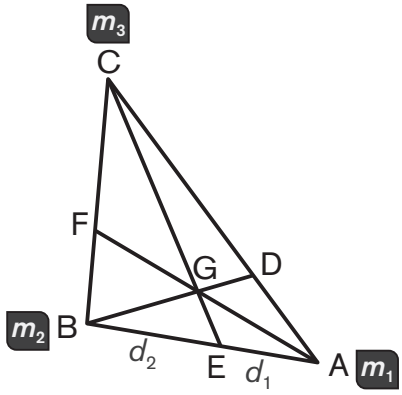
Consider a seesaw, with a fulcrum at B, that has objects at A and C. As shown, the object at A has a mass of m_1, and its distance from B is d_1. The object at C has mass m_2, and its distance from B is d_2. These examples show how the position of the fulcrum determines whether the seesaw is balanced. The mass at B is $m_1 + m_2$, the sum of the masses at A and C.



In this first example, B is positioned so that $d_1 = d_2 = 5$. Notice that $m_1 \times d_1 = 2 \times 5 = 10$ and $m_2 \times d_2 = 3 \times 5 = 15$. Although the objects at A and C are equidistant from B, the object at C is lower than the object at A because $m_1 \times d_1 < m_2 \times d_2$.

In this example, B is positioned so that $d_1 = 6$ and $d_2 = 4$. Here, $m_1 \times d_1 = 2 \times 6 = 12$ and $m_2 \times d_2 = 3 \times 4 = 12$. This time, the seesaw is balanced because $m_1 \times d_1 = m_2 \times d_2$. In this case, the position of B is known as the *center of mass*.

A *cevian* is a line segment that joins a vertex of a triangle with a point on the opposite side. Mass point geometry is a technique used to solve problems involving triangles and intersecting cevians by applying center of mass principles. Because triangle ABC, shown here, has cevians AF, BD and CE that intersect at point G, we can apply the center of mass principles presented. For example, side AB is balanced on point E when $m_1 \times d_1 = m_2 \times d_2$.



A *mass point*, denoted *m*P, consists of point P and its associated mass, *m*. Assume point G is the center of mass on which the entire triangle balances. Then the mass at G is the sum of the masses at the endpoints for each cevian and *m*G = *m*A + *m*F = *m*B + *m*D = *m*C + *m*E.

Suppose BF:CF = 3:4 and AD:CD = 2:5, and we are asked to determine the ratios AE:BE, AG:FG and BG:DG.

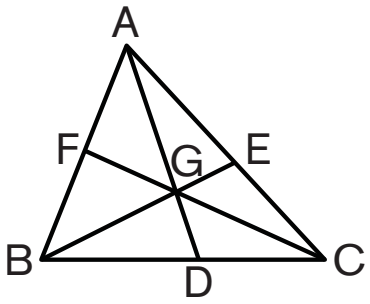
Start by finding *m*B and *m*C for side BC, which is balanced on point F. We know $m_2 \times 3 = m_3 \times 4$. We can let $m_2 = 4$ and $m_3 = 3$, so 4B + 3C = (4 + 3)F = 7F.

Next, find *m*A for side AC, which is balanced on point D. We know $m_1 \times 2 = m_3 \times 5$. Since $m_3 = 3$, it follows that $m_1 \times 2 = 3 \times 5$ and $m_1 = 15/2$. Rather than having mass point (15/2)A, we can multiply 4B, (15/2)A, 3C and 7F by 2 to get the following mass points: 8B, 15A, 6C and 14F. Now the mass at each point is of integer value.

Now, there is enough information to find *m*D and *m*E, since 15A + 6C = (15+ 6)D = 21D and 15A + 8B = (8 + 15)E = 23E. Therefore, given mass points 15A and 8B, it follows that side AB is balanced on point E when AE:BE = 8:15. In addition, given mass points 21D and 14F, we see that cevians AF and BD both are balanced on point G when AG:FG = 14:15 and BG:DG =21:8.

Solve the following problems by using mass point geometry. Express ratio answers as common fractions.

Triangle ABC, shown here, has cevians AD, BE and CF intersecting at point G, with AF:BF = 3:2 and BD:CD = 5:3.

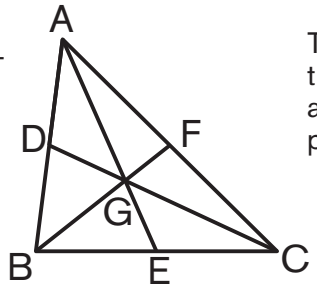


291._____ What is the ratio of AE to CE?

292._____ What is the ratio of BG to EG?

293._____ What is the ratio of DG to AG?

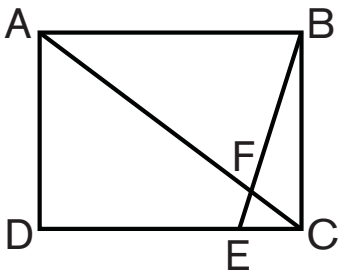
294._____



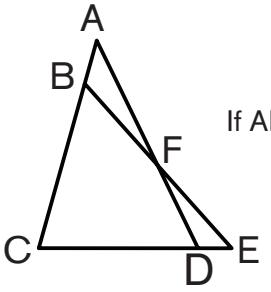
The medians of a triangle intersect at a point in the interior of the triangle as shown. What is the ratio of the lengths of the shorter and longer segments into which each median is divided at the point of intersection?

295._____

In rectangle ABCD, point E is on side DC such that BC = 8, BE = 10 and AC = 17. If segments AC and BE intersect at F, what is the ratio of the area of triangle CFE to the area of triangle AFB?



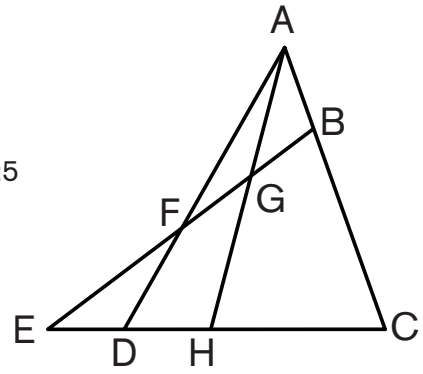
296._____



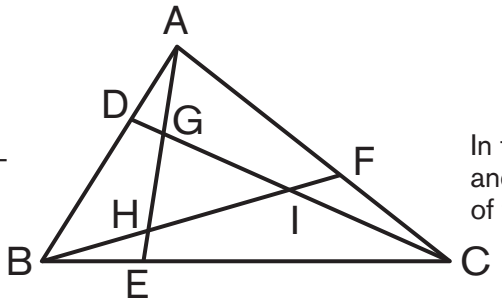
If AB:BC = 1:4 and AF:DF = 5:4, what is the ratio of DE to CD?

297._____

For integers x, y and z, if AB:BC = 1:4, AG:GH = 3:5 and AF:DF = 5:4, then CH:DH:DE = x:y:z. What is the value of $x + y + z$?



298._____

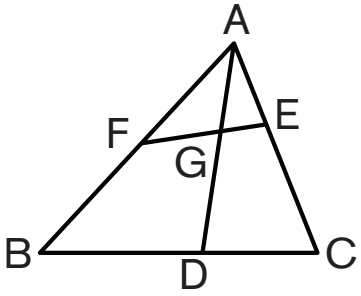


In triangle ABC, AD:BD = 1:2, BE:EC = 1:3 and AF:CF = 3:2. What is the ratio of the area of triangle GHI to the area of triangle ABC?

Triangle ABC, shown here, has cevian AD and transversal EF intersecting at G, with AE:CE = 1:2, AF:BF = 5:4 and BD:CD = 3:2.

299._____ What is the ratio of AG to DG?

300._____ What is the ratio of EG to FG?



ANSWERS

In addition to the answer, we have provided a difficulty rating for each problem. Our scale is 1-7, with 7 being the most difficult. These are only approximations, and how difficult a problem is for a particular student will vary. Below is a general guide to the ratings:

Difficulty 1/2/3 - One concept; one- to two-step solution; appropriate for students just starting the middle school curriculum.
4/5 - One or two concepts; multistep solution; knowledge of some middle school topics is necessary.
6/7 - Multiple and/or advanced concepts; multistep solution; knowledge of advanced middle school topics and/or problem-solving strategies is necessary.

Warm-Up 1

Answer	Difficulty		
1. 495	(1)	6. 26	(3)
2. 11	(2)	7. B & C	(4)
3. 70	(2)	8. 11,304	(2)
4. $3\frac{23}{24}$	(3)	9. 5/9	(4)
5. 16	(3)	10. 12	(4)

Warm-Up 2

Answer	Difficulty		
11. 4	(3)	16. 1000 or 1000.00	(3)
12. 5	(4)	17. 0	(2)
13. 8	(3)	18. (3, 3)	(4)
14. 50	(3)	19. 5/9	(4)
15. 14	(4)	20. $\sqrt{21}$	(4)

Workout 1

Answer	Difficulty		
21. 49	(2)	26. 17	(4)
22. 1.225×10^9	(3)	27. 174	(3)
23. 12	(4)	28. 0.04	(5)
24. 3.34	(3)	29. 5	(3)
25. 400	(4)	30. 1/6	(3)

Warm-Up 3

Answer	Difficulty		
31. 5	(2)	36. 234 or 234.00	(2)
32. 2	(2)	37. 10	(3)
33. 1*	(3)	38. 15	(3)
34. 6	(3)	39. 4.80	(3)
35. 7/36	(3)	40. 1/4	(3)

Warm-Up 4

Answer	Difficulty		
41. a^2	(3)	46. 5	(3)
42. 1/8	(3)	47. 3456	(3)
43. 37	(4)	48. 78	(5)
44. 20	(3)	49. 60	(3)
45. $1\frac{1}{2}$	(3)	50. 4	(3)

Workout 2

Answer	Difficulty		
51. 32	(5)	56. 2 or 2.00	(3)
52. 58	(2)	57. 6	(2)
53. 27.7	(4)	58. 89	(3)
54. 10	(5)	59. 2008	(3)
55. 25	(5)	60. 8	(4)

** The plural form of the units is always provided in the answer blank, even if the answer appears to require the singular form of the units.*

Warm-Up 5

Answer	Difficulty		
61. 300 or 300.00	(3)	66. 60	(4)
62. 56	(4)	67. 2.4	(3)
63. 2015	(3)	68. 16	(4)
64. 0	(4)	69. 10	(4)
65. 16/15	(4)	70. 2015	(3)

Warm-Up 6

Answer	Difficulty		
71. 11/6	(3)	76. 1459	(3)
72. 80	(4)	77. 3	(4)
73. $15\sqrt{3}$	(4)	78. 12	(3)
74. 810	(3)	79. 46	(3)
75. 9	(4)	80. 12	(3)

Warm-Up 7

Answer	Difficulty		
91. 72	(3)	96. 56	(3)
92. 18π	(4)	97. 3/2	(3)
93. 9	(3)	98. 54	(2)
94. 50	(3)	99. 2	(3)
95. 9/8	(3)	100. 45	(3)

Warm-Up 8

Answer	Difficulty		
101. 9	(2)	106. 682	(3)
102. 12	(2)	107. 48	(3)
103. 85	(2)	108. 11	(3)
104. 35	(4)	109. 35,800 or 35,800.00	(3)
105. 5	(3)	110. 2	(4)

Workout 3

Answer	Difficulty		
81. 0	(4)	86. 2	(3)
82. 51	(3)	87. 41	(2)
83. 0.5	(3)	88. 31	(4)
84. 2900	(4)	89. 460	(4)
85. 3	(4)	90. 7	(4)

Workout 4

Answer	Difficulty		
111. 79	(4)	116. 1353	(4)
112. 331,776	(5)	117. 32.2	(4)
113. 47	(2)	118. $72\sqrt{2}$	(4)
114. 6	(2)	119. 1207	(4)
115. 22.5	(3)	120. 6	(5)

Warm-Up 9

Answer	Difficulty	Answer	Difficulty
121. 120	(3)	126. 1024	(4)
122. 10	(3)	127. 17	(2)
123. 400 or 400.00	(2)	128. 14,580 or 14,580.00	(4)
124. 2	(2)	129. 27	(3)
125. 5	(3)	130. 6	(3)

Warm-Up 10

Answer	Difficulty	Answer	Difficulty
131. 64 + 32π or 32π + 64	(4)	136. 27	(5)
132. 792	(3)	137. 2279	(5)
133. $\sqrt{3}/2$	(3)	138. 2	(3)
134. 30	(4)	139. 16	(4)
135. 4	(3)	140. 80	(4)

Warm-Up 11

Answer	Difficulty	Answer	Difficulty
151. 32	(3)	156. 8/27	(5)
152. 60	(4)	157. (−3, 1)	(4)
153. 4/9	(4)	158. 16	(3)
154. $3\frac{1}{3}$	(4)	159. 12	(5)
155. 78	(4)	160. 648	(4)

Warm-Up 12

Answer	Difficulty	Answer	Difficulty
161. 1024	(3)	166. 60	(5)
162. 34	(3)	167. 24	(5)
163. $2\sqrt{7}$	(5)	168. 1	(4)
164. 4/5	(4)	169. $144w^3$	(3)
165. 15	(3)	170. 9	(5)

Workout 5

Answer	Difficulty	Answer	Difficulty
141. 7.5	(3)	146. 2.3	(4)
142. 2.7	(5)	147. 8:12	(3)
143. 66,660	(5)	148. 2001	(4)
144. 22	(3)	149. 78	(3)
145. 0.3	(5)	150. 10	(3)

Workout 6

Answer	Difficulty	Answer	Difficulty
171. 76	(4)	176. 12	(3)
172. 251.6	(5)	177. 29	(5)
173. 1012	(5)	178. 4576	(4)
174. 3.4	(4)	179. 121/282	(5)
175. 33	(4)	180. 4	(4)

Warm-Up 13

Answer	Difficulty		
181. 36	(4)	186. 14	(3)
182. $\sqrt{5}/2$	(4)	187. 1/12	(4)
183. $2 + \sqrt{2}$ or $\sqrt{2} + 2$	(4)	188. 18	(3)
		189. $-3/4$	(3)
184. 25	(3)	190. 16	(4)
185. 17	(2)		

Warm-Up 14

Answer	Difficulty		
191. 324π	(4)	196. 3	(4)
192. 37/64	(4)	197. $4\frac{2}{7}$	(4)
193. 7/3	(4)	198. 6	(4)
194. 22	(3)	199. 1/6	(5)
195. 1008/2015	(5)	200. 757	(4)

Workout 7

Answer	Difficulty		
201. 25	(4)	206. 4.57	(4)
202. 13/55	(3)	207. 403	(3)
203. 46.1	(6)	208. 2	(4)
204. 42.7	(4)	209. 9	(3)
205. -5050	(4)	210. 12	(4)

Warm-Up 15

Answer	Difficulty		
211. 20	(4)	216. 1/100	(4)
212. 148	(5)	217. 61	(2)
213. -7	(3)	218. 55	(4)
214. $3 - \sqrt{3}$ or $-\sqrt{3} + 3$	(6)	219. 1 or 1.00	(3)
		220. 10	(4)
215. 60	(4)		

Warm-Up 16

Answer	Difficulty		
221. 0	(3)	226. 10	(4)
222. $6 + \pi$ or $\pi + 6$	(5)	227. 12,288	(3)
		228. 1358	(3)
223. 14	(4)	229. 10	(4)
224. 60	(4)	230. 55	(5)
225. 7/37	(5)		

Workout 8

Answer	Difficulty		
231. $\sqrt{6}$	(4)	236. 4.5	(5)
232. 7200 or 7200.00	(3)	237. -406	(4)
		238. 3.7	(4)
233. 0.155	(5)	239. 78.5	(5)
234. 84	(5)	240. 53	(4)
235. 0.18	(5)		

Warm-Up 17

Answer	Difficulty	Answer	Difficulty
241. 12	(5)	246. 44	(4)
242. 12	(4)	247. 19	(6)
243. $3 - \sqrt{3}$ or $-\sqrt{3} + 3$	(5)	248. 20	(5)
244. $1 + \sqrt{3}$ or $\sqrt{3} + 1$	(5)	249. 5	(6)
245. 40	(4)	250. 60	(4)

Warm-Up 18

Answer	Difficulty	Answer	Difficulty
251. $(47\sqrt{3})/2$	(5)	256. 1/27	(5)
252. $1 + \sqrt{2}$ or $\sqrt{2} + 1$	(4)	257. 5/14	(5)
253. 9	(6)	258. 11/140	(5)
254. 20	(5)	259. 27	(5)
255. 17/24	(4)	260. 75	(4)

Workout 9

Answer	Difficulty	Answer	Difficulty
261. 1	(4)	266. 397.6	(4)
262. $c^2/(2ab)$	(4)	267. 76	(4)
263. 32.9	(5)	268. 14.4	(4)
264. 7/16	(6)	269. 6	(5)
265. 0.049	(6)	270. 45	(4)

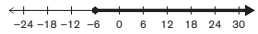
Logic Stretch

Answer	Difficulty	Answer	Difficulty
271. 7	(2)	276. 1038	(3)
272. 15	(3)	277. 39	(3)
273. 9	(3)	278. 8	(2)
274. 2	(1)	279. 3/7	(4)
275. 12	(3)	280. 972	(4)

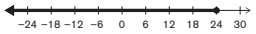
Solving Inequalities Stretch

Answer **Difficulty**

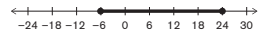
281. $x \geq -6$ (3)



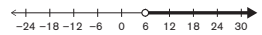
282. $x \leq 24$ (3)



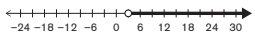
283. $-6 \leq x \leq 24$ (4)
or $x \geq -6$ and $x \leq 24$



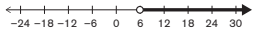
284. $x > 6$ (3)



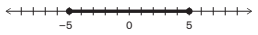
285. $x > 3$ (3)



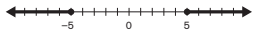
286. $x > 6$ (4)



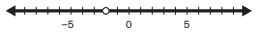
287. $-5 \leq x \leq 5$ (5)
or $x \geq -5$ and $x \leq 5$



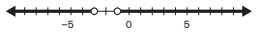
288. $x \leq -5$ or $x \geq 5$ (5)



289. $x \neq -2$ (6)
or $x < -2$ or $x > -2$



290. $x < -3$ or $x > -1$ (6)



Mass Point Geometry Stretch

Answer	Difficulty	Answer	Difficulty
291. 5/2	(5)	296. 1/4	(6)
292. 7/3	(5)	297. 60	(6)
293. 1/4	(5)	298. 16/105	(7)
294. 1/2	(5)	299. 25/38	(7)
295. 4/25	(6)	300. 2/5	(7)

MATHCOUNTS Problems Mapped to Common Core State Standards (CCSS)

Forty-three states, the District of Columbia, four territories and the Department of Defense Education Activity (DoDEA) have adopted the Common Core State Standards (CCSS). As such, MATHCOUNTS considers it beneficial for teachers to see the connections between the *2014-2015 MATHCOUNTS School Handbook* problems and the CCSS. MATHCOUNTS not only has identified a general topic and assigned a difficulty level for each problem but also has provided a CCSS code in the Problem Index (pages 83-84). A complete list of the Common Core State Standards can be found at www.corestandards.org.

The CCSS for mathematics cover K-8 and high school courses. MATHCOUNTS problems are written to align with the NCTM Standards for Grades 6-8. As one would expect, there is great overlap between the two sets of standards. MATHCOUNTS also recognizes that in many school districts, algebra and geometry are taught in middle school, so some MATHCOUNTS problems also require skills taught in those courses.

In referring to the CCSS, the Problem Index code for each or the Standards for Mathematical Content for grades K-8 begins with the grade level. For the Standards for Mathematical Content for high school courses (such as algebra or geometry), each code begins with a letter to indicate the course name. The second part of each code indicates the domain within the grade level or course. Finally, the number of the individual standard within that domain follows. Here are two examples:

- *6.RP.3 → Standard #3 in the Ratios and Proportional Relationships domain of grade 6*
- *G-SRT.6 → Standard #6 in the Similarity, Right Triangles and Trigonometry domain of Geometry*

Some math concepts utilized in MATHCOUNTS problems are not specifically mentioned in the CCSS. Two examples are the Fundamental Counting Principle (FCP) and special right triangles. In cases like these, if a related standard could be identified, a code for that standard was used. For example, problems using the FCP were coded 7.SP.8, S-CP.8 or S-CP.9 depending on the context of the problem; SP → Statistics and Probability (the domain), S → Statistics and Probability (the course) and CP → Conditional Probability and the Rules of Probability. Problems based on special right triangles were given the code G-SRT.5 or G-SRT.6, explained above.

There are some MATHCOUNTS problems that either are based on math concepts outside the scope of the CCSS or based on concepts in the standards for grades K-5 but are obviously more difficult than a grade K-5 problem. When appropriate, these problems were given the code SMP for Standards for Mathematical Practice. The CCSS include the Standards for Mathematical Practice along with the Standards for Mathematical Content. The SMPs are (1) Make sense of problems and persevere in solving them; (2) Reason abstractly and quantitatively; (3) Construct viable arguments and critique the reasoning of others; (4) Model with mathematics; (5) Use appropriate tools strategically; (6) Attend to precision; (7) Look for and make use of structure and (8) Look for and express regularity in repeated reasoning.

PROBLEM INDEX

It is difficult to categorize many of the problems in the *MATHCOUNTS School Handbook*. It is very common for a MATHCOUNTS problem to straddle multiple categories and cover several concepts. This index is intended to be a helpful resource, but since each problem has been placed in exactly one category and mapped to exactly one Common Core State Standard (CCSS), the index is not perfect. In this index, the code **9 (3) 7.SP.3** refers to problem 9 with difficulty rating 3 mapped to CCSS 7.SP.3. For an explanation of the difficulty ratings refer to page 78. For an explanation of the CCSS codes refer to page 82.

Measurement

3	(2)	6.RP.3
15	(4)	6.RP.3
22	(3)	8.EE.4
38	(3)	6.RP.3
39	(3)	6.RP.3
23	(4)	7.G.4
28	(5)	7.G.4
40	(3)	7.G.6
111	(4)	8.G.7
117	(4)	8.G.7
126	(4)	G-SRT.6
131	(4)	7.G.6
133	(3)	G-SRT.6
139	(4)	G-SRT.6
146	(4)	G-SRT.6
159	(5)	G-SRT.6
167	(5)	G-SRT.6
174	(4)	G-SRT.6
177	(5)	8.G.7
182	(4)	8.G.7
191	(4)	8.G.7
206	(4)	8.G.7
214	(6)	G-SRT.6
233	(5)	G-SRT.6
243	(5)	G-SRT.6
244	(5)	G-SRT.6
247	(6)	G-SRT.6
251	(5)	G-SRT.6
263	(5)	G-SRT.6
269	(5)	8.G.7

Coordinate Geometry

18	(4)	8.EE.8
29	(3)	8.EE.8
49	(3)	6.G.3
100	(3)	8.F.3
157	(4)	8.F.3
204	(4)	F-IF.2
220	(4)	8.G.8
228	(3)	8.F.3
238	(4)	G-SRT.6
239	(5)	SMP
262	(4)	8.F.3

Sequences, Series & Patterns

34	(3)	F-BF.2
71	(3)	F-BF.2
95	(3)	F-BF.2
103	(2)	F-BF.2
114	(2)	F-BF.2
137	(5)	F-BF.2
170	(5)	F-BF.2
171	(4)	SMP
205	(4)	F-BF.2
229	(4)	F-BF.2
241	(5)	F-BF.2

Plane Geometry

9	(4)	8.G.7
20	(4)	8.G.7
53	(4)	7.G.4
75	(4)	SMP
92	(4)	7.G.4
101	(2)	SMP
121	(3)	8.G.5
136	(5)	G-CO.10
145	(5)	G-C.2
163	(5)	G-C.2
212	(5)	G-C.2
222	(5)	7.G.4
223	(4)	G-C.2
225	(5)	G-GMD.3
248	(5)	G-C.2
249	(6)	G-C.2
252	(4)	8.G.7
253	(6)	8.G.7
260	(4)	G-C.2

Solid Geometry

11	(3)	G-GMD.3
77	(4)	G-GMD.3
79	(3)	7.G.6
88	(4)	7.G.6
106	(3)	7.G.6
118	(4)	8.G.9
119	(4)	7.G.6
130	(3)	SMP
134	(4)	G-GMD.3
155	(4)	7.G.6
169	(3)	6.G.2
178	(4)	7.G.6
194	(3)	7.G.6
224	(4)	SMP
226	(4)	SMP
227	(3)	G-GMD.3
236	(5)	G-GMD.3
256	(5)	SMP
266	(4)	G-GMD.3
268	(4)	7.G.4

Probability, Counting & Combinatorics

5	(3)	7.SP.8
19	(4)	7.SP.8
30	(3)	7.SP.8
35	(3)	7.SP.8
42	(3)	7.SP.8
78	(3)	7.SP.8
80	(3)	SMP
143	(5)	S-CP.9
151	(3)	SMP
153	(4)	S-CP.8
160	(4)	S-CP.8
179	(5)	7.SP.8
187	(4)	7.SP.8
211	(4)	S-CP.9
216	(4)	7.SP.8
232	(3)	S-CP.9
258	(5)	7.SP.8
264	(6)	7.SP.8
265	(6)	7.SP.8

Percents & Fractions

10	(4)	6.RP.3
16	(3)	7.RP.3
24	(3)	7.RP.3
25	(4)	6.RP.3
27	(3)	6.RP.3
52	(2)	6.RP.3
56	(3)	7.RP.3
59	(3)	6.RP.3
65	(4)	6.RP.3
66	(4)	6.RP.3
67	(3)	6.RP.3
72	(4)	7.RP.3
74	(3)	6.RP.3
82	(3)	7.RP.3
89	(4)	7.RP.3
94	(3)	6.RP.3
98	(2)	6.RP.3
102	(2)	6.NS.1
109	(3)	7.RP.3
128	(4)	7.RP.3
140	(4)	6.RP.3
188	(3)	6.RP.3
215	(4)	7.RP.3
217	(2)	7.RP.3
257	(5)	7.RP.3
267	(4)	6.RP.3

Proportional Reasoning

54	(5)	7.G.6
73	(4)	7.G.6
83	(3)	7.G.6
97	(3)	6.G.1
113	(2)	7.G.6
142	(5)	7.RP.2
156	(5)	7.G.6
164	(4)	G-SRT.5
183	(4)	6.G.1
197	(4)	G-SRT.5
203	(6)	7.G.6
291	(5)	6.RP.1
292	(5)	6.RP.1
293	(5)	6.RP.1
294	(5)	6.RP.1
295	(6)	6.RP.1
296	(6)	6.RP.1
297	(6)	6.RP.1
298	(7)	6.RP.1
299	(7)	6.RP.1
300	(7)	6.RP.1
45	(3)	6.RP.3
61	(3)	6.RP.3
84	(4)	6.RP.3
115	(3)	6.RP.3
173	(5)	G-C.2
185	(2)	6.RP.3
186	(3)	6.RP.3
219	(3)	6.RP.3
250	(4)	6.EE.7