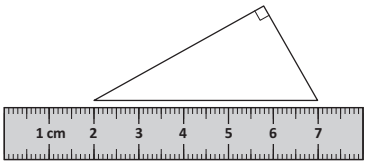
Warm-Up 1

1. _____ cm What is the length, to the nearest centimeter, of the hypotenuse of the right triangle shown?



2. _____ cm If the ratio of the length of a rectangle to its width is $\frac{9}{4}$ and its length is 18 cm, what is the width of the rectangle?

3. _____ bins Mike bought $2\frac{3}{4}$ pounds of rice. He wants to distribute it among bins that each hold $\frac{1}{3}$ pound of rice. How many bins can he completely fill?

4. _____ : _____ p.m. It took Jessie 15 minutes to drive to the movie theater from home. He waited 10 minutes for the movie to start, and the movie lasted 1 hour 43 minutes. After the movie ended, Jessie immediately went home. It took Jessie 25 minutes to drive home from the theater. If he left for the movie at 4:05 p.m., at what time did he get home?

5. $ _____ A carnival pass costs $15 and is good for 10 rides. This is a savings of $2.50 compared to paying the individual price for 10 rides. What is the individual price of a ride without the pass?

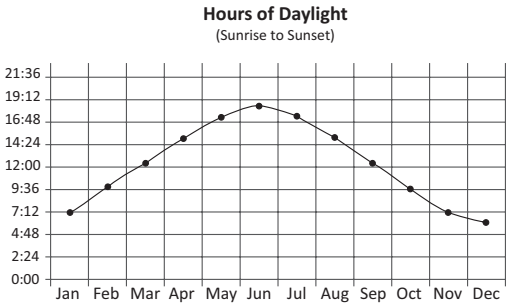
6. _____ If $x + y = 7$ and $x - y = 1$, what is the value of the product $x \cdot y$?

7. _____ Mrs. Stephens has a bag of candy. The ratio of peppermints to chocolates is 5:3, and the ratio of peppermints to gummies is 3:4. What is the ratio of chocolates to gummies? Express your answer as a common fraction.

8. _____ degrees The angles of a triangle form an arithmetic progression, and the smallest angle is 42 degrees. What is the degree measure of the largest angle of the triangle?

9. _____ Each of the books on Farah's shelves is classified as sci-fi, mystery or historical fiction. The probability that a book randomly selected from her shelves is sci-fi equals 0.55. The probability that a randomly selected book is mystery equals 0.4. What is the probability that a book selected at random from Farah's shelves is historical fiction? Express you answer as a decimal to the nearest hundredth.

10. _____



According to the graph shown, which of the other eleven months has a number of daylight hours most nearly equal to the number of daylight hours in April?

Warm-Up 2

11. _____ Consider the following sets: A = {2, 5, 6, 8, 10, 11}, B = {2, 10, 18} and C = {10, 11, 14}. What is the greatest number in either of sets B or C that is also in set A?

12. _____ °F The temperature is now 0 °F. For the past 12 hours, the temperature has been decreasing at a constant rate of 3 °F per hour. What was the temperature 8 hours ago?

13. _____ What is the value of x if $\frac{1}{x} + \frac{1}{2x} = \frac{1}{2}$?

14. _____ In June, Casey counted the months until he would turn 16, the minimum age at which he could obtain his driver's license. If the number of months Casey counted until his birthday was 45, in what month would Casey turn 16?

15. _____ buckets It takes 1 gallon of floor wax to cover 600 ft^2. If floor wax is sold only in 1-gallon buckets, how many buckets of floor wax must be purchased to wax the floors of three rooms, each measuring 20 feet by 15 feet?

16. _____ Consider the pattern below:

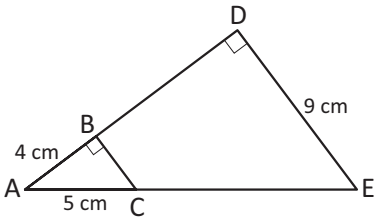
$22^2 = 121 \times (1 + 2 + 1)$
$333^2 = 12{,}321 \times (1 + 2 + 3 + 2 + 1)$
$4444^2 = 1{,}234{,}321 \times (1 + 2 + 3 + 4 + 3 + 2 + 1)$

For what positive value of n will $n^2 = 12{,}345{,}654{,}321 \times (1 + 2 + 3 + 4 + 5 + 6 + 5 + 4 + 3 + 2 + 1)$?

17. _____ times If United States imports increased 20% and exports decreased 10% during a certain year, the ratio of imports to exports at the end of the year was how many times the ratio at the beginning of the year? Express your answer as a common fraction.

18. _____ James needs $150 to buy a cell phone. In January, he saved $5. He saved twice as much in February as he saved in January, for a total savings of $15. If James continues to save twice as much each month as he saved the previous month, in what month will his total savings be enough to purchase the cell phone?

19. _____ cm What is the perimeter of △ADE shown here?



20. _____ females The following table shows the results of a survey of a random sample of people at a local fair. If there are 1100 people at the fair, how many females would you expect to prefer the Flume?

Favorite Ride	Male	Female
Ferris Wheel	15	20
Roller Coaster	24	14
Carousel	6	10
Flume	5	6

Workout 1

21. _____ hours It takes Natasha nine hours to mow six lawns. On average, how many hours does it take her to mow each lawn? Express your answer as a decimal to the nearest tenth.

22. _____ What is the value of $(\pi^4 + \pi^5)^{\frac{1}{6}}$ when expressed as a decimal to the nearest hundredth?

23. _____ cm What is the length of a diagonal that cuts through the center of a cube with edge length 4 cm? Express your answer in simplest radical form.

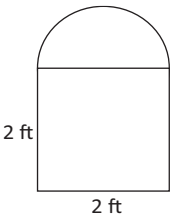
24. $ _____ Carol finds her favorite brand of jeans on sale for 20% off at the mall. If the jeans are regularly $90 and the tax is 7.5%, how much will she pay for one pair of jeans?

25. _____ What is the value of 1 + 1 when written in base 2?

26. _____ euros In May 2002, the exchange rate for converting U.S. dollars to euros was 1 dollar = 1.08 euros. At this rate, 250 U.S. dollars could be exchanged for how many euros?

27. _____ units Two sides of a right triangle have lengths 5 units and 12 units. If the length of its hypotenuse is not 13 units, what is the length of the third side? Express your answer in simplest radical form.

28. _____ ft^2



A Norman window has the shape of a rectangle on three sides, with a semicircular top. This particular Norman window includes a 2-foot by 2-foot square. What is the area of the whole window? Express your answer as a decimal to the nearest hundredth.

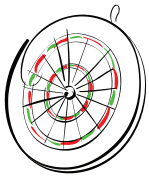
29. _____ A fair coin is flipped, and a standard die is rolled. What is the probability that the coin lands heads up and the die shows a prime number? Express your answer as a common fraction.

30. _____ in^3 Bailey is estimating the volume of a container. The container is a cube that measures 2 feet 7 inches on each edge. Bailey estimates the volume by using 3 feet for each edge. In cubic inches, what is the positive difference between Bailey's estimate and the actual volume?

Warm-Up 3

31. _____ If the ratio of a to b is $\frac{7}{3}$, what is the ratio of $2a$ to b? Express your answer as a common fraction.

32. _____ Remy throws three darts and Rita throws one dart at a dartboard. Each dart lands at a different distance from the center. Assuming Remy and Rita are equally skilled at darts, what is the probability that the dart closest to the center is one that Remy threw? Express your answer as a common fraction.



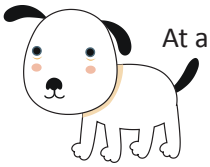
33. _____ Grace had an average test score of exactly 89 in her algebra class after the first three tests. After the fourth test, her average was exactly 91. What was Grace's score on the fourth test?

34. _____ cm^3 What is the volume, in cubic centimeters, of a cube that has a surface area of 96 cm^2?

35. _____ If 45% of the students at South Park High School were born at South Park Hospital, what is the ratio of the number of students who were not born at South Park Hospital to the number of students who were born at South Park Hospital? Express your answer as a common fraction.

36. _____ If $\frac{c}{d} = 4$, what is the value of $\frac{d}{c} + \frac{1}{2}$? Express your answer as a common fraction.

37. _____ dogs



At a pet store, there are 23 animals. Among the animals in the store, 15 are white, 5 are white dogs and 7 animals are neither dogs nor white. How many dogs are at the pet store?

38. _____ Yoon is expecting an important phone call today at a randomly selected time from 2:00 p.m. to 3:30 p.m. What is the probability that he will receive the call before 2:15 p.m.? Express your answer as a common fraction.

39. _____ quarts Donatella's recipe for punch calls for the following ingredients:

$\frac{1}{2}$ gallon of apple juice
3 cups of lemon-lime soda
64 fluid ounces of pineapple juice
2 quarts of cold water
1 cup of lemon juice

One gallon = 4 quarts = 8 pints = 16 cups = 128 fluid ounces. How many quarts of punch will this recipe produce?

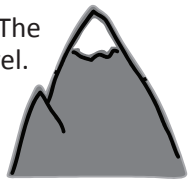
40. _____ cookies Jude ate 100 cookies in five days. On each day, he ate 6 more than on the previous day. How many cookies did he eat on the fifth day?

Warm-Up 4

41. _____ If the point (−3, 5) is reflected across the x-axis, what is the sum of the coordinates of the image?

42. _____ Let @x@ be defined for all positive integer values of x as the product of all of the factors of $2x$. For example, @7@ = 14 × 7 × 2 × 1 = 196. What is the value of @3@?

43. _____ feet The peak of Mount Everest is approximately 29,000 feet above sea level. The bottom of the Mariana Trench is approximately 36,201 feet below sea level. What is the vertical distance, to the nearest thousand, from the base of the Mariana Trench to the peak of Mount Everest?



44. _____ What is the mean of $7\frac{1}{2}$, $-3\frac{1}{4}$, 4, $-5\frac{1}{4}$ and 2?

45. _____ If $45 + \sqrt{c} = 49$, what is the value of $c^2 - 21$?

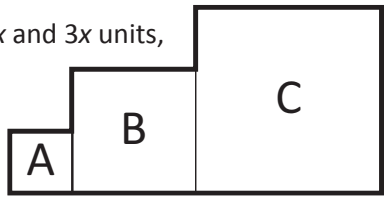
46. _____ weeks Spending at a rate of 100 dollars every minute, how many weeks will it take Janelle to spend one million dollars? Express your answer to the nearest whole number.

47. _____ What is the value of $(-20) + (-17) + (-14) + \cdots + 13 + 16 + 19 + 22$?

48. _____ cm The area of a right triangle is 36 cm^2. If the length of one leg of this triangle is 8 cm, what is the length of the other leg, in centimeters?

49. _____ There is a $\frac{2}{3}$ chance of rain for each of three days. If the weather on each day is independent of the weather on the other two days, what is the probability that it will rain on none of the three days? Express your answer as a common fraction.

50. _____ units Squares A, B and C, shown here, have sides of length x, $2x$ and $3x$ units, respectively. What is the perimeter of the entire figure? Express your answer in terms of x.



Workout 2

51. _____ The line passing through points (1, c) and (−5, 3) is parallel to the line passing through the points (4, 3) and (7, −2). What is the value of c?

52. _____ minutes

The book of *Guinness World Records* states that Fuatai Solo set a world record in 1980 by climbing a coconut tree 29 feet 6 inches tall in 4.88 seconds. At that rate, how many minutes would it take Fuatai to climb the 1454 feet to the top of the Empire State Building? Express your answer to the nearest whole number.

53. _____ cents At a store, a four-pack of 16-oz cans of soup costs \$3.20 and a three-pack of 24-oz cans costs \$3.60. How many cents are in the absolute difference between the price per ounce of a four-pack and the price per ounce of a three-pack?

54. _____ What is the closest integer to the real number x such that $2^x=1000$?

55. _____ feet A wheel that makes 10 revolutions per minute takes 18 seconds to travel 15 feet. In feet, what is the diameter of the wheel? Express your answer as a decimal to the nearest tenth.

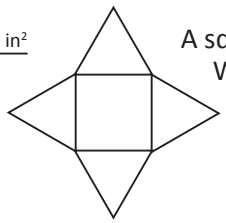
56. _____ years The average age of a group of 12 people is 26 years. If 8 new people are added to the group, the average age of the group increases to 32 years. In years, what is the average age of the 8 new people?

57. _____ Connie and her little brother like to play a number game. When Connie says a number, her brother then says the number that is 3 less than half of Connie's number. If Connie says a number, and her brother gives the correct response, 9, what number did Connie say?

58. _____ pounds Arnold, Benji and Celal found an old scale. When Arnold and Benji stepped on the scale, it showed a weight of 158 pounds. When Benji and Celal stepped on the scale, it showed a weight of 176 pounds. When all three of them stepped on the scale, it accurately showed a weight of 250 pounds but then promptly broke under the strain. However, they already had enough information to determine each of their weights. How much does Benji weigh?

59. _____ odd numbers Of all three-digit natural numbers less than 523, how many of the odd numbers contain no 5?

60. _____ in^2



A square of side length 4 inches has four equilateral triangles attached as shown. What is the total area of this figure? Express your answer in simplest radical form.

Warm-Up 5

61. _____ What number must be added to the set {5, 10, 15, 20, 25} to increase the mean by 5?

62. _____ For each pair (x, y) in the table shown, $y = \frac{c}{x}$ where c is a constant. What is the value of c?

x	$-\frac{1}{2}$	-1	-2	-4
y	-16	-8	-4	-2

63. _____ : _____ p.m. Sinclair is going to visit her family in New York. She lives 90 miles away in New Jersey. Assuming that there are no traffic delays and she can travel at an average speed of 45 mi/h for the entire trip, at what time should she leave if she needs to meet her family at 4:00 p.m.?

64. _____ knots A ship is 108 feet long and travels on open water at a speed of 30 knots. A model of the ship that is 12 feet long is used to test its hydrodynamic properties. To replicate the wave pattern that appears behind a ship, the speed of the model, r, should be equal to $r = s\sqrt{\frac{m}{a}}$, where s is the speed of the actual ship, a is the length of the actual ship and m is the length of the model. What speed, in knots, should be used for the model to simulate travel in open water?

65. _____ What fraction of 45 is 60% of 50? Express your answer as a common fraction.

66. _____ The integer x is the sum of three different positive integers, each less than 10. The integer y is the sum of three different positive integers, each less than 20. What is the greatest possible value of $\frac{y}{x}$?

67. _____ In the four by four grid shown, move from the 1 in the lower left corner to the 7 in the upper right corner. On each move, go up, down, right or left, but do not touch any cell more than once. Add the numbers as you go. What is the maximum possible value that can be obtained, including the 1 and the 7?

4	5	6	7
3	4	5	6
2	3	4	5
1	2	3	4

68. _____ complete pages

If a printer prints at a uniform rate of 3 complete pages every 40 seconds, how many complete pages will it print in 3 minutes?

69. _____ sides The measure of an interior angle of a regular polygon is eight times the measure of one of its exterior angles. How many sides does the polygon have?

70. _____ The number 101 is a three-digit palindrome because it remains the same when its digits are reversed. What is the ratio of the number of four-digit palindromes to the number of five-digit palindromes? Express your answer as a common fraction.

Warm-Up 6

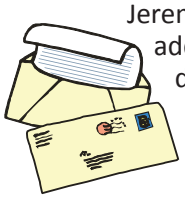
71. _____ values For how many nonzero values of x does $x^{2x} = 1$?

72. (,) The function $y = 3x + 6$ is graphed in the coordinate plane. At what point on the graph is the y-value double the x-value? Express your answer as an ordered pair.

73. _____ degrees The typical person spends 8 hours a day sleeping. In a circle graph that shows how 24 hours in a day are spent, how many degrees are in the central angle for sleeping?

74. _____ The average of a, b and c is 15. The average of a and b is 18. What is the value of c?

75. _____



Jeremiah has written four letters, one to each of four different people, and he has an addressed envelope for each person. If Jeremiah randomly places each letter in a different one of the four envelopes, what is the probability that two letters are in the correct envelopes and the other two are not? Express your answer as a common fraction.

76. _____ If the points $(-2, 5)$, $(0, y)$ and $(5, -16)$ are collinear, what is the value of y?

77. _____ If $(2x - 5)(2x + 5) = 5$, what is the value of $4x^2$?

78. $ _____ Arturo invests $5000 in a mutual fund that gains 20% of its value in the first month, and then loses 20% of its value the following month. In dollars, how much is Arturo's investment worth at the end of the second month?

79. _____ What is the sum of the 31st through 36th digits to the right of the decimal point in the decimal expansion of $\frac{4}{7}$?

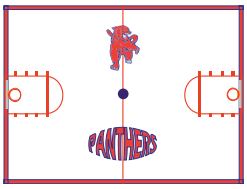
80. _____ What numeral in base 8 is equivalent to 332_5 (denoting 332 base 5)?

Workout 3

81. _____ mi/h A pilot flew a small airplane round-trip between his home airport and a city 720 miles away. The pilot logged 5 hours of flight time and noted that there was no wind during the flight to the city, but he did encounter a headwind on his return flight. If the pilot was able to maintain a speed of 295 mi/h during the flight to the city, what was his average speed during the return flight, in miles per hour? Express your answer as a decimal to the nearest hundredth.

82. _____ For nonzero numbers a, b and c, b is $\frac{1}{3}$ of a, and c is twice b. What is the value of $\frac{a^2}{c^2}$? Express your answer as a decimal to the nearest hundredth.

83. _____ ft^2 A rectangular basketball court had an area of 1200 ft^2. The court was enlarged so that its length was increased by 40% and its width by 50%. How many square feet larger than the original court is the new court?



84. _____ There are 300 members of the eighth-grade class at Woodlawn Beach Middle School, of whom 28 have Mr. Jackson for Algebra 1. Two members of the eighth-grade class will be selected at random to represent the school at an upcoming event. What is the probability that neither of the students selected will be from Mr. Jackson's Algebra 1 class? Express your answer as a decimal to the nearest hundredth.

85. $_____ Matthew earns a regular pay rate of $8.80 per hour, before deductions, at his full-time job. If he works more than 40 hours in a week, he earns overtime at $1\frac{1}{2}$ times his normal pay rate for any time worked beyond 40 hours. All of his deductions combined are 35% of his gross pay. How much does Matthew earn after deductions if he works 48 hours in one week?

86. _____ books According to one estimate, a new book is published every 13 minutes in the United States. Based on this estimate, how many books will be published in the year 2014? Express your answer to the nearest whole number.

87. _____ feet Stephen took a ride on a circular merry-go-round. The horse Stephen rode was at a distance of 15 feet from the center of the merry-go-round. If the ride made exactly $2\frac{3}{4}$ revolutions, how many feet did Stephen travel? Express your answer as a common fraction in terms of π.

88. _____ degrees The absolute difference between the measure of an acute angle and the measure of its supplement is 136 degrees. What is the degree measure of the acute angle?

89. _____ For what fraction of the day is the hour hand or minute hand (or both the hour and minute hands) of an analog clock in the upper half of the clock? Express your answer as a common fraction.

90. _____ m What is the height of a right square pyramid whose base measures 48 m on each side and whose slant height is 72 m? Express your answer as a decimal to the nearest hundredth.

Warm-Up 7

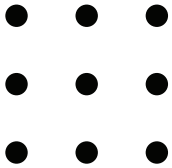
91. _____ If positive integers p, q and $p + q$ are all prime, what is the least possible value of pq?

92. _____ Two concentric circles have radii of x and $3x$. The absolute difference of their areas is what fraction of the area of the larger circle? Express your answer as a common fraction.

93. _____ To unlock her mobile device, Raynelle must enter the four different digits of her security code in the correct order. Raynelle remembers the four different digits in her security code. However, since she can't recall their order, she enters the four digits in a random order. What is the probability that the security code Raynelle enters will unlock her device? Express your answer as a common fraction.

94. _____ Penny has $4x$ apples and $7y$ oranges. If she has the same number of apples and oranges, what is the ratio of x to y? Express your answer as a common fraction.

95. _____ sides A polygon is made in this grid of 9 dots, by connecting pairs of dots with line segments. At each vertex there is a dot joining exactly two segments. What is the greatest possible number of sides of a polygon formed in this way?



96. _____ If $f(x) = x^2 + 3x - 4$ and $g(x) = \frac{3}{4}x + 6$, what is $g(-8) - f(-2)$?

97. _____ hours As Gregory enters his room for the night, he glances at the clock. It says 9:12 p.m. He listens to music and checks his social media page for half an hour. He then spends 15 minutes getting ready for bed. If he falls asleep 8 minutes after he climbs into bed and wakes up at 8:00 a.m. the next day, for how many hours was he asleep? Express your answer as a mixed number.

98. _____ If $\frac{x}{y} = \frac{3}{4}$ and $\frac{x}{z} = \frac{1}{8}$, what is the value of $\frac{y}{z}$? Express your answer as a common fraction.

99. $ _____ Bright Middle School has budgeted $10,000 to purchase computers and printers. Using the full amount budgeted, the school can buy 10 computers and 10 printers or 12 computers and 2 printers. What is the cost of 1 computer, in dollars?

100. $ _____ Last year, David earned money by performing odd jobs for his neighbors, and he had no other source of income. The combined amount David earned during January, February and March was $\frac{1}{12}$ of his total income. During April, May and June, combined, he earned $\frac{1}{6}$ of his total income. David earned $\frac{1}{2}$ of his total income during July, August and September. If the combined amount he earned during October, November and December was $2,000, what was his total income last year?

Warm-Up 8

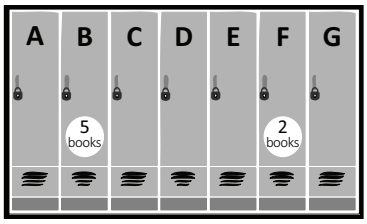
101. ____segments Sebi has a string that is 1.75 m long. What is the greatest number of segments, each 10 cm in length, that he can cut from this string?

102. ____children A group of children stopped to buy ice cream from a stand that sold 9 different flavors of ice cream. When every child in the group had purchased one double-scoop cone with two different flavors, every possible two-flavor combination had been served exactly once. If none of the children purchased the same two flavors, how many children were there in the group?

103. _____ The result of multiplying a number by 5 is the same as adding it to 5. What is the number? Express your answer as a common fraction.

104. ____feet For a certain rectangle, its perimeter, in feet, and area, in square feet, are numerically equal. If the length of the rectangle is 8 feet, what is its width? Express your answer as a mixed number.

105. ____books Lockers A through G are arranged side by side as shown, with lockers B and F containing exactly 5 books and exactly 2 books, respectively. In one of the other five lockers, there are exactly 8 books, in another exactly 7 books, in another locker exactly 5 books, in one other only 2 books, and one locker contains no books. The number of books in each of the lockers A through G is such that the total number of books contained in any two adjacent lockers is different from the number of books in each of the other five lockers. For example, the total number of books contained in lockers A and B is different from the number of books in each of the lockers labeled C, D, E, F and G. What is the combined number of books in lockers A and G?



106. ____units The center of a circle in a rectangular coordinate system has the coordinates (−8, −3). What is the radius of the circle if the circle touches the *y*-axis at only one point?

107. ____dollars Barbara's allowance is *x* cents per day. How many dollars in allowance will Barbara receive during the month of June? Express your answer as a common fraction in terms of *x*.

108. ____units^2 What is the absolute difference between the largest and smallest possible areas of two rectangles that each have a perimeter of 46 units and integer side lengths?

109. ____men In a group of 212 men and women, there were 32 more men than women. How many men were in the group?

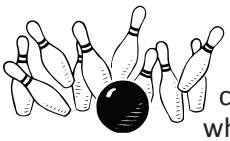
110. ____socks A drawer contains five brown socks, five black socks and five gray socks. Randomly selecting socks from this drawer, what is the minimum number of socks that must be selected to guarantee at least two matching pairs of socks? A matching pair is two socks of the same color.

Workout 4

111. _____ inches The Pine Lodge Ski Resort had exactly 200 inches of snowfall in 2000. The table shows the percent change in total snowfall for each year compared with the previous year. After 2003, what was the total snowfall, in inches, the year that the total snowfall first exceeded 200 inches? Express your answer as a decimal to the nearest hundredth.

Year	% Change
2001	+10
2002	−5
2003	−10
2004	+4
2005	+4
2006	+4
2007	+4
2008	+4

112. _____ games



Country Bowl charges $2.60 for bowling shoe rental and $4.00 for each game of bowling, with no charge for using their bowling balls. Super Bowl charges $2.50 per game, but its charge for shoe and ball rental is $7.10. For what number of games is the price the same at the two bowling alleys?

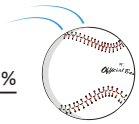
113. _____ dates A particular date is called a difference date if subtracting the month number from the day gives you the two-digit year. For example, June 29, 2023 and January 1, 2100 are difference dates since 29 − 6 = 23 and 1 − 1 = 00. Including these two dates, how many dates during the 21st century (January 1, 2001 to December 31, 2100) can be classified as difference dates?

114. _____ If the median of the ordered set $\{0, \frac{2}{5}x, x, 11.5x, 5, 9\}$ is 2, what is the mean? Express your answer as a decimal to the nearest hundredth.

115. $ _____ Carmen bought new software for her computer for $133.38, including 8% tax. What was the cost for the software before the tax was added?

116. _____ boxes Square tiles measuring 6 inches by 6 inches are sold in boxes of 10 tiles. What is the minimum number of boxes of tiles needed to exactly cover a rectangular floor that has dimensions 12 feet by 13 feet if only whole boxes can be purchased?

117. _____ pounds A giant panda bear must eat about 38% of its own weight in bamboo shoots or 15% of its own weight in bamboo leaves and stems each day. A male panda at the local zoo requires 49.35 pounds of bamboo leaves and stems daily. How much does the male panda weigh?

118. _____ %  Suppose the yarn wrapped around the rubber core inside a major league baseball is 450 feet long. In 1991, Cecil Fielder made a home run by hitting a baseball an amazing 502 feet. By what percent does the length of Fielder's home run exceed the length of yarn used to create a major league baseball? Express your answer to the nearest hundredth.

119. _____ newtons The formula $P = F/A$ indicates the relationship between pressure (P), force (F) and area (A). In newtons, what is the maximum force that could be applied to a square area with side length 4 meters so that the pressure does not exceed 25 newtons per square meter?

120. _____ in^2 A cylindrical can has a label that completely covers the lateral surface of the can with no overlap. If the can is 6 inches tall and 4 inches in diameter, what is the area of the label? Express your answer as a decimal to the nearest tenth.

Warm-Up 9

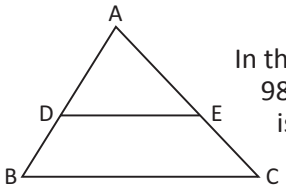
121. _____ feet A flea can jump 350 times the length of its own body. If a human were able to jump 350 times his or her height, how many feet would an average American, whose height is 5 feet 6 inches, be able to jump?

122. _____ people Of the 3 million people who auditioned for a television talent competition in the past 10 years, only 1% of 1% were selected to be contestants. How many people were selected to be contestants in this talent competition in the past 10 years?

123. _____ cans A rectangular room has a length, width and height of 15 feet, 12 feet and 8 feet, respectively. The room has one 30-inch by 60-inch window on each of the four walls. One wall also contains two 3-foot by 7-foot doors. If a can of paint is enough to cover an area of 100 ft^2, what is the minimum number of whole cans of paint needed to paint the walls and ceiling in this room?

124. _____ dollars Together Brianna and Shanita have \$24.00. If Brianna has \$3.00 more than twice the amount of money Shanita has, how many more dollars than Shanita does Brianna have?

125. _____ m



In the figure, segment DE is parallel to segment BC. The area of $\triangle ABC$ is 98 m^2. The area of $\triangle ADE$ is 50 m^2. The perimeter of $\triangle ADE$ is 55 m. What is the perimeter, in meters, of $\triangle ABC$?

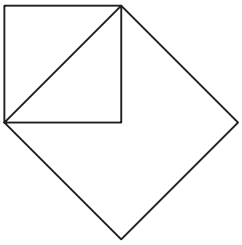
126. _____ fractions Two standard six-sided dice are rolled. One of the dice represents the numerator and the other represents the denominator of a fraction. The fraction is simplified, if possible. How many distinct fractions less than one can be generated by this method?

127. _____ dimes Rayshon has 51 coins consisting of dimes and nickels that total \$3.55. How many dimes does he have?

128. (____, ____) A circle passes through the origin and (8, 0). It has a radius of 5, and its center is in the first quadrant. What are the coordinates of its center? Express your answer as an ordered pair.

129. _____ If $f(x) = -(x - 1)^2 + 2$, what is the greatest possible value of $f(x) + 3$?

130. _____ $units^2$ In the figure shown, a side of the larger square is a diagonal of the smaller square. If the area of the smaller square is 1 square unit, what is the area of the larger square?



Warm-Up 10

131. _____ A 2-cup mixture consists of $\frac{2}{3}$ cup of flour and the rest is nuts. If 1 cup of flour is added to make a 3-cup mixture, what fraction of the 3-cup mixture is flour? Express your answer as a common fraction.

132. _____ years Marshall's age is 53, and Cody's age is 17. How many years ago was Marshall four times as old as Cody was?

133. _____ houses The houses on Main Street have three-digit house numbers that begin with either 7 or 9. If the remaining digits must contain one even and one odd digit and cannot contain a 0, what is the greatest number of houses that could be on Main Street?

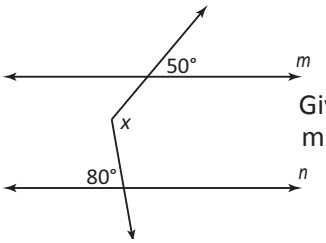
134. _____ mi/h What is the average speed of a cyclist who bikes up a hill at 6 mi/h but then bikes back along the same path down the hill at 12 mi/h?

135. _____ hours Shimdra is on vacation and wants to drive from Melbourne, Florida to Miami Beach, Florida.

The scale on the map is 1 inch = 16 miles. The map distance from Melbourne to Miami Beach is $11\frac{1}{4}$ inches. If Shimdra's average speed is 60 mi/h, how many hours will it take Shimdra to make the trip?

136. _____ What is the value of $\frac{(1.4\times10^{-7})(2.4\times10^{8})}{1.2\times10^{9}}$ when written in simplest form? Express your answer in scientific notation to two significant digits.

137. _____ degrees

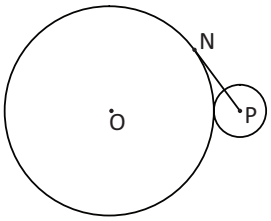


Given parallel lines *m* and *n* and the degree measures of the two marked angles, what is the degree measure of the angle marked *x*?

138. _____ students Three-year-old Sally attends a preschool class every weekday. One day, five new students were added to her class, after which there were $\frac{3}{2}$ as many students in Sally's preschool class as before. How many students were in the class before the addition of five new students?

139. _____ slugs Two cobbles and 3 burreys cost 19 slugs. If you subtract the cost of 5 cobbles from the cost of 37 slugs, you get the cost of 4 burreys. What is the total cost, in slugs, of 1 cobble and 1 burrey?

140. _____ cm



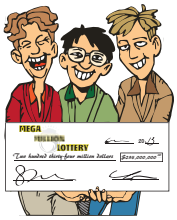
Circles O and P, of radius 16 cm and 4 cm, respectively, are tangent, as shown. Segment NP is tangent to circle O at point N. What is the length of segment NP?

Workout 5

141. _____ units One cube has a volume that is 728 units^3 larger than that of a second cube. If the smaller cube has edge length 10 units, what is the number of units in the edge length of the larger cube?

142. _____ cm^2 A rectangle is inscribed in a circle of radius 5 cm. The base of the rectangle is 8 cm. What is the area of the rectangle?

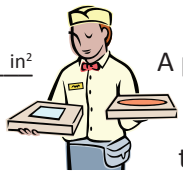
143. _____ % Three Maryland educators will split equally $234 million from the Mega Million Lottery. Each will collect about $53 million after taxes. What percentage of tax will be paid by each of the winners if the taxes also are split equally among the winners? Express your answer to the nearest whole number.



144. _____ A merchant alternately reduces and then increases the price of an item by 20%. After six price changes, the item is priced at *a*% of its original price. What is the value of *a*? Express your answer as a decimal to the nearest tenth.

145. _____ degrees When the sum of the degree measures of the acute angles of a scalene right triangle is divided by 8, what is the value of the quotient? Express your answer as a decimal to the nearest hundredth.

146. _____ in^2 A pizzeria sells a rectangular 18-inch by 24-inch pizza for the same price as its large round pizza with a 24-inch diameter. How many more square inches of pizza do you get with the round pizza for the same amount of money? Express your answer to the nearest whole number.



147. $_____ Kate notices that the cost of a week of electricity for air conditioning her house varies directly with the week's average outdoor temperature in degrees Fahrenheit. For a week in May, the average outdoor temperature was 81 °F and the air conditioning electricity bill was $32.40. What will Kate's air conditioning electricity bill be for a week in August when the average outdoor temperature is 96 °F?

148. _____ feet Columbus ran one time around the perimeter of a rectangular field that measures 40 feet by 70 feet. Pythagoras ran from one corner to the opposite corner and back. How much farther did one of them run than the other? Express your answer as a decimal to the nearest tenth.

149. _____ children A couple getting married today can be expected to have 0, 1, 2, 3, 4 or 5 children with probabilities of 20%, 20%, 30%, 20%, 8% and 2%, respectively. What is the mean number of children a couple getting married today can be expected to have? Express your answer to the nearest whole number.

150. _____ quarters A collection of nickels, dimes and quarters is worth $5.30. There are two more dimes than nickels and four more quarters than dimes. How many quarters are in this collection of coins?

Warm-Up 11

151. _____ The mean of seven numbers is 9. What is the new mean if each of the numbers is doubled?

152. _____ What is the 2013th digit after the decimal point when $\frac{1}{7}$ is expressed as a decimal?

153. _____ A number z is chosen at random from the set of positive integers less than 20. What is the probability that $\frac{19}{z} \geq z$? Express your answer as a common fraction.

154. _____ If $\frac{3}{4} = \frac{a}{36} = \frac{36}{b}$, what is the value of $a + b$?

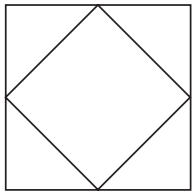
155. _____ degrees Some Mathletes® bought a circular pizza for \$10.80. Pat's share was \$2.25. Each student contributed to its cost based on the area of the fractional part he or she received. In degrees, what was the measure of the central angle of Pat's part?

156. _____ minutes

A pedestrian averages 3 mi/h on the streets of Manhattan, and a subway train averages 30 mi/h. If each city block is $\frac{1}{20}$ of a mile, how many more minutes than the subway train does it take for a pedestrian to travel 60 blocks in Manhattan?

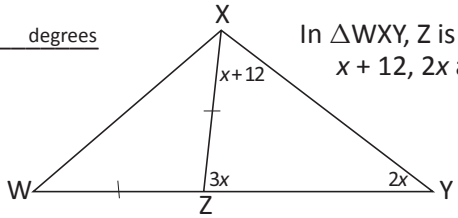
157. _____ Two different integers are randomly selected from the set of positive integers less than 10. What is the probability that their product is a perfect square? Express your answer as a common fraction.

158. _____ inches The figure shown here is to be made from a single piece of yarn. What is the shortest length of yarn that can be used to make the figure if each side of the outer square is 12 inches long and the vertices of the inner square each bisect a side of the outer square? Express your answer in simplest radical form.



159. _____ mi/h After driving along at a certain speed for 5 hours, Rich realizes that he could have covered the same distance in 3 hours if he had driven 20 mi/h faster. What is his current speed?

160. _____ degrees



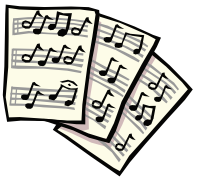
In $\triangle$WXY, Z is on side WY and WZ = XZ. If the angles of $\triangle$XYZ have measures $x + 12$, $2x$ and $3x$, as shown, what is the degree measure of $\angle$W?

Warm-Up 12

161. _____ km^2 If 1000 cubic meters of pine mulch can fertilize 0.02 square kilometers of soil, how many square kilometers of soil can be fertilized by 10^8 cubic meters of pine mulch?

162. _____ If $9^c = 27^{c-1}$, what is the value of c?

163. _____ Each term after the first term of the sequence 2, 4, 8, … is 2 times the preceding term. Each term after the first of sequence 10, 20, 30, … is 10 more than the preceding term. What is the least value of n such that the nth term of the first sequence is greater than the nth term of the second sequence ?



164. _____ combinations For the orchestra contest, Mrs. Treble is going to select 4 pieces of music from the recommended list of 20 pieces. How many combinations of 4 pieces of music are possible?

165. _____ integers How many of the first 500 positive integers are multiples of all three integers 3, 4 and 5?

166. _____ inches

The distance from the center of a clock to the tip of the minute hand is 4 inches. Between 2:45 p.m. and 7:15 p.m., what is the total distance traveled by the tip of the minute hand? Express your answer in terms of π.

167. _____ Nine people are forming three teams of three people each for a game of XFlag. Each team has one player who is the captain. The nine participants are Alana, Benny, Chico, Danzig, Elias, Frederico, Gina, Hsin-Hsin and Illiana. They are very particular about which players can be on a team together. Frederico must be with Hsin-Hsin or Illiana. Elias, Frederico and Gina must be on different teams. Hsin-Hsin and Illiana must be on different teams. Chico and Danzig must be on the same team; neither is a captain. Danzig cannot be on a team with Gina as captain. Frederico cannot be on a team with Alana as captain. Hsin-Hsin cannot be on a team with Elias as captain. Alana and Benny are captains. Who is the third captain?

168. _____ ft^2 In Figure 1, four congruent circles are inscribed in a square, and in Figure 2, sixteen congruent circles are inscribed in a square. Both squares measure 4 feet by 4 feet. What is the absolute difference between the shaded area in Figure 1 and the shaded area in Figure 2?

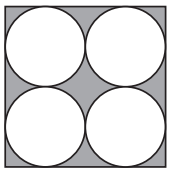


Figure 1

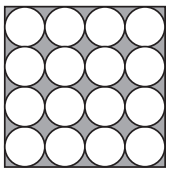


Figure 2

169. _____ If $x^2 + \frac{1}{x^2} = 3$, what is the value of $x^4 + \frac{1}{x^4}$?

170. _____ $units^2$ What is the area of the △JKL in the coordinate plane with vertices J(−3, 2), K(−1, −2) and L(5, 6)?

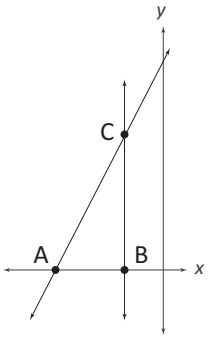
Workout 6

171. _____ Juwan purchased several coats from a manufacturer. He wants to sell the coats at his store for 60% more than he paid. By what number should he multiply the manufacturer's price to determine his price? Express your answer as a decimal to the nearest tenth.

172. _____ % What is the percent of increase in the volume of a cube when its edge length is increased by 50%? Express your answer to the nearest tenth.

173. _____ inches What is the perimeter of a 60-degree slice of a pizza with a 7-inch radius? Express your answer as a decimal to the nearest tenth.

174. _____



In the *xy*-coordinate plane, AB = 5, AC = 13, and lines BC and AB are perpendicular, as shown. The coordinates of point B are (−4, 0), and point A is on the *x*-axis. If the *y*-intercept of line CA has coordinates (0, *k*), what is the value of *k*? Express your answer as a decimal to the nearest tenth.

175. _____ The quadratic equation $x^2 - 7x + 5 = 0$ has two real roots, *m* and *n*. What is the value of $\frac{1}{m} + \frac{1}{n}$? Express your answer as a common fraction.

176. _____ To win a certain lottery, one must match three different numbers chosen from the integers 1 through 25, in any order. Liam buys 500 tickets, each with a unique combination of numbers. What is his probability of winning? Express your answer as a decimal to the nearest hundredth.

177. $ _____ Jayden buys two belts and four scarves for $59.70. Katrina buys three belts and five scarves for $80.60. Assuming that all belts cost the same and all scarves cost the same, what is the cost of one belt?

178. _____ inches A piece of wood is to be sawed into 15 smaller pieces, each 2 inches in length. If each cut eliminates $\frac{1}{8}$ inch of wood as sawdust, how many inches long must the original piece of wood be? Express your answer as a decimal to the nearest hundredth.

179. _____ penguins There are 190 penguins standing near the beach, and then one penguin dives into the sea, followed by two penguins, then three penguins, and so on, with one more penguin in each group. How many penguins will be in the last group?

180. _____ cm^2 A circle of radius 1 cm is inscribed in a square, as shown in Figure 1. In Figure 2, a circle of radius 1 cm circumscribes a square. What is the absolute difference between the shaded area in Figure 1 and the shaded area in Figure 2? Express your answer as a decimal to the nearest hundredth.

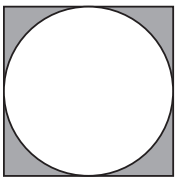


Figure 1

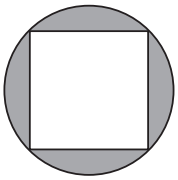


Figure 2

Warm-Up 13

181. _____ ordered pairs A bag contains eight red tags numbered from 1 to 8 and two blue tags numbered 9 and 10. If Sid selects two tags from the bag at random, with replacement, how many different ordered pairs of tags can he select with at least one red tag?

182. _____ While working with a drawing program, Rochelle changed the dimensions of a rectangle so that the new rectangle was $\frac{1}{2}$ as tall and $\frac{1}{4}$ as wide as the original rectangle. What is the ratio of the area of the new rectangle to the area of the original rectangle? Express your answer as a common fraction.

183. _____ The graph of a linear equation contains the points (1, a), (2, b) and (4, 18). What is the value of $\frac{3}{2}b - a$?

184. _____ marbles Sue, Kim and Donna each entered into a drawing to guess the number of marbles in a jar. Sue estimated 300, Kim estimated 325 and Donna estimated 360. One estimate was off by 49 marbles, another estimate was off by 24 marbles and the remaining estimate was off by 11 marbles. How many marbles are in the jar?

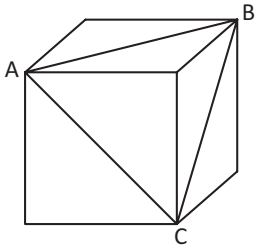
185. _____ A cylinder has height 9 inches and diameter 8 inches. The volume of the largest sphere that can fit in the cylinder is $\frac{\pi x}{3}$ in^3. What is the value of x?

186. $ _____ Briana's retail store had a $1000 surplus last quarter, so she awarded employee bonuses. She divided the entire surplus among her sales team, which includes a sales manager, five full-time sales associates and four junior associates. Each junior associate received the same amount of money. Each full-time sales associate received twice as much as each junior associate. The sales manager received three times as much as each full-time sales associate. What was the amount of the sales manager's bonus?

187. _____ Given that $x^2 + 2xy + y^2 = 81$, $x > 0$ and $y > 0$, what is the mean of x and y? Express your answer as a decimal to the nearest tenth.

188. _____ What is the value of $1 - 5 \cdot 2 + 10 \cdot 2^2 - 10 \cdot 2^3 + 5 \cdot 2^4 - 2^5$?

189. _____ bricks Vincent is building a structure out of LEGO® bricks. The top level (first level) is a rectangle of 1 brick by 2 bricks. The level directly below the top level (second level) is a rectangle of 2 bricks by 3 bricks. The third level is a 3-by-4 rectangle. If the pattern continues, how many bricks are needed to build a structure 12 levels high?



190. _____ in^2 Vertices A, B and C of a cube are connected as shown. If the volume of the cube is 27 in^3, what is the area of $\triangle$ABC? Express your answer as a common fraction in simplest radical form.

Warm-Up 14

191. _____ What is the units digit of $1! + 2! + 3! + \cdots + 2014!$?

192. _____ units^2 In a square of side length 12 units, what is the area of the region of points that are closer to the center of the square than to any vertex?

193. _____ The product of the integers from 1 through 10 is equal to $2^a \cdot 3^b \cdot 5^2 \cdot 7$, where a and b are positive integers. What is the value of $a + b$?

194. _____ km/h Ava and Lizzy were both competing in long-distance bike races. Ava's race was 150 km long, and Lizzy's race was 180 km long. They completed their races in the same time. If Lizzy's average rate was 2 km/h faster than Ava's, what was Ava's average rate?

195. _____ The product of the first three terms of an arithmetic sequence of integers is a prime number. What is the sum of the three numbers?

196. _____ feet A triangle has a base of 3 feet and a height of 6 feet. If its area is increased to 35 ft^2 by increasing the base and height by the same number of feet, what is the sum of the new base and height?

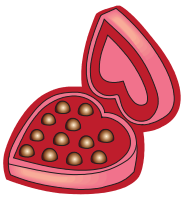
197. _____ lists How many lists of integers that contain only powers of 2 have a sum of 9, if the order within each list is not important and numbers can be repeated?

198. _____ Anthony took five tests in physics last semester, each with a maximum score of 100. Anthony's five integer test scores have a mean of 85, a median of 87 and a unique mode of 92. What is the lowest possible score Anthony could have for one of his five tests?

199. _____ marbles Sara told Jo, "If you give me three of your marbles, I will have twice as many as you." Jo responded, "If you give me just two of your marbles, I will have twice as many as you." How many marbles does Sara have?

`

200. _____ Angel opens a box containing a dozen chocolate-covered candies. Each candy has a vanilla, raspberry or lemon crème filling. Angel likes the vanilla candies, but she can't tell which candies have vanilla filling because the candies in the box look identical. If four of the candies have vanilla filling, and Angel randomly selects two candies from the box, what is the probability that both candies have vanilla filling? Express your answer as a common fraction.



Workout 7

201. $ _____ Twenty of Ms. Oliver's math students each agreed to contribute the same amount of money toward the purchase of her retirement gift. The students would have had exactly enough for the gift they selected, but four students forgot to bring in their contributions. If each of the other students gives an additional $1.50, they will still have exactly enough to purchase the gift that was selected. What is the total amount of money the students need to purchase the gift they selected for Ms. Oliver?

202. _____ When $\dfrac{(a^2b^{-3}c)^2(a^{-1}b^3c^3)^{-3}}{(a^{-1}b^3c^{-2})^2(a^4bc^{-3})^4}$ is rewritten as $a^xb^yc^z$, what is the value of $x + y + z$?

203. _____ miles The engineers at an auto manufacturer pay students $0.08 per mile plus $25 per day to road test their new vehicles. Archer earned $48 on one day by road testing a new car. How far did he drive? Express your answer as a decimal to the nearest tenth.

204. _____ times In 1982, Albert Rayner skipped a rope 128 times in 10 seconds. If he could have continued at that rate, how many times would he have skipped the rope in 1 hour?

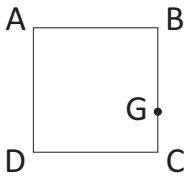
205. _____ If Andrew and Beth each randomly think of an integer from 1 to 10 inclusive, what is the probability that their numbers are relatively prime? Note: Every integer is relatively prime to 1. Express your answer as a common fraction.

206. _____ hours The number of *E. coli* bacteria in a rich medium is increasing at a rate of 50% every 3 hours. How many hours does it take for 960 bacteria to increase to 4860 bacteria?

207. _____ spheres A pile of clay can be molded to form a solid cube with edges that measure 10 cm. What is the maximum number of solid spheres of radius 2 cm that can be made from the same amount of clay?

208. _____ There are exactly $n!$ seconds in 6 weeks. What is the value of n?

209. _____ m^2 A goat is fastened by a rope to an exterior wall of a square building ABCD at point G, shown here. The distance from the goat's collar to the end of the rope fastened to the wall is 3 m. If G is 1 m from C and 2 m from B, how many square meters of the large yard surrounding ABCD can the goat cover? Express your answer to the nearest integer.



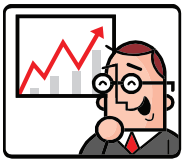
210. _____ For the integer 263, neither pair of consecutive digits (the pair 2 and 6 and the pair 6 and 3) is relatively prime. But the nonconsecutive digits 2 and 3 are relatively prime. What is the greatest positive integer n that satisfies the following three conditions?

(1) None of the digits is zero.
(2) No pair of consecutive digits is relatively prime.
(3) All non-consecutive digits are relatively prime.

Warm-Up 15

211. _____ hours A cylindrical tank starts with an unspecified amount of water in it, and water is added to it at a constant rate. After 8 hours the depth of the water in the tank is 34 feet. After 12 hours, the water depth is 35 feet. After how many hours in all will the water depth in the tank be 37 feet?

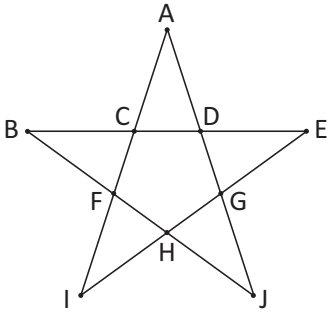
212. $ _____



The closing price of a stock on Tuesday was double its closing price on Monday. Its closing price on Wednesday was 3 times its closing price on Tuesday. Its closing price on Thursday was 10 times its closing price on Wednesday. Its closing price on Thursday was $72,000. What was its closing price on Monday?

213. _____ ft^2 In the Mexican jungle, an archaeologist finds a right square pyramid with a base edge length of 60 yards. The pyramid has lateral edge length 50 yards. In square feet, what is the total surface area of the lateral faces of the pyramid?

214. _____ arrangements The game board for Star Line-Up is shown. Players take turns placing markers on the lettered points. The first player to get three markers on the same line is the winner. For instance, a player would win for covering A, D and G or for covering A, D and J. How many winning arrangements of three markers are there?



215. _____ If $\frac{64^8}{32^{-3}} = 2^k$, what is the value of k?

216. _____ A wooden cube is painted on each of its faces and then cut into n^3 unit cubes. If 216 of those smaller cubes are painted on exactly one face, what is the value of n?

217. _____ Line m is tangent to a circle at the point (3, 7). If (1, 4) is the center of the circle, what is the slope of line m? Express your answer as a common fraction.

218. _____ If $f(x) = \begin{cases} \sqrt{x} & \text{if } x \geq 16 \\ x+1 & \text{if } 10 < x < 16 \\ \frac{x}{2} - 10 & \text{if } 0 \leq x \leq 10 \\ x^2 & \text{if } x < 0 \end{cases}$, what is the value of $f(f(f(f(f(f(f(f(f(f(f(f(13))))))))))))$?

219. _____ days

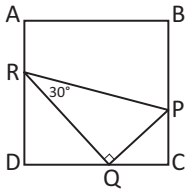
The *ides* occur on the 15th day of March, May, July and October, but the ides occur on the 13th day of every other month. What is the maximum number of days strictly between two ides?

220. _____ units There exist two non-congruent right triangles for which the length of the shorter leg in each triangle is 9 units and all sides have integer lengths. What is the sum of the lengths of the longer legs of these two triangles?

Warm-Up 16

221. _____ In the decimal representation of $\frac{1}{17} = 0.\overline{058m235294117647}$, what is the value of the digit m?

222. _____



Quadrilateral ABCD is a square. What is the ratio of the area of $\triangle$PQC to the area of $\triangle$RQD? Express your answer as a common fraction.

223. _____ teams In a basketball league of n teams in which each team plays every other team twice, the total number of games played is $n^2 - n$. How many teams are in the league if 56 games were played?

224. _____ The odds against the Patriots playing in the championship game are 4:1. The odds against the Texans playing in the championship game are 7:1. What is the probability that those two teams will play each other in the championship game? Express your answer as a common fraction.

225. _____ ordered pairs How many ordered pairs of integers (x, y) satisfy $xy = 144$?

226. _____ yards A boat can hold three people, one of whom needs to row to cross a river that is 20 yards wide. What is the minimum distance the boat must travel to transport 9 people from the left bank of the river to the right bank?

227. _____ Point C(4, 2) is on circle O, with center (4, −2). Segment CD is a diameter of circle O. If circle O intersects the x-axis at A and B, what is the ratio of the length of $\overset{\frown}{ACB}$ to the length of $\overset{\frown}{ADB}$? Express your answer as a common fraction.

228. _____ On October 12, 2006, Michael Cresta scored 830 points in a game of Scrabble. At one point in the game, the letter R was on the board, and he had the letters I, O, Q, U and X on his rack. In the bag of tiles, there were 3 Ts, 1 Y and 52 other tiles. He needed to draw a T and a Y in two draws, without replacement, to have the letters to make the word QUIXOTRY. What is the probability of getting a T and a Y when 2 tiles are randomly selected, without replacement, from the bag of tiles described? Express your answer as a common fraction.

229. _____ gallons A 5-gallon and a 20-gallon jug can be used to measure exactly 5 gallons or 20 gallons of water, respectively. They can also be used to measure 15 gallons by filling the 20-gallon jug with water, dumping 5 gallons into the 5-gallon jug and having 15 gallons of water left in the 20-gallon jug. Using similar processes, what is the sum of all positive integer numbers of gallons that can be obtained using only the 5-gallon and the 20-gallon jugs?

230. _____ cars As a used-car salesperson, Noah has a monthly sales quota, which is the minimum number of cars he must sell each month. Noah had not sold any cars in June, as of the 24th of the month. However, on June 25th, Noah sold half of the number of cars in his monthly quota, plus one more car. On June 26th, he sold half of the remaining number of cars he needed to sell, plus one more car. The same pattern continued until June 30th, when Noah sold half of the remaining cars he needed to sell, plus one more car and reached his monthly sales quota. Noah has a monthly sales quota to sell how many cars?

Workout 8

231. _____ Given that 12, v, w, x, y, z is an arithmetic sequence whose median is 20.75, what is the sum of these six numbers? Express your answer as a decimal to the nearest tenth.

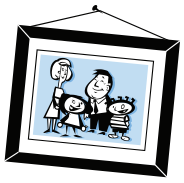
232. _____ ways In a beauty contest, 51 contestants must be narrowed down to a first-, second- and third-place finalist. In how many ways can the three finalists be chosen?

233. _____ minutes

A runner completes the first mile of a 26-mile race in 5 minutes. After that, each mile takes 1% longer than the previous mile. How many minutes does it take the runner to complete 26 miles? Express your answer to the nearest whole number.

234. _____ combinations In the land of Binaria, the currency consists of coins worth 1¢, 2¢, 4¢, 8¢, 16¢, 32¢ and 64¢. Bina has two of each coin. How many combinations of her coins have a combined value of 50¢?

235. _____ in^2 John has an 8-inch by 10-inch photo that he wants to shrink so that its perimeter is exactly 27 inches. After the photo has been reduced in size, what will be the area of the new photo?



236. _____ cm Each rectangle in a collection has a length 1 cm more than three times its width. What is the maximum possible width of one of these rectangles if its perimeter is less than or equal to 150 cm? Express your answer as a decimal to the nearest tenth.

237. _____ % If $x > 0$, then 2% of 5% of $3x$ equals what percent of x? Express your answer to the nearest tenth.

238. _____ ways

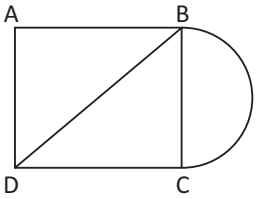
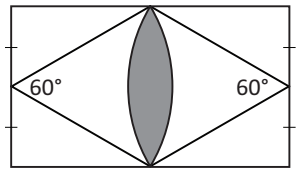


Figure ABCD can be drawn, without retracing, in one continuous pen stroke. If the stroke must begin at A, B, C or D, in how many different ways can this be done?

239. _____ $units^2$ A rectangle measures $2 \times 2\sqrt{3}$ units. Two arcs are drawn with their centers at the midpoints of the shorter sides, as shown. What is the area of the shaded region? Express your answer as a decimal to the nearest hundredth.



240. _____ % If four people each randomly pick an integer from 1 to 10, inclusive, what is the probability that at least two of the people pick the same integer? Express your answer to the nearest tenth.

Warm-Up 17

241. _____ students At one school, the ratio of students who have one or more younger siblings to those who have no younger siblings is 6 to 5. If 180 students do not have a younger sibling, how many students at this school have one or more younger siblings?

242. _____ inches A regular octagon has side length $2\sqrt{2}$ inches. What is the median length of all its diagonals? Express your answer in simplest radical form.

243. _____ Kevin and Evan have a set of 10 freshly inked stamps, one for each digit 0 through 9. When freshly inked, each stamp makes exactly 20 impressions. Kevin and Evan will stamp consecutive integers beginning with 1 and continuing until not enough ink remains to stamp the next consecutive number. What is the last number Kevin and Evan will be able to stamp?

244. _____ Each box in the expression below is to be filled in with one of the symbols +, −, × or ÷, with each symbol used exactly once. If no parentheses are inserted, what is the least possible absolute value of the resulting number? Express your answer as a decimal to the nearest tenth.

$$1 \square 2 \square 3 \square 4 \square 5$$

245. _____ units What is the greatest possible distance between some point on a square of side length 2 units and some point on its inscribed circle? Express your answer in simplest radical form.

246. _____ If $f(x) = x(x - 1)(x + 1)$, what is the product of the nonzero real numbers x such that $f(x) = x$?

247. _____ inches The integer sides of a triangle are in the ratio 3:4:6. If the perimeter of the triangle is 26 inches, what is the length of the longest side?

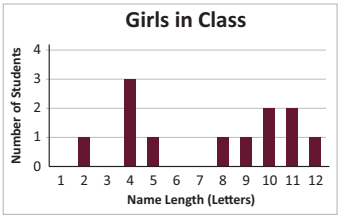
248. _____ seconds There is a moving sidewalk in the local shopping mall. When Marlow stands still on the moving sidewalk, it takes her 180 seconds to get from one end of the sidewalk to the other end. Walking beside the moving sidewalk at a constant rate, it takes Marlow 90 seconds to travel the same distance. If Marlow were to get on the sidewalk and walk at her same rate, in the same direction as the moving sidewalk, how many seconds would it take her to get from one end of the sidewalk to the other end?

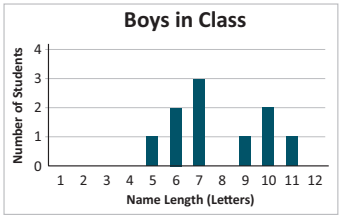
249. _____ ten-dollar bills Sam went to the bank to withdraw $440.00. He received 30 bills altogether. There were some five-, some ten- and some twenty-dollar bills. Sam received four times as many twenty-dollar bills as five-dollar bills. How many ten-dollar bills did Sam receive?

250. _____ Using the number keypad shown, it is possible to convert any three-digit number to various three-letter strings by choosing one letter for each number. For example, 223 can be used to make *BAD*, *ACE*, *CCF*, and 24 others, some of which are real words and some of which are not. Using only the buttons 2 through 9, what is the smallest three-digit number such that none of its possible three-letter strings are real words? (Exclude proper nouns and abbreviations.)

Warm-Up 18

251. _____ The bar charts below show the number of letters in the first names of the girls and boys in Mrs. Rodriguez's class. If one girl and one boy are chosen at random, what is the probability that the chosen students have the same number of letters in their first names? Express your answer as a common fraction.





252. _____ positive integers How many positive integers are in the domain of $f(x) = \dfrac{\sqrt{12 - x}}{4 - x^2}$?

253. _____ Two standard six-sided dice are to be rolled. If the sum is an even number greater than 7, then what is the probability that both dice are even? Express your answer as a common fraction.

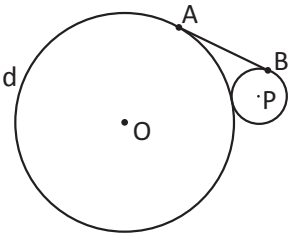
254. _____ units^3 A cylinder's radius is equal to its height. If its surface area is 100π units^2, what is its volume? Express your answer in terms of π.

255. _____ A number n is randomly chosen from the set {1, 2, 3, …, 24, 25}. What is the probability that the equation $x^2 + nx + 24 = 0$ has two integer solutions? Express your answer as a common fraction.

256. _____ : _____ a.m. Silas has a clock that gains 15 minutes each hour; for instance, if it shows the correct time at 2:00 p.m., one hour later it will show a time of 3:15 p.m. when it should show 3:00 p.m.

Last night, Silas set the clock to the correct time at 10:00 p.m. While he was sleeping, the clock stopped working, and it showed a time of 4:00 a.m. That was 4 hours before he woke up. At what time did Silas wake up?

257. _____ cm Circle O and circle P are tangent to each other. Circle O has radius 8 cm and is tangent to segment AB at A, as shown. Circle P has radius 2 cm and is tangent to segment AB at B. What is the length of segment AB?



258. _____ The six integers 1, 3, 5, 7, 9 and 11 form an arithmetic sequence. If three of the integers are selected randomly without replacement, what is the probability that they form an arithmetic sequence in the order they are selected? Express your answer as a common fraction.

259. _____ The first three terms of an arithmetic sequence are p, 6 and $2p - 3$. What is the tenth term of this sequence?

260. _____ If RATS $\times$ 4 = STAR, and each letter represents a different digit from 0 to 9, inclusive, what is the value of S + T + A + R?

Workout 9

261. _____

Subtract → ↓		
17	10	7
8	5	3
9	5	4

Figure 1

In Figure 1, numbers were subtracted vertically and horizontally until a value was found for the shaded box. For instance, 17 – 10 = 7 and 8 – 5 = 3 were the results from the first two rows, with 7 – 3 = 4 in the third column. If the partially completed grid in Figure 2 follows the same rules, what is the value of $m + n$?

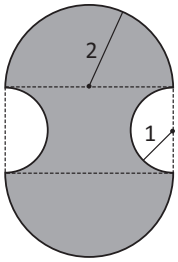
Subtract → ↓		
263	m	
n	34	
		104

Figure 2

262. _____ If $(a + b)^c = 1024$ for positive integers $a < b < c$, what is the value of b?

263. _____ seconds The time, t seconds, that it takes for a rock to fall a distance, d meters, is approximately $t = 0.45\sqrt{d}$. How many seconds does it take a rock to fall 200 m? Express your answer as a decimal to the nearest tenth.

264. _____ units^2 Congruent semicircles are placed on the top and bottom of a rectangle (dotted segments), and congruent semicircles are removed from the left and right sides of the rectangle, as shown. How many square units are in the area of the shaded figure? Express your answer as a decimal to the nearest hundredth.

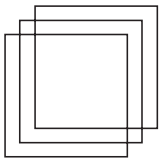


265. _____ in^2 An ice cream cone is 5 inches tall with a radius of 1.5 inches. The outer surface area of the cone is covered with chocolate. How many square inches of cone are covered with chocolate? Express your answer as a decimal to the nearest tenth.

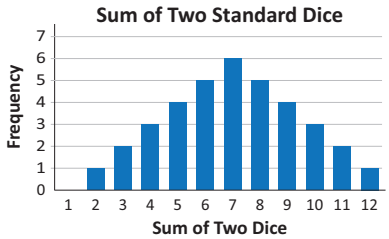
266. _____ In Bag A are 9 red balls and 1 green ball. A player gets one draw from Bag A and wins if the green ball is selected. In Bag B are 99 red balls and 1 green ball. A player gets 20 draws from Bag B and wins if the green ball is selected, but if the selected ball is red, it must be returned to the bag before the next draw. What is the absolute difference between the probability of winning with Bag A and the probability of winning with Bag B? Express your answer as a decimal to the nearest thousandth.

267. _____ elements How many elements are in the set $\{x^2 - 2x + 1 \mid x = -5, -4, \ldots, 4, 5\}$?

268. _____ regions When three squares are arranged as shown, seven unique regions are formed. What is the maximum number of regions that can be formed by three congruent, overlapping squares?



269. _____ Two standard six-sided dice, each with faces numbered with the positive integers 1 through 6, have the probability distribution shown for the sum of the top-facing values on the dice. Two non-standard but fair six-sided dice can be numbered differently with nonnegative integers on each face and still yield the same probability distribution. Though a number may be on both dice, a number may not appear more than once on either die. If a is the sum of the six numbers on one of these non-standard dice, and b is the sum of the six numbers on the other die, what is the value of the product $a \times b$?



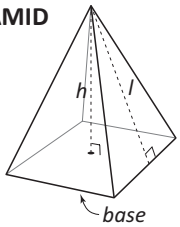
270. _____ The digits 2, 0, 1 and 4 are used to create every possible positive four-digit integer, with each digit used exactly once in each integer. What is the arithmetic mean of all these integers?

Surface Area & Volume Stretch

This activity involves determining the surface area (SA) and volume (V) of various geometric solids. For the purposes of these exercises, all solids are assumed to be right (the height is perpendicular to the base at its center). Below is an example of each solid along with the formulas for determing its surface area and volume.

B = base area | *P* = base perimeter | *h* = height | *l* = slant height | *r* = radius

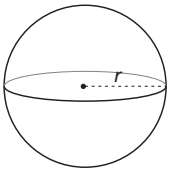
PYRAMID



$SA = B + \frac{1}{2}Pl$

$V = \frac{1}{3}Bh$

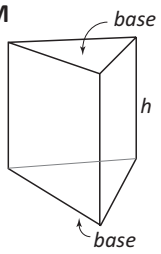
SPHERE



$SA = 4\pi r^2$

$V = \frac{4}{3}\pi r^3$

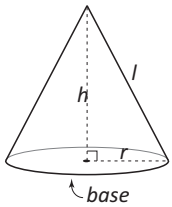
PRISM



$SA = 2B + Ph$

$V = Bh$

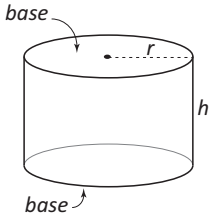
CONE: *Pyramid with circular base*



$SA = \pi r^2 + \pi rl$

$V = \frac{1}{3}\pi r^2 h$

CYLINDER: *Prism with circular bases*



$SA = 2\pi r^2 + 2\pi rh$

$V = \pi r^2 h$

For 271 and 272, find the surface area and volume of the geometric solid.

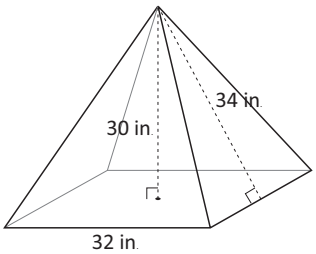
271. SA = _____ in^2

V = _____ in^3

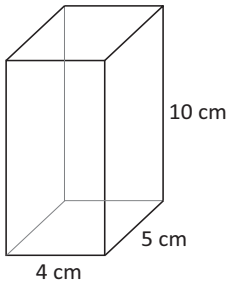
272. SA = _____ cm^2

V = _____ cm^3

271. square pyramid



272. rectangular prism



For 273-275, find the surface area and volume of the geometric solid. Express your answer in terms of π.

273. SA = _____ m^2

V = _____ m^3

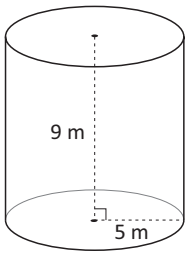
274. SA = _____ cm^2

V = _____ cm^3

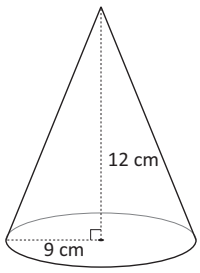
275. SA = _____ ft^2

V = _____ ft^3

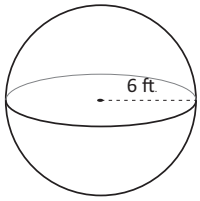
273. cylinder



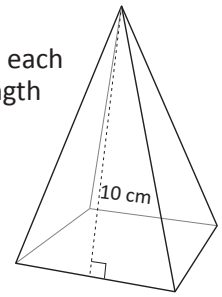
274. cone



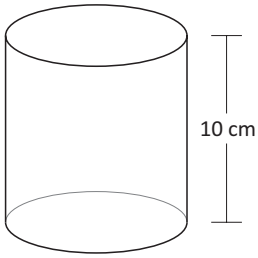
275. sphere



276. _____ cm A right square pyramid has lateral faces with slant heights that are each 10 cm. If the surface area of this pyramid is 96 cm^2, what is the length of one of the edges of the base?

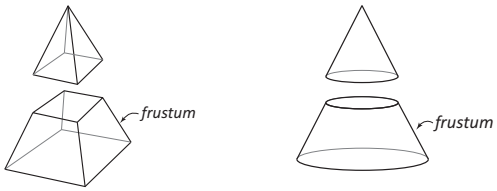


277. _____ cm^2

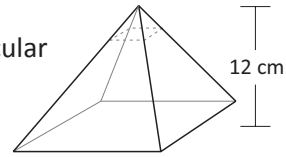


What is the surface area, in square centimeters, of a cylinder with volume 250π cm^3 and height 10 cm? Express your answer in terms of π.

The frustum of a cone or a pyramid is that part of the solid left when the top portion is cut off by a plane parallel to its base.



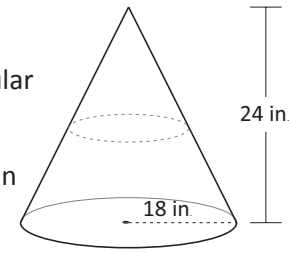
278. A pyramid with height 12 cm has a square base with area 64 cm^2. A plane perpendicular to the height intersects the pyramid 3 cm from its apex.



a. _____ cm^3 What is the volume of the resulting frustum?

b. _____ cm^2 What is the surface area of the frustum? Express your answer in simplest radical form.

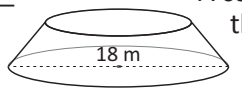
279. A cone with a height of 24 inches has a base with radius 18 inches. A plane perpendicular to the height intersects the cone halfway between its apex and base.



a. _____ in^3 What is the volume of the resulting frustum? Express your answer in terms of π.

b. _____ in^2 What is the surface area of the frustum? Express your answer in terms of π.

280. _____ m^3

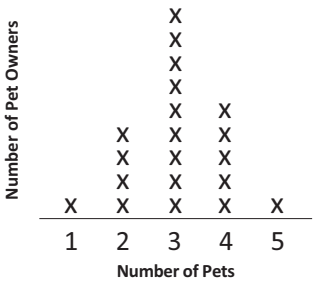


A cone of height 9 m was cut parallel to its base at 3 m above its base. If the base of the original cone had diameter 18 m, what is the volume of the resulting frustum? Express your answer in terms of π.

Data & Statistics Stretch

281. _____ pets As part of a survey, 20 pet owners indicated the total number of pets they currently own, and the results are displayed in the line plot shown. What is the mean of the median and the mode of the data?

Total Number of Pets per Owner



282. _____ mi/h The maximum speed of the fastest roller coaster at 20 different amusement parks is shown in this stem-and-leaf plot, where 10|7 represents 107 mi/h. What is the absolute difference between the mean and the median of the data? Express your answer as a decimal to the nearest tenth.

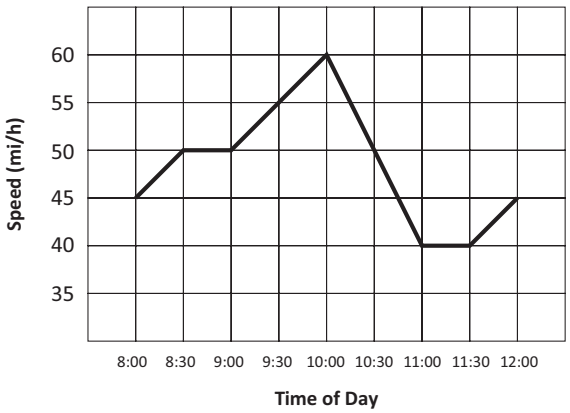
Maximum Speed of 20 Roller Coasters (mi/h)

Stem	Leaf				
9	0	2	3	5	6
10	0	0	7		
11	6	9			
12	0	8			
13	0	5	6	9	
14	1	3	8		
15	0				

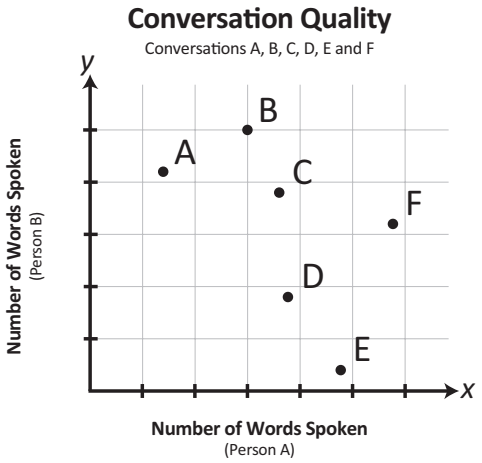
283. _____ mi/h The line graph shows one driver's speed, in miles per hour, from 8:00 a.m. to 12:00 noon. Based on the graph, what was the driver's average speed from 10:30 to 11:30? Express your answer as a decimal to the nearest tenth.

Driver's Speed

8:00 a.m. to 12:00 noon



284. __point_____ The quality of a conversation between two people can be described by the ratio of the number of words spoken by each person. The closer the ratio is to 1, the higher the quality. Each point in the graph shown represents a two-person conversation, with the number of words spoken by one person on the *x*-axis and the number of words spoken by the other person on the *y*-axis. If both axes use the same scale, which point in the graph represents the conversation with the highest quality?



285. _____ If the mean of five values is 27, what is the sum of the five values?

286. _____ When each of five numbers is doubled, the mean of the five new numbers is 60. What was the mean of the five original numbers?

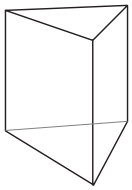
287. _____ A list of 20 numbers has a mean of 37. When two numbers are removed from the list, the new mean is 38. What is the mean of the two numbers that were removed?

288. _____ The mean, median and unique mode of six positive integers are 8, 7 and 3, respectively. What is the maximum possible value for the range of the six numbers?

289. _____ The mean of three consecutive terms in an arithmetic sequence is 10, and the mean of their squares is 394. What is the largest of the three original terms?

290. _____ Six different positive integers add to 66. If one of them is the mean and another is the range, what is the largest possible number in the set?

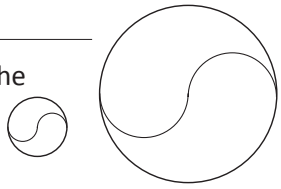
Geometric Proportions Stretch



291. _____ smaller prisms

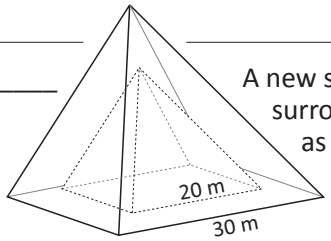
Each edge of the smaller triangular prism shown is $\frac{1}{2}$ the corresponding edge length of the larger triangular prism. How many of the smaller prisms combined have a total volume equal to the volume of the larger prism?

292. _____ A plane, parallel to the bases, slices the smaller prism $\frac{3}{4}$ of the way from one base to the other, dividing it into two smaller prisms. Of the three prisms, what fraction of the volume of the largest prism is the volume of the smallest prism? Express your answer as a common fraction.



293. _____ For the two similar circles shown, the area of the large circle is nine times the area of the small circle. What is the ratio of the radius of the small circle to the radius of the large circle? Express your answer as a common fraction.

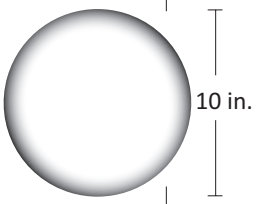
294. _____ cm If the S-curve in the large circle has length 3.12 cm, what is the length of the S-curve in the small circle? Express your answer as a decimal to the nearest hundredth.



295. _____ A new solid pyramid with a square base of side length 30 m will be constructed surrounding an existing solid pyramid with a square base of side length 20 m, as shown. If the existing and new pyramids are similar, what is the ratio of the total volume of the new pyramid to the volume of the old pyramid? Express your answer as a common fraction.

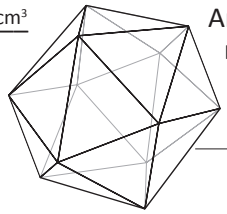
296. _____ years It took 16 years to completely build the original pyramid. Adding to the original pyramid, at the same volume-per-year rate, in how many years will construction of the new, larger pyramid be completed?

297. _____ in^3 A glassblower starts with a solid glass sphere that is 2 inches in diameter. What is the volume of the glass sphere? Express your answer as a decimal to the nearest hundredth.



298. _____ The glassblower will heat the glass sphere and blow air into it to create a hollow sphere 10 inches in diameter of uniform thickness. What is the ratio between the surface area of the original sphere and that of the new, hollow sphere? Express your answer as a common fraction.

299. _____ In the finished hollow sphere, what is the ratio of the volume of glass to the volume of enclosed air? Express your answer as a common fraction.



300. _____ cm^3 An icosahedron sculpture is being installed at a state fair. On the architect's model of the sculpture, each of the 20 equilateral triangular faces has area 2.85 cm^2. The actual sculpture has a total surface area of 4617 cm^2. If the volume of the model is 34 cm^3, what is the volume of the actual sculpture?

ANSWERS

In addition to the answer, we have provided a difficulty rating for each problem. Our scale is 1-7, with 7 being the most difficult. These are only approximations, and how difficult a problem is for a particular student will vary. Below is a general guide to the ratings:

Difficulty 1/2/3 - One concept; one- to two-step solution; appropriate for students just starting the middle school curriculum.
4/5 - One or two concepts; multistep solution; knowledge of some middle school topics is necessary.
6/7 - Multiple and/or advanced concepts; multistep solution; knowledge of advanced middle school topics and/or problem-solving strategies is necessary.

Warm-Up 1

Answer	Difficulty	Answer	Difficulty
1. 5	(1)	6. 12	(3)
2. 8	(2)	7. 9/20	(4)
3. 8	(2)	8. 78	(4)
4. 6:38	(2)	9. 0.05	(3)
5. 1.75	(2)	10. August	(1)

Warm-Up 2

Answer	Difficulty	Answer	Difficulty
11. 11	(2)	16. 666,666	(2)
12. 24	(2)	17. 4/3	(4)
13. 3	(3)	18. May	(3)
14. March	(2)	19. 36	(3)
15. 2	(2)	20. 66	(4)

Workout 1

Answer	Difficulty	Answer	Difficulty
21. 1.5	(1)	26. 270	(2)
22. 2.72	(2)	27. √119	(4)
23. 4√3	(5)	28. 5.57	(4)
24. 77.40	(3)	29. 1/4	(3)
25. 10	(3)	30. 16,865	(3)

Warm-Up 3

Answer	Difficulty	Answer	Difficulty
31. 14/3	(2)	36. 3/4	(2)
32. 3/4	(2)	37. 6	(4)
33. 97	(3)	38. 1/6	(3)
34. 64	(4)	39. 7	(3)
35. 11/9	(4)	40. 32	(4)

Warm-Up 4

Answer	Difficulty	Answer	Difficulty
41. −8	(3)	46. 1*	(3)
42. 36	(3)	47. 15	(3)
43. 65,000	(2)	48. 9	(4)
44. 1	(2)	49. 1/27	(4)
45. 235	(4)	50. 18*x*	(2)

Workout 2

Answer	Difficulty	Answer	Difficulty
51. −7	(4)	56. 41	(5)
52. 4	(3)	57. 24	(2)
53. 0	(3)	58. 84	(4)
54. 10	(3)	59. 144	(4)
55. 1.6	(5)	60. 16 + 16√3 or 16√3 + 16	(4)

** The plural form of the units is always provided in the answer blank, even if the answer appears to require the singular form of the units.*

Warm-Up 5

Answer	Difficulty		
61. 45	(3)	66. 9	(4)
62. 8	(3)	67. 62	(4)
63. 2:00	(2)	68. 13	(3)
64. 10	(3)	69. 18	(5)
65. 2/3	(3)	70. 1/10	(5)

Warm-Up 6

Answer	Difficulty		
71. 2	(4)	76. −1	(4)
72. (−6, −12)	(4)	77. 30	(4)
73. 120	(3)	78. 4800 or 4800.00	(3)
74. 9	(3)	79. 27	(3)
75. 1/4	(4)	80. 134	(2)

Workout 3

Answer	Difficulty		
81. 281.32	(4)	86. 40,431	(3)
82. 2.25	(4)	87. (165π)/2	(3)
83. 1320	(4)	88. 22	(4)
84. 0.82	(4)	89. 3/4	(3)
85. 297.44	(3)	90. 67.88	(4)

Warm-Up 7

Answer	Difficulty		
91. 6	(2)	96. 6	(3)
92. 8/9	(4)	97. $9\frac{11}{12}$	(3)
93. 1/24	(4)	98. 1/6	(4)
94. 7/4	(3)	99. 800	(4)
95. 7	(3)	100. 8000	(4)

Warm-Up 8

Answer	Difficulty		
101. 17	(2)	106. 8	(3)
102. 36	(4)	107. 3*x*/10	(4)
103. 5/4	(4)	108. 110	(5)
104. $2\frac{2}{3}$	(4)	109. 122	(4)
105. 7	(2)	110. 6	(4)

Workout 4

Answer	Difficulty		
111. 203.45	(3)	116. 63	(4)
112. 3	(3)	117. 329	(3)
113. 299	(6)	118. 11.56	(3)
114. 3.02	(6)	119. 400	(4)
115. 123.50	(3)	120. 75.4	(4)

Warm-Up 9

Answer	Difficulty		
121. 1925	(3)	126. 11	(3)
122. 300	(2)	127. 20	(4)
123. 6	(5)	128. (4, 3)	(5)
124. 10 or 10.00	(4)	129. 5	(4)
125. 77	(5)	130. 2	(4)

Warm-Up 10

Answer	Difficulty		
131. 5/9	(5)	136. 2.8×10^{-8}	(3)
132. 5	(3)	137. 130	(4)
133. 80	(4)	138. 10	(3)
134. 8	(3)	139. 8	(5)
135. 3	(3)	140. 12	(5)

Workout 5

Answer	Difficulty		
141. 12	(4)	146. 20	(3)
142. 48	(4)	147. 38.40	(3)
143. 32	(3)	148. 58.8	(3)
144. 88.5	(4)	149. 2	(4)
145. 11.25	(3)	150. 15	(4)

Warm-Up 11

Answer	Difficulty		
151. 18	(2)	156. 54	(3)
152. 2	(3)	157. 1/9	(4)
153. 4/19	(3)	158. 48 + 24√2 or 24√2 + 48	(4)
154. 75	(3)	159. 30	(4)
155. 75	(4)	160. 42	(5)

Warm-Up 12

Answer	Difficulty		
161. 2000	(3)	166. 36π	(5)
162. 3	(4)	167. Elias	(5)
163. 6	(3)	168. 0	(4)
164. 4845	(4)	169. 7	(6)
165. 8	(3)	170. 20	(5)

Workout 6

Answer	Difficulty		
171. 1.6	(2)	176. 0.22	(5)
172. 237.5	(4)	177. 11.95	(4)
173. 21.3	(4)	178. 31.75	(3)
174. 21.6	(5)	179. 19	(3)
175. 7/5	(5)	180. 0.28	(5)

Warm-Up 13

Answer	Difficulty		
181. 96	(3)	186. 300 or 300.00	(4)
182. 1/8	(2)	187. 4.5	(5)
183. 9	(4)	188. −1	(2)
184. 349	(3)	189. 728	(4)
185. 256	(4)	190. (9√3)/2	(4)

Warm-Up 14

Answer	Difficulty		
191. 3	(4)	196. 17	(4)
192. 72	(4)	197. 10	(4)
193. 12	(3)	198. 68	(3)
194. 10	(4)	199. 7	(3)
195. −3	(4)	200. 1/11	(3)

Workout 7

Answer	Difficulty		
201. 120 or 120.00	(3)	206. 12	(4)
202. −23	(4)	207. 29	(3)
203. 287.5	(3)	208. 10	(3)
204. 46,080	(2)	209. 18	(5)
205. 63/100	(5)	210. 968	(5)

Warm-Up 15

Answer	Difficulty		
211. 20	(3)	216. 8	(5)
212. 1200 or 1200.00	(2)	217. −2/3	(5)
213. 43,200	(4)	218. 7	(4)
214. 20	(4)	219. 31	(4)
215. 63	(4)	220. 52	(6)

Warm-Up 16

Answer	Difficulty		
221. 8	(2)	226. 140	(3)
222. 1/3	(5)	227. 1/2	(5)
223. 8	(3)	228. 3/1540	(5)
224. 1/40	(5)	229. 75	(4)
225. 30	(4)	230. 126	(6)

Workout 8

Answer	Difficulty		
231. 124.5	(5)	236. 18.5	(4)
232. 124,950	(4)	237. 0.3	(3)
233. 148	(5)	238. 32	(4)
234. 12	(4)	239. 0.72	(7)
235. 45	(4)	240. 49.6	(6)

Warm-Up 17

Answer	Difficulty		
241. 216	(2)	246. −2	(5)
242. 4 + 2√2 or 2√2 + 4	(5)	247. 12	(3)
243. 99	(4)	248. 60	(5)
244. 0.2	(5)	249. 10	(4)
245. 1 + √2 or √2 + 1	(5)	250. 225	(4)

Warm-Up 18

Answer	Difficulty		
251. 1/15	(4)	256. 6:48	(5)
252. 11	(4)	257. 8	(6)
253. 2/3	(4)	258. 1/10	(5)
254. 125π	(4)	259. 14	(4)
255. 4/25	(4)	260. 18	(4)

Workout 9

Answer	Difficulty		
261. 193	(4)	266. 0.082	(5)
262. 3	(4)	267. 7	(3)
263. 6.4	(2)	268. 25	(5)
264. 17.42	(4)	269. 405	(7)
265. 24.6	(4)	270. 2506	(6)

Surface Area & Volume Stretch

Answer	Difficulty		
271. SA = 3200 V = 10,240	(3)	276. 4	(5)
272. SA = 220 V = 200	(3)	277. 150π	(5)
273. SA = 140π V = 225π	(3)	278a. 252 b. 68 + 60√10 or 60√10 + 68	(5)
274. SA = 216π V = 324π	(3)	279a. 2268π b. 810π	(5)
275. SA = 144π V = 288π	(3)	280. 171π	(5)

Data & Statistics Stretch

Answer	Difficulty		
281. 3	(3)	286. 30	(3)
282. 0.6	(4)	287. 28	(4)
283. 42.5	(4)	288. 17	(5)
284. C	(3)	289. 31	(6)
285. 135	(3)	290. 25	(5)

Geometric Proportions Stretch

Answer	Difficulty		
291. 8	(4)	296. 38	(4)
292. 1/32	(4)	297. 4.19	(3)
293. 1/3	(4)	298. 1/25	(4)
294. 1.04	(3)	299. 1/124	(5)
295. 27/8	(4)	300. 24,786	(6)

SOLUTIONS

The solutions provided here are only *possible* solutions. It is very likely that you or your students will come up with additional—and perhaps more elegant—solutions. Happy solving!

Warm-Up 1

1. The hypotenuse of a right triangle is the side that is opposite the right angle, and it is always the longest side. In the figure, the hypotenuse starts at the 2-cm mark and ends at the 7-cm mark, so it is 7 – 2 = **5** cm long.

2. A ratio is a comparison by division. If we compare the known length of 18 cm to the unknown width of x cm, the ratio needs to equal 9/4, as given. In other words, $18/x = 9/4$. Since 18 is twice 9, x must be twice 4, which means the width is **8** cm.

3. With 2 pounds of rice, Mike can completely fill 2 ÷ (1/3) = 2 × 3 = 6 bins. Since (3/4) ÷ (1/3) = (3/4) × 3 = 9/4 = 2 ¼, Mike can completely fill another 2 bins with rice. Therefore, Mike can completely fill 6 + 2 = **8** bins.

4. If Jessie left for the movie at 4:05 p.m., he arrived at the theater at 4:05 + 0:15 = 4:20 p.m., and the movie started at 4:20 + 0:10 = 4:30 p.m. The movie ended at 4:30 + 1:43 = 5:73 = 6:13, and he got home at 6:13 + 0:25 = **6:38** p.m. Alternatively, we can add the four times to find that he was gone for a total of 0:15 + 0:10 + 1:43 + 0:25 = 1:93 = 2:33, which is 2 hours 33 minutes after 4:05, or **6:38** p.m.

5. Without the savings of $2.50, the 10 carnival rides would cost 15 + 2.50 = $17.50. Dividing $17.50 by 10 rides, we get a price per ride of $**1.75**.

6. Adding the two equations, we get $(x + y) + (x - y) = 7 + 1 \rightarrow 2x = 8 \rightarrow x = 4$. Substituting this into the first equation, we see that $4 + y = 7 \rightarrow y = 3$. Therefore, $xy = 3 \times 4 =$ **12**. We could also approach the solution a different way. The algebra tells us that the sum of two numbers is 7 and their difference is 1. Suppose the bigger number is decreased by 1. Then we would have two equal numbers that add up to 6, which would be 3 and 3. The two numbers must be $x = 4$ and $y = 3$, and the product $xy = 3 \times 4 =$ **12**.

7. We should use the least common multiple of 5 and 3 to make equivalent ratios with the same number of peppermints. We rewrite the ratio of peppermints to chocolates (5:3) as 15/9 and the ratio of peppermints to gummies (3:4) as 15/20. This means that for every 15 peppermints there are 9 chocolates and 20 gummies. The desired ratio is **9/20**.

8. In an arithmetic progression, the difference between consecutive terms is constant. If we let d represent the difference between successively larger angles, then the three angles would be 42 degrees, $(42 + d)$ degrees and $(42 + d + d)$ degrees. The sum of the angles in a triangle is 180 degrees, so we know that $3 \times 42 + 3d = 180$. This means $3d = 180 - 3 \times 42 = 180 - 126 = 54$, and $d = 54 \div 3 = 18$. The three angles are 42 degrees, 42 + 18 = 60 degrees and 60 + 18 = 78 degrees, the largest being **78** degrees.

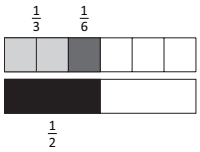
9. The total probability must be 1, so the probability that a book on historical fiction is selected at random is 1 – 0.55 – 0.4 = **0.05**.

10. The number of daylight hours in April appears to be a little more than 14 hours 24 minutes. If we look to the right along the same horizontal line, we see that the dot representing the daylight hours in **August** is about the same.

Warm-Up 2

11. The numbers that are in set B or set C, or both, are {2, 10, 11, 14, 18}. Beginning with the largest number, we see that neither 18 nor 14 is an element in set A. Therefore, the greatest number in either of sets B or C is **11**.

12. The temperature decreased 3 °F during each of the last 8 hours for a total drop of 3 × 8 = 24 °F. It is 0 °F now, so it must have been **24** °F before the drop.



13. We are looking for two unit fractions that add up to 1/2. One of the fractions has twice the denominator of the other, so it is half the size. If we divide 1/2 into three equal parts, we will have what we need. These parts are sixths, so $1/x = 1/3 = 2/6$ and $1/(2x) = 1/6$. The value of x is **3**.

Algebraically, we can write the equation $1/x + 1/(2x) = 1/2$. Rewriting the left-hand side with the common denominator $2x$ and simplifying, we have $2/(2x) + 1/(2x) = 1/2 \rightarrow 3/(2x) = 1/2$. Crossmultiplying and solving for x, we get $2x = 6 \rightarrow x =$ **3**.

14. Forty-five months is equivalent to 3 years and 45 – 36 = 9 months, or 3 months less than 4 years. We can count 9 months forward from June or 3 months backward from June to arrive at **March**.

15. Each of the three rooms has a floor with area 20 × 15 = 300 ft^2. That's a total area to be waxed of 3 × 300 = 900 ft^2. Each bucket contains enough floor wax to cover an area of 600 ft^2. Therefore, to wax the floors in the three rooms, **2** buckets of floor wax must be purchased, and there will be enough floor wax left over to wax another room of the same size.

16. In each example given, the single digit in the center of the palindrome on the right side is both the digit that is repeated and the number of times it appears in the numeral. The middle digit in the palindrome 12,345,654,321 is 6. So, n = **666,666**. Note that this pattern will fail after nine 9s, because there will be some numbers that "carry" into the next place.

17. The increase of 20% on imports means that imports were 120%, or 12/10, of what they had been at the beginning of the year. The decrease of 10% on exports means that exports were 90%, or 9/10, of what they had been. The ratio of imports to exports was (12/10)/(9/10) = (12/10) × (10/9) = 12/9 = **4/3** times what it had been at the beginning of the year.

18. The table shows the amount James saves each month and the sum of his monthly savings. As shown, James will have enough to buy the cell phone once he has saved $155, which will occur in **May**.

	Jan	Feb	Mar	Apr	May
Monthly Savings	$5	$10	$20	$40	$80
Total Savings	$5	$15	$35	$75	$155

19. Triangles ABC and ADE are similar (Angle-Angle Theorem). Using the Pythagorean Theorem, we can determine the length of segment BC, represented by x. We have $x^2 + 4^2 = 5^2 \rightarrow x^2 + 16 = 25 \rightarrow x^2 = 9 \rightarrow x = 3$ cm. You may have recognized, without using the Pythagorean Theorem, that the lengths of the sides of $\triangle$ABC are the Pythagorean Triple 3-4-5. Because corresponding sides of similar triangles are in proportion, we know that the side lengths of $\triangle$ADE are a multiple of 3-4-5. Since DE/BC = 9/3, it follows that DE = 3 × BC. That means AD = 3 × AB = 3 × 4 = 12 cm, and AE = 3 × AC = 3 × 5 = 15 cm. The perimeter of $\triangle$ADE is AD + DE + AE = 12 + 9 + 15 = **36** cm.

20. Based on the table data, we see that a total of 100 people (50 males and 50 females) were surveyed. Six of those people were females who preferred the Flume. Think of the 1100 people at the fair as 11 groups of 100 people. We would expect that 6 females out of each of the 11 groups of 100 people prefer the Flume. That would be 6 × 11 = **66** females.

Workout 1

21. If we divide the number of hours by the number of lawns, we get the average number of hours it takes to mow each lawn. In this case, we get 9 ÷ 6 = **1.5** hours per lawn.

22. To find this value, you should find the π (pi) key on your calculator and learn how to raise a number to a power. Depending on the calculator, you might have a key that shows x^y or y^x, or you might have the symbol ^. If you have never seen fractional exponents before, now is a good time to learn that a base raised to the m/n is equivalent to the nth root of the mth power of the base. In this problem, we have the 1/6 power, which is the sixth root. If you enter this correctly on your calculator, you should get **2.72**, to the nearest hundredth.

23. First we will use the Pythagorean Theorem to find the length of a surface diagonal: $4^2 + 4^2 = c^2 \rightarrow c = \sqrt{32}$. Now, with this surface diagonal and an edge of the cube, we have another right triangle, so we use the Pythagorean Theorem again: $4^2 + (\sqrt{32})^2 = d^2 \rightarrow d^2 = 16 + 32 \rightarrow d = \sqrt{48}$. Now we write this value in simplest radical form as follows: $\sqrt{48} = \sqrt{(16 \times 3)} = \sqrt{16} \times \sqrt{3}$ = **4√3** cm. Notice that the length of this space diagonal is √3 times the edge length of the cube. This is true in general.

24. When an item is sold at 20% off, the customer pays 100 – 20 = 80% of the original cost. To calculate with percents, we have to convert them to either a fraction or a decimal. The discounted price is $90 × 0.80 = $72. Then we add a tax of 7.5% to the discounted price. One nice shortcut is to calculate 107.5% directly so that we won't have to add the tax amount to the price. Multiplying $72 by 1.075, we see that, after tax, Carol paid $**77.40** for the pair of jeans.

25. Since 1 + 1 = 2, we need to determine how to write 2 in base 2, which uses only two digits, 0 and 1. Every number is represented by a string of one or more of these digits. As in base 10, the right-most digit is the units digit, which tells us how many units there are. The next place, to the immediate left of the units place, is the twos place, which tells us how many twos there are. So to write 2 in base 2, we need a 1 in the twos place to indicate that there is one of the value of that place, and we need a 0 in the units place to indicate that there are no units. Therefore, 2 in base 10 is equivalent to **10** in base 2, also written 2_{10} = $\mathbf{10}_2$.

26. Every U.S. dollar was equal to 1.08 euros. At this rate, 250 U.S. dollars could be exchanged for 250 × 1.08 = **270** euros.

27. The triple 5-12-13 is a well-known "Pythagorean Triple," which is a set of three positive integers that satisfy the Pythagorean Theorem. In other words, since $5^2 + 12^2 = 13^2$, we know that three segments of lengths 5, 12 and 13 units can be used to form a right triangle. But we are told that 13 is *not* the length of the hypotenuse. Since the hypotenuse is the longest side of a right triangle, it follows that 12 units is the length of the hypotenuse. If we call the missing side length a, then we have $a^2 = 12^2 - 5^2 \rightarrow a^2 = 144 - 25 \rightarrow a^2 = 119 \rightarrow a = \sqrt{119}$. Since 119 = 7 × 17 has no perfect square factors, the length of the third side, in simplest radical form, is **√119** units.

28. The area of the square part of the window is 2 × 2 = 4 ft^2. The area of the top part of the window is one-half of the area of a circle with a radius of 1 foot, or $1/2 \times \pi \times 1^2 = \pi/2 \approx 1.57$. Thus, the entire window has a total area of **5.57** ft^2.

29. A coin can land with either of 2 sides up, and a die can land with any one of 6 sides up, so there are 2 x 6 = 12 possible outcomes. Of these 12 outcomes, the 3 ways to get a favorable outcome are (heads up, 2), (heads up, 3) and (heads up, 5). The probability of getting one of these outcomes is 3/12, or **1/4**. *Recall that the number 1 is neither prime nor composite; it is the multiplicative identity.*

30. Bailey's estimate was 3 × 3 × 3 = 27 ft^3. Since each 3-foot side is equal to 36 inches, the estimate is equivalent to 36 × 36 × 36 = 46,656 in^3. Since 2 feet 7 inches is equivalent to 24 + 7 = 31 inches, the actual volume is 31 × 31 × 31 =29,791 in^3. The absolute difference between the estimate and the actual volume is 46,656 − 29,791 = **16,865** in^3.

Warm-Up 3

31. If $a/b = 7/3$, $a = (7/3)b$ and $2a = (14/3)b$. So, the ratio $2a/b$ = **14/3**. Or simply, the ratio of $2a$ to b is twice the ratio of a to b, or 2(7/3) = **14/3**.

32. The dart closest to the center is equally likely to be any of the four darts. Since Remy threw three darts, the probability that a dart thrown by Remy is closest to the center is **3/4** (and the probability that Rita's dart is closest is 1/4).

33. Since the average of Grace's first three test scores was 89, the sum of the three scores was 89 × 3 = 267. With an average of 91 after the fourth test, the sum of all four test scores was 91 × 4 = 364. The difference between these two sums is Grace's fourth test score. So, Grace scored 364 − 267 = **97** on her fourth test.

34. There are 6 faces on a cube, so the area of each face must be 96 ÷ 6 = 16 cm^2. Since each face is a square with side length x, we have $x^2 = 16$, and the cube has edges of length x = 4 cm. The volume of the cube must be 4 × 4 × 4 = **64** cm^3.

35. If 45% percent of the students were born at South Park Hospital, then the remaining 55% were not born there. Notice that the question asks for the ratio of the number of students *not* born there to the number of students born there, which is 55/45 = **11/9**.

36. If we rewrite 4 as 4/1, it's easier to see that if $c/d = 4/1$, the reciprocal is $d/c = 1/4$. Now we add 1/4 to 1/2 and get 1/4 + 2/4 = **3/4**.

37. We know that 7 animals are neither dogs nor white. We are told that there are 15 white animals, 5 of which are dogs. That leaves 10 animals that are white but not dogs. Since 10 + 7 = 17 of the animals at the store are not dogs, it follows that the pet store has 23 − 17 = **6** dogs.

38. From 2:00 to 2:15 is 15 minutes. There are 90 minutes from 2:00 to 3:30. The probability that Yoon will get the call before 2:15 is 15/90 = **1/6**.

39. The question asks for a number of quarts, so we should convert all values to quarts. There are 1/2 gallon = 2 quarts of apple juice, 3 cups = 3/4 quart of lemon-lime soda, 64 fluid ounces = 2 quarts of pineapple juice, 2 quarts of cold water and 1 cup = 1/4 quart of lemon juice. The total is 2 + 3/4 + 2 + 2 + 1/4 = **7** quarts of punch.

40. Let's say that Jude ate x cookies on the first day. Then he ate $x + 6$ on the second day, $x + 12$ on the third day, $x + 18$ on the fourth day and $x + 24$ on the fifth day. The total for the five days is $5x + 60$ and we know that he ate a total of 100 cookies. That means that $5x$ must be 40 and x must be 8. On the fifth day, Jude ate $x + 24$ cookies, or 8 + 24 = **32** cookies.

Warm-Up 4

41. Any point (a, b) has image $(a, -b)$ when it is reflected across the x-axis. So, when reflected across the x-axis, the point (–3, 5) has the image (–3, –5). The sum of the image's coordinates is –3 + (–5) = **–8**.

42. To calculate @3@, we first must determine the factors of 2 × 3 = 6. The factors of 6 are 1, 2, 3 and 6, and their product is 1 × 2 × 3 × 6 = **36**.

43. If you were to start at the bottom of the Mariana Trench and climb to the height of Mount Everest, you would travel 36,201 feet up to sea level and then an additional 29,000 feet. That's a total of 36,201 + 29,000 = 65,201 feet in elevation, which, rounded to the nearest thousand, is **65,000** feet.

44. To determine the arithmetic mean, or average, we add the numbers and divide by 5. In this case, we have some positive numbers and some negative numbers. The sum of the positive numbers is 7 ½ + 4 + 2 = 13 ½. The sum of the negative numbers is –3 ¼ + (–5 ¼) = –8 ½. The sum of all the numbers is 13 ½ + (–8 ½) = 5. Dividing 5 by 5, we get a mean of **1**.

45. Since 45 + 4 = 49, the value of $\sqrt{c}$ must be 4. Squaring 4, we find that the value of c must be 16. Now we have to calculate $c^2 - 21$, which is $16^2 - 21 = 256 - 21$ = **235**.

46. If Janelle spends \$100 every minute, then she will spend 100 × 60 = \$6000 every hour, 6000 × 24 = \$144,000 every day and 144,000 × 7 = \$1,008,000 every week. It will take just **1** week.

47. Filling in the missing addends, we see that we are calculating the sum of 15 integers. If we add pairs of these integers beginning with the 1st and 15th integers, then the 2nd and 14th, and so on, we see that the sum of each pair is 2, with the 8th addend, 1, not being paired. That's 7 sums of 2, plus 1, or 7 × 2 + 1 = **15**.

48. The area of a triangle equals $(1/2)bh$, where b and h represent the length of the triangle's base and its height, respectively. Suppose this right triangle has as its base the leg of length 8 cm. Its height is the length of the other leg, and we can write $(1/2)(8)h = 36 \rightarrow 4h = 36 \rightarrow h = 9$. The length of the other leg, then, is **9** cm.

49. Because there is a 2/3 chance of rain on any given day, there is a 1 – 2/3 = 1/3 chance that it will not rain. The probability that it will not rain on any of those days is 1/3 × 1/3 × 1/3 = **1/27**.

50. Square A, with side length x units, has 3 sides that account for $3x$ of the figure's perimeter. Square B, with side length $2x$ units, has 2 complete sides and half of a side that make up $2x + 2x + x = 5x$ units of the perimeter. Square C, with side length $3x$ units, has 3 complete sides and a third of another side that make up $3x + 3x + 3x + x = 10x$ units of the perimeter. The perimeter of the entire figure is $3x + 5x + 10x =$ **$18x$** units.

Workout 2

51. The slope of the line passing through (4, 3) and (7, –2) is (–2 – 3)/(7 – 4) = –5/3. A line parallel to this line must have the same slope, so we can write the equation $(3 - c)/(-5 - 1) = -5/3$. To solve this, we multiply both sides of the equation by –6 to get $3 - c = 10$. The value of c must be **–7**, since 3 – (–7) = 10.

52. Since 4.88 seconds is equivalent to 4.88/60 minutes, we can write the rate as 29.5 ÷ (4.88/60) ft/min. To climb to the top of the Empire State Building at this rate would take 1454 ÷ (29.5 ÷ (4.88/60)) ≈ 4.0088 ≈ **4** minutes.

53. The four-pack of 16-oz cans of soup contains a total of 4 × 16 = 64 oz, with a unit price of 3.20 ÷ 64 = $0.05 per ounce. The three-pack of 24-oz cans of soup contains a total of 3 × 24 = 72 oz, with a unit price of 3.60 ÷ 72 = $0.05 per ounce. The absolute difference in unit prices is **0** cents.

54. Knowing the powers of 2 can be very useful. The first ten powers of 2 are 2, 4, 8, 16, 32, 64, 128, 256, 512 and 1024. Because 1024, which is 2^{10}, is very close to 1000, it follows that x is very close to **10**. (In fact the value of x is about 9.966.)

55. Eighteen seconds is 18/60 = 3/10 minute. That means it takes 3 complete revolutions for the wheel to travel 15 feet, and in 1 revolution it travels 15/3 = 5 feet. Since the distance traveled in a revolution is equivalent to the wheel's circumference, we can solve $\pi d = 5$ to determine that the wheel's diameter is $d = 5/\pi \approx$ **1.6** feet.

56. Although we can't know the individual ages of the 12 people, we know that the sum of their ages is 12 × 26 = 312 years. When the 8 people are added, the sum of the ages is 20 × 32 = 640 years. The difference of 640 – 312 = 328 must be the sum of the ages of the 8 new people. It follows that the average age for the 8 new people is 328 ÷ 8 = **41** years.

57. If 9 is 3 less than half of Connie's number, then, working backward, she must have said (9 + 3) × 2 = 12 × 2 = **24**.

58. If we subtract the combined weight of Arnold and Benji from the weight of all three people, we will get the weight of Celal, which is 250 – 158 = 92 pounds. Now we can subtract Celal's weight from the combined weight of Benji and Celal to get Benji's weight, which is 176 – 92 = **84** pounds.

59. If we look at the possible values for each of the three digits, we see that there are 4 possible values for the hundreds digit (1, 2, 3, 4). There are 9 possible values for the tens digit (0, 1, 2, 3, 4, 6, 7, 8, 9), and there are 4 possible values for the units digit (1, 3, 7, 9). That's a total of 4 × 9 × 4 = **144** odd numbers that do not contain a 5.

60. Using the formula for the area of an equilateral triangle, we see that each triangle has an area of $(s^2\sqrt{3})/4 = (4^2\sqrt{3})/4 = (16\sqrt{3})/4 = 4\sqrt{3}$ in^2. The total area of all four triangles is $4 \times 4\sqrt{3} = 16\sqrt{3}$ in^2. The area of a square with side length 4 inches is 4 × 4 = 16 in^2. Thus, the area of the entire figure is **$16 + 16\sqrt{3}$** in^2.

Warm-Up 5

61. The current sum is 5 + 10 + 15 + 20 + 25 = 75, so the mean (average) of the 5 numbers is 75/5 = 15. If we want the mean to become 20 when we add another number, we will need a sum of 6 x 20 = 120. The new number has to be 120 – 75 = **45**.

62. If we multiply both sides of the equation $y = c/x$ by x, we get $xy = c$. In other words, c is the constant product of x and y. Looking at the table, we can see that the product for each pair (x, y) is (−1/2) × (–16) = (–1) × (–8) = (–2) × (–4) = (–4) × (–2) = **8**.

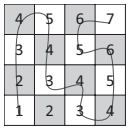
63. It will take Sinclair 90 ÷ 45 = 2 hours to drive 90 miles at an average speed of 45 mi/h. If she needs to arrive in New York at 4:00 p.m., she should leave at 4:00 – 2:00 = **2:00** p.m.

64. When we substitute the given values into the formula and calculate, we get $r = s\sqrt{(m/a)} = 30 \times \sqrt{(12/108)} = 30 \times \sqrt{(1/9)} = 30 \times (1/3) = 10$. Therefore, the model ship should travel at **10** knots.

65. Sixty percent of 50 is 30, and 30 is 30/45 = **2/3** of 45.

66. To get the greatest possible value of y/x, we would like to have the greatest possible numerator, which is 17 + 18 + 19 = 54, and the least possible denominator, which is 1 + 2 + 3 = 6. So we have 54/6 = **9**.

67. After trying a few routes through the grid, you'll see that is is not possible to move from the 1 to the 7 by passing through every cell. Notice that each move is from an unshaded cell to a shaded cell, or vice versa. To pass through all 16 cells, you would need to end on a shaded cell if you began on an unshaded cell. However, the cells with the 1 and the 7 are both unshaded, so it is impossible to move from the 1 to the 7 by passing through all 16 cells. But there are many ways to pass through all but one of the squares. Since we want to maximize the sum and we have to start with the 1, we should choose to leave out one of the 2s. One possible route is shown at right. The sum of the numbers we pass through is **62**.



68. Three minutes is 3 × 60 = 180 seconds. In 40 seconds, 3 pages are printed. In 180 seconds, 3 pages will be printed 180 ÷ 40 = 4.5 times, for a total of 13.5 pages. That's **13** complete pages.

69. The sum of an interior angle and an exterior angle has to be 180 degrees since they form a straight angle. If we take the 8 parts on the interior and the 1 part on the exterior, then the 9 parts must each be 180 ÷ 9 = 20 degrees. Since the polygon is regular, all of the exterior angles must be 20 and they have to total 360 degrees. There must be 360 ÷ 20 = **18** sides on this polygon.

70. Four-digit palindromes must have the form ABBA. Since A cannot be zero and B can be any digit, there are 9 × 10 = 90 four-digit palindromes. Five-digit palindromes must have the form ABCBA. There are the same 90 possibilities for A and B, and C can be any of the 10 digits. This means there are 90 × 10 = 900 five-digit palindromes. The desired ratio is 90/900 = **1/10**.

Warm-Up 6

71. Since 1 raised to any power is 1, we know that $x = 1$ satisfies $x^{2x} = 1$. Also, since −1 raised to any even power is 1, we know that this equation is also satisfied when $x = -1$. These are the only **2** values for x that satisfy the equation.

72. Since we want to have $y = 2x$, we can substitute to get the following equation: $2x = 3x + 6$. Solving, we get $x = -6$. We expect y to be 2 × (−6) = −12. Substituting, we confirm that when $x = -6$, $y = 3(-6) + 6 = -18 + 6 = -12$. The ordered pair is **(−6, −12)**.

73. Of the 24 hours in a day, 8 hours represents 8/24 = 1/3 of the hours. So the section of the pie chart labeled *Sleeping* would take up 1/3 of the circle. Since a circle has 360 degrees, it follows that the central angle for this section measures 360 × (1/3) = **120** degrees.

74. Because the average of a, b and c is 15, the sum of the three numbers is 3 × 15 = 45. Similarly, the sum of a and b must be 2 × 18 = 36. The value of c must be 45 – 36 = **9**.

75. There are 4! = 4 × 3 × 2 × 1 = 24 ways to place the four letters in the four envelopes. If we are going to have exactly two of the letters in wrong envelopes, then let's imagine that the letters are all in the correct envelopes and that we are choosing two of them to swap. To calculate "4 choose 2," we imagine that we could select any of the 4 letters first and then any of the 3 remaining letters second. We divide these 4 × 3 = 12 choices by 2, because every pair of letters is counted twice. This means there are 6 out of 24 ways to get two letters in the right envelopes and two letters in wrong envelopes. The probability is 6/24 = **1/4**.

76. Let's examine the slope, or rise/run, of the line containing the points A(−2, 5) and C(5, −16). From A to C, the rise is −16 − 5 = −21 and the run is 5 − (−2) = 7, giving us a slope of −21/7 = −3. Since the point B(0, y) is also on this line, the rise/run from A to B, $(y - 5)/(0 - (-2)) = (y - 5)/2$, also must be −3. We have $(y - 5)/2 = -3 \rightarrow y - 5 = -6 \rightarrow y =$ **−1**.

77. Notice that the left-hand side of the equation has the form $(a - b)(a + b)$. Recall this is equivalent to $a^2 - b^2$. So we can rewrite the equation as $(2x)^2 - 5^2 = 5 \rightarrow 4x^2 - 25 = 5 \rightarrow 4x^2 =$ **30**.

78. We know that after two months Arturo will have less money than his initial investment, because the loss of 20% is a greater dollar amount than the gain of 20%. At the end of the first month, Arturo has 100% of his initial \$5000, plus 20% of the \$5000, or 1.2 times as much, which is 1.2 × 5000 = \$6000. At the end of the second month, Arturo has 20% less than this new amount of \$6000, or 80% of \$6000, which is 0.8 × 6000 = \$**4800**.

79. An interesting feature of the decimal expansion of 1/7, 2/7, 3/7, 4/7, 5/7 and 6/7 is that they all have the same six-digit repeating pattern; the pattern just starts in different places. For the decimal expansion of 4/7, which is $0.\overline{571428}$, the six digits that repeat are 5, 7, 1, 4, 2 and 8. Since 36 is a multiple of 6, we know that the 36th digit to the right of the decimal point is 8. It follows, then, that the 31st through 36th digits to the right of the decimal point consist of all six repeating digits, and the sum of the digits is 5 + 7 + 1 + 4 + 2 + 8 = **27**. In fact, since the decimal repeats every six consecutive digits, this will be the sum of any six consecutive digits to the right of the decimal point.

80. Let's convert the base-5 number 332_5 to base 10 and then convert the base-10 number to base 8. In base 5, from right to left the place values are units, fives, twenty-fives, and so on, and in base 8, they are units, eights, sixty-fours, and so on. So, $332_5 = (\underline{3} \times 25) + (\underline{3} \times 5) + (\underline{2} \times 1) = 92$ in base 10. Starting with the greatest power of 8, less than 92, we see that $92 = (\underline{1} \times 64) + (\underline{3} \times 8) + (\underline{4} \times 1)$. So in base 8, 332_5 is written as **134**.

Workout 3

81. The pilot must have taken about 720/295 = 144/59 hours to get to the city. Since the pilot logged 5 hours total, the return trip must have taken approximately 5 – (144/59) = (295/59) − 144/59 = 151/59 hours. His average speed for the return trip must have been about 720/(151/59) = 720 × 59/151 = 42,480/151 ≈ **281.32** mi/h.

82. Because $c = 2b$ and $b = (1/3)a$, $c = 2(1/3)a$ and $a/c = 3/2$. Therefore, $a^2/c^2 = 9/4 =$ **2.25**.

83. We don't actually need to know the length and width of the original rectangular basketball court to find the area of the new court. Since one dimension increased by 40% and the other increased by 50%, the area of the new court is 1.4 × 1.5 = 2.1 times as large as the original court. So the area of the new basketball court is 2.1 × 1200 = 2520 ft^2. That's an increase of 2520 – 1200 = **1320** ft^2.

84. Because 28 of the 300 eighth-graders are in Mr. Jackson's Algebra 1 class, 300 – 28 = 272 students are not in that class. The probability that neither of the students selected will be from that class is (272/300) × (271/299) ≈ **0.82** to the nearest hundredth.

85. Matthew earns 40 x 8.80 = \$352 for the first 40 hours and 8 × 1.5 x 8.80 = \$105.60 for the 8 hours of overtime. Thus his gross pay is \$352 + \$105.60 = \$457.60. Since 35% of this amount gets deducted from his pay, Matthew gets to keep the other 65%, which is 0.65 × \$457.60 = \$**297.44**. The overtime pay of 1.5 times as much money is sometimes referred to as "time-and-a-half" pay. In other words, Matthew gets paid as if he worked 40 + 1.5 × 8 = 52 hours at his normal hourly rate. Multiplying 52 by \$8.80 per hour, we get the same \$457.60 in gross pay.

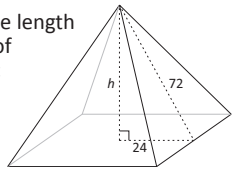
86. There are 365 days × 24 hours/day × 60 minutes/hour = 525,600 minutes in a non-leap year such as 2014. If a new book is published every 13 minutes, then we can expect about 525,600 ÷ 13 ≈ **40,431** books to be published in 2014.

87. The distance around a circle is a little more than three times the distance across the circle. More specifically, the circumference is π times the diameter. Since Stephen was 15 feet from the center of the merry-go-round, the diameter of the circular path he traveled was 2 × 15 = 30 feet. Multiplying this diameter by π and by the 2 ¾ revolutions, we see that Stephen traveled **(165π)/2** feet.

88. The sum of the measures of an angle and its supplement is 180 degrees. In other words, if an angle has degree measure x, then the supplement of x has degree measure $180 - x$. We are told that the difference in measure between an angle and its supplement is 136 degrees. From the information given, we can determine that $(180 - x) - (x) = 136 \rightarrow 180 - 2x = 136 \rightarrow 2x = 44 \rightarrow x =$ **22** degrees.

89. Consider when neither hand is in the upper half of an analog clock. This occurs between 15 minutes and 45 minutes after the hour, every hour from 3:00 p.m. to 9:00 p.m. and from 3:00 a.m. to 9:00 a.m. That's a total of 12 × 30 = 360 minutes out of the 24 × 60 = 1440 minutes in a day, which is 360/1440 = 1/4 of the day. That means during the other **3/4** of the day, at least one hand is in the upper half of the clock.

90. To find the height, h, of the pyramid, we visualize a right triangle with its right angle at the center of the square base. The length of one leg of this triangle is half the side length of the square, or 24 m, and the length of the hypotenuse is the slant height of 72 m. The height of the pyramid is the length of the other leg, which we can find using the Pythagorean Theorem as follows: $24^2 + h^2 = 72^2 \rightarrow 576 + h^2 = 5184 \rightarrow h^2 = 5184 - 576 \rightarrow h^2 = 4608 \rightarrow h = \sqrt{4608}$. Using the square root button on a calculator, we get a height of about **67.88** m.



Warm-Up 7

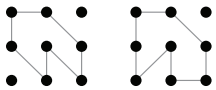
91. It turns out that the least two primes work. When $p = 2$ and $q = 3$, $p + q = 2 + 3 = 5$. The value of pq is 2 × 3 = **6**.

92. The area of the larger circle is $\pi \times (3x)^2 = 9x^2\pi$, and the area of the smaller circle is $x^2\pi$. The absolute difference of their areas is $|9x^2\pi - x^2\pi| = 8x^2\pi$. This difference is exactly $(8x^2\pi)/(9x^2\pi) =$ **8/9** of the area of the larger circle.

93. There are 4! = 4 × 3 × 2 × 1 = 24 ways to order the four different digits, but only one of them can be used to unlock her device. Therefore, the probability that the security code Raynelle enters will unlock her device is **1/24**.

94. Since the number of apples, $4x$, and the number of oranges, $7y$, are the same, we can equate the two expressions to get the equation $4x = 7y$. To solve for the ratio x/y, we divide both sides of the equation by 4, and then by y, and we see that $x/y =$ **7/4**.

95. Two examples are given of polygons with **7** sides.



96. $f(-2) = (-2)^2 + 3(-2) - 4 = 4 - 6 - 4 = -6$, and $g(-8) = (3/4)(-8) + 6 = -6 + 6 = 0$, so $g(-8) - f(-2) = 0 - (-6) =$ **6**.

97. Gregory falls asleep 30 + 15 + 8 = 53 minutes after 9:12 p.m., which is 10:05 p.m. From 10:05 p.m. to 8:05 a.m. is 10 hours, but we need to subtract 5 minutes since Gregory woke at 8:00 a.m. That means Gregory slept for 9 hours 55 minutes, which is $\mathbf{9\frac{11}{12}}$ hours.

98. If we divide x/z by x/y, we will get the ratio y/z, which is (1/8) ÷ (3/4) = (1/8) × (4/3) = **1/6**.

99. Let c represent the cost of a computer and p represent the cost of a printer. Since it costs \$10,000 to purchase 10 computers and 10 printers or 12 computers and 2 printers, we can write the following equation: $10c + 10p = 12c + 2p$. Solving for c, we get $c = 4p$, which we can substitute into the equation $10c + 10p = 10{,}000$ to get $10(4p) + 10p = 10{,}000 \rightarrow 40p + 10p = 10{,}000 \rightarrow 50p = 10{,}000$. Solving, we see that $p =$ \$200. So the cost of a computer is 4 × 200 = \$**800**.

100. If we add the fractions that are given, we see that from the beginning of January to the end of September, David earned 1/12 + 1/6 + 1/2 = 1/12 + 2/12 + 6/12 = 9/12 = 3/4 of his total income last year. Therefore, the \$2000 David earned during October, November and December represents 1 − 3/4 = 1/4 of his total income. It follows, then, that his total income for last year was 4 × 2000 = \$**8000**.

Warm-Up 8

101. Since there are 100 cm in a meter, 1.75 m equals 175 cm, and 175 ÷ 10 = 17.5. So Sebi can cut at most **17** segments of length 10 cm from the string, and there will be a 5-cm segment left over.

102. There are 9 different flavors to choose from for the first scoop and then 8 different flavors for the second scoop. There are 9 × 8 = 72 possible ways to make a double-scoop cone. This includes the double-scoop cone with Flavor A as the first scoop and Flavor B as the second scoop, as well as the cone with these scoops reversed. However, we are told that none of the children purchased the same two flavors. That means the total number of two-flavor combinations is 72 ÷ 2 = 36 combinations. Therefore, the group consisted of **36** children.

103. Let's call the unknown number n. Then we have $5n = 5 + n$. Solving for n, we see that $4n = 5 \rightarrow n =$ **5/4**.

104. Let l and w represent the length and width of the rectangle, respectively. Then the perimeter of the rectangle is $2l + 2w = 2 \times 8 + 2w = 16 + 2w$ feet. The area of the rectangle is $lw = 8w$ ft^2. It should be noted that these two quantities are very different: perimeter is a length and area is a measure of surface. However, since the two are numerically equal, we can write the equation $16 + 2w = 8w$. Solving for w, we get $6w = 16 \rightarrow w = 16/6 = 8/3 =$ $\mathbf{2\frac{2}{3}}$ feet.

105. Let a, b, c, d, e, f and g be the numbers of books in lockers A through G, respectively. For lockers A and C, $a \neq 0$ and $c \neq 0$; otherwise we would have $a + b = 5$ or $b + c = 5$, and one of the other lockers contains 5 books. Similarly, for lockers E and G, $e \neq 0$ and $g \neq 0$. Therefore, $d = 0$, which means neither c nor e can equal 5 or 2. That means the combined number of books in lockers A and G must be 2 + 5 = **7** books.

106. When a circle touches a line at just one point, we say the line is *tangent* to the circle. A line tangent to a circle is perpendicular to the radius that is drawn from the center of the circle to the point of tangency. In the coordinate plane, it is clear that the radius perpendicular to the y-axis is the horizontal segment from the center (–8, –3) to the point of tangency at (0, –3). So, the length of this radius is **8** units.

107. There are 30 days in June, so that month Barbara will receive allowance in the amount of $30x$ cents. Since there are 100 cents in a dollar, that's $30x/100 =$ $\mathbf{3x/10}$ dollars.

108. If the perimeter of a rectangle is 46 units, then its semiperimeter (half the perimeter) is 46 ÷ 2 = 23 units. The rectangle with integer side lengths and with a perimeter of 46 units that has the least area is a 1-unit by 22-unit rectangle, with area 1 × 22 = 22 units^2. The rectangle with the same perimeter that has the greatest area is the one with dimensions closest to those of a square. With integer side lengths, the closest we can get to a square is an 11-unit by 12-unit rectangle, with an area of 11 × 12 = 132 units^2. The absolute difference between these two areas is $|22 - 132| =$ **110** units^2.

109. If 32 women had been added to the group, it would have had 212 + 32 = 244 people, half of them men and half of them women. It follows, then, that the actual group had 244 ÷ 2 = **122** men (and 122 – 32 = 90 women).

110. If the first three socks selected are three different colors, the fourth sock selected will create a matching pair with one of them. It is possible that the fifth sock selected is the same color as the fourth sock selected. With only one matching pair, we need to select another sock from the drawer. The sixth sock we select will be the same color as the fifth sock selected, or it will be the same color as one of the other first three socks selected. In either case, there will be another matching pair after the sixth sock is selected. Suppose, on the other hand, that the first three socks selected are all the same color, and the fourth and fifth socks selected are each a different one of the other two colors. Again, the sixth sock selected will be the same color as the first three socks selected, or it will match the fourth or fifth sock selected, resulting in a second matching pair. Therefore, the minimum number of socks that must be selected is **6** socks.

Workout 4

111. In 2001, the total snowfall increased 10% from the previous year. That was an additional 200 × 0.10 = 20 inches, for a total of 200 + 20 = 220 inches. In 2002, the total snowfall decreased 5%, or 220 × 0.05 = 11 inches, resulting in a total snowfall of 220 − 11 = 209 inches. In 2003, the total snowfall decreased 10%, or 209 × 0.10 = 20.9 inches, resulting in a total snowfall of 209 − 20.9 = 188.1 inches. In 2004, the total snowfall increased 4%, or 188.1 × 0.04 = 7.524 inches, resulting in a total snowfall of 188.1 + 7.524 = 195.624 inches. It would take just over 4.5 inches of additional snowfall to have a total snowfall over 200 inches. In 2005, the total snowfall again increased 4%, or 195.624 × 0.04 = 7.82496 inches. Therefore, in 2005, when the total snowfall first exceeded 200 inches, the total snowfall was about **203.45** inches.

Games	1	2	3	4	5
Country Bowl $4n + 2.60$	6.60	10.60	14.60	18.60	22.60
Super Bowl $2.5n + 7.10$	9.60	12.10	14.60	17.10	19.60

112. Creating the table shown, we can compare the cost of bowling n games at Country Bowl and Super Bowl. According to the table, the cost is the same at Country Bowl and Super Bowl for **3** games. An algebraic approach would be to set the expression for the cost of bowling n games at Country Bowl, $4n + 2.60$, equal to the expression for the cost of bowling n games at Super Bowl, $2.5n + 7.10$. Doing so, we have $4n + 2.60 = 2.5n + 7.10 \rightarrow 1.5n = 4.5 \rightarrow n = 3$. After solving, we see that the cost to bowl **3** games is the same at the two places.

113. Let us consider the number of difference dates each month during the 21st century. Since January has 31 days, the greatest difference is 31 − 1 = 30. That's 30 difference dates, one each January from 2001 through 2030. The greatest difference in February is 29 − 2 = 27, but 2027 is not a leap year. So, the greatest difference is 28 − 2 = 26. That's 26 difference dates, one each February from 2001 through 2026. The number of difference dates per month is summarized in this table. So, from 2001 through 2030, there are 287 difference dates. But we also need to count the 12 difference dates in 2100: 1/1, 2/2, 3/3, 4/4, 5/5, 6/6, 7/7, 8/8, 9/9, 10/10, 11/11, 12/12. So, in the 21st century, 287 + 12 = **299** dates can be classified as difference dates.

	MAX DIFFERENCE	DIFFERENCE DAYS
January	31 – 1	30
February	28 – 2	26
March	31 – 3	28
April	30 – 4	26
May	31 – 5	26
June	30 – 6	24
July	31 – 7	24
August	31 – 8	23
September	30 – 9	21
October	31 – 10	21
November	30 – 11	19
December	31 – 12	19
	TOTAL:	**287**

114. Since the set contains six numbers, the median is the average of the two middle numbers, x and $11.5x$. We can write the equation $(x + 11.5x)/2 = 2 \rightarrow x + 11.5x = 4 \rightarrow 12.5x = 4$. Solving, we see that $x = 4/12.5 = 0.32$. It follows that $(2/5)x = (2/5) \times 0.32 = 0.128$, and $11.5x = 11.5 \times 0.32 = 3.68$. Therefore, the mean of the set is $(0 + 0.128 + 0.32 + 3.68 + 5 + 9)/6 = 18.128/6 \approx$ **3.02**.

115. The price \$133.38 is 108% of the cost of the software since it is 100% of the price plus the 8% tax. Dividing 133.38 by 1.08, we find that the price of the software was \$**123.50**.

116. Every square foot of floor space will require $2 \times 2 = 4$ of the 6-inch by 6-inch tiles. The floor has an area of $12 \times 13 = 156$ ft^2, so $156 \times 4 = 624$ tiles will be needed. Since there are 10 tiles in each box and $624 \div 10 = 62.4$ boxes, the minimum number of boxes required is **63** boxes.

117. The 49.35 pounds of bamboo leaves and stems must be 15% of the panda's weight. It follows that the panda weighs $49.35 \div 0.15 =$ **329** pounds.

118. The 502 feet that Cecil Fielder hit his home run is $502/450 \approx 1.1156$, or 111.56%, of the length of yarn in the baseball. The percent by which the home run exceeds the length of the yarn is 111.56% – 100% = **11.56**%.

119. The area of a square with a side length of 4 m is $4 \times 4 = 16$ m^2. Substituting into $P = F/A$, we have $25 = F/16$. Solving, we see that the maximum force that can be applied to 16 m^2 without exceeding 25 newtons per square meter is $F = 25 \times 16 =$ **400** newtons.

120. If you unwrapped the label from the cylindrical can, you would find that the label is a rectangle with a width equal to the height of the can and a length equal to the circumference of the can. The circumference of a circle is πd, so this can has a circumference of 4π inches. Therefore, the label has area $4\pi \times 6 = 24\pi \approx$ **75.4** in^2.

Warm-Up 9

121. If a human were able to jump 350 times his or her height, the average American, at 5 feet 6 inches, would be able to jump $350 \times 5.5 =$ **1925** feet. By way of comparison, that's more than the height of the Empire State Building in New York City.

122. One percent of 3 million people is $3{,}000{,}000 \times 0.01 = 30{,}000$ people. One percent of 30,000 people is $30{,}000 \times 0.01 =$ **300** people.

123. There are two walls that are 12 feet by 8 feet, two walls that are 15 feet by 8 feet and the ceiling is 12 feet by 15 feet. If there were no windows and doors, there would be an area of $2 \times (12 \times 8) + 2 \times (15 \times 8) + (12 \times 15) = 192 + 240 + 180 = 612$ ft^2 to paint. Since 30 inches is 2.5 feet, and 60 inches is 5 feet, we can calculate the area of the 4 windows as $4 \times (2.5 \times 5) = 50$ ft^2. The area of the 2 doors is $2 \times (3 \times 7) = 42$ ft^2. The total area to be painted is $612 - 50 - 42 = 520$ ft^2. If each can of paint covers an area of 100 ft^2, we get $520 \div 100 = 5.2$. So, **6** cans of paint are needed.

124. If Shanita has s dollars, then Brianna has $2s + 3$ dollars. Together, they have $s + 2s + 3 = 3s + 3$ dollars. We are told this amount is \$24, so we have $3s + 3 = 24 \rightarrow 3s = 21$. Solving, we see that Shanita has $s = 7$ dollars, and Brianna has $2 \times 7 + 3 = 17$ dollars. Brianna has $17 - 7 =$ **10** dollars more than Shanita has.

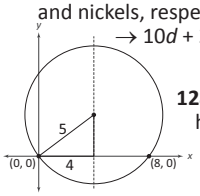
125. Because corresponding angles in $\triangle$ABC and $\triangle$ADE are equal, the two triangles are similar. That means that the ratios of corresponding sides are equal. It also means that the ratio of their areas is the square of the ratio of their perimeters. If we let p represent the perimeter of $\triangle$ABC, we can set up the proportion $p^2/55^2 = 98/50 \rightarrow p^2/55^2 = 49/25$. Taking the square root of both sides yields $p/55 = 7/5 \rightarrow 5p = 385 \rightarrow p =$ **77** m.

126. In order for the fraction to be less than one, the numerator must be less than the denominator. The fractions less than one that can occur when each of the possible numerators is rolled are:

1: 1/2, 1/3, 1/4, 1/5, 1/6
2: 2/3, 2/5 (2/4 = 1/2 and 2/6 = 1/3 included above)
3: 3/4, 3/5 (3/6 = 1/2 included above)
4: 4/5 (4/6 = 2/3 included above)
5: 5/6

That's a total of **11** fractions.

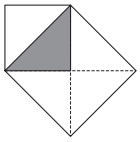
127. If the \$3.55 were entirely nickels, there would be $355 \div 5 = 71$ coins. If we trade two nickels for one dime, we keep the same amount of money but lose one coin. We need to make 20 of these trades to get from 71 coins down to 51 coins, so Rayshon must have **20** dimes (and $71 - 2 \times 20 = 31$ nickels). Algebraically, we can write the equations $10d + 5n = 355$ and $d + n = 51$, where d and n represent the number of dimes and nickels, respectively. Solving the second equation for n yields $n = 51 - d$. Substituting this into the first equation, we have $10d + 5(51 - d) = 355 \rightarrow 10d + 255 - 5d = 355 \rightarrow 5d = 100 \rightarrow d = 20$. Rayshon has **20** dimes.



128. The chord connecting the two given points (0, 0) and (8, 0) is 8 units long. The perpendicular bisector of this chord, shown here as a dashed line, passes through the center of the circle and, with the help of this line, we can form a 3-4-5 right triangle. The center of the circle is at (**4**, **3**).

129. First, we will determine the maximum value of $f(x)$. Let's start by considering $x - 1$. Since the expression will be squared, we know $(x - 1)^2 \geq 0$, and when $x = 1$, $(x - 1)^2 = 0$. Now, examining the expression $-(x - 1)^2$, we notice that $-(x - 1)^2 \leq 0$. That means when $-(x - 1)^2 = 0$, we will get the maximum value of $f(x)$, which is 2. So, the greatest value of $f(x) + 3$ would be $2 + 3 =$ **5**.

130. Let's extend two sides of the unit square until they intersect two vertices of the larger square, as shown. We see that the larger square is composed of four triangles congruent to the one shaded, while the smaller square is composed of only two of those triangles. That means the larger square has an area that is twice that of the smaller square, or **2** units^2.



Warm-Up 10

131. At the start, the 2-cup mixture contains 2/3 cup of flour and 4/3 cups of nuts. When 3/3 cup of flour is added, the mixture is now 5/3 cups of flour and 4/3 cups of nuts. We now have 5 parts flour out of 5 + 4 = 9 parts, so the mixture is **5/9** flour.

132. The difference between Marshall's and Cody's ages is 53 – 17 = 36 years. In any other year (at this time of year), this difference is the same. When Marshall was four times as old as Cody, the difference of 36 would have been three times Cody's age, so Cody must have been 36 ÷ 3 = 12 years old, and Marshall must have been 4 × 12 = 48 years old. That was 17 – 12 = **5** years ago.

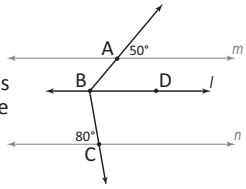
133. The hundreds digit of a house number can be either 7 or 9, so there are 2 possibilities for each of the following cases. Case 1: If the tens digit is even, there are 4 possibilities (zero is not allowed) followed by 5 possible odd digits for the units place. Case 2: If the tens digit is odd, there are 5 possibilities followed by 4 possible even digits for the units place. The greatest number of houses that could be on Main Street is 2 × (4 × 5) + 2 × (5 × 4) = 40 + 40 = **80** houses.

134. The cyclist spends twice as much time biking up the hill at a rate of 6 mi/h as biking down the hill at a rate of 12 mi/h, so the average speed will be closer to 6 mi/h than to 12 mi/h; it's *not* 9 mi/h. The uphill time can be written $t_1 = d/6$. The downhill time can be written $t_2 = d/12$. The total time is $t_1 + t_2 = d/6 + d/12 = 2d/12 + d/12 = 3d/12 = d/4$. The total distance is $2d$. So, if the average rate of the total trip is r_T, we have $r_T(d/4) = 2d \rightarrow r_T = 2d \times (4/d) =$ **8** mi/h.

135. The distance of 11 ¼ = 45/4 inches on the map represents 45/4 × 16 = 180 miles. Averaging 60 mi/h, the trip will take 180 ÷ 60 = **3** hours.

136. Rewrite the expression as $\left(\frac{1.4 \times 2.4}{1.2}\right)\left(\frac{10^{-7} \times 10^{8}}{10^{9}}\right)$. Simplifying, we have $(1.4 \times 2)(10^{-7} \times 10^{-1}) =$ $\mathbf{2.8 \times 10^{-8}}$.

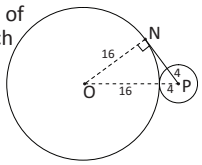
137. Draw line *l* parallel to lines *m* and *n* and through the vertex of ∠ABC and point D, as shown. Line *l* divides ∠ABC into angles ABD and CBD. Since $l \parallel m$, ∠ABD corresponds to the angle with measure 50 degrees. Thus, $m\angle ABD = 50$ degrees. Similarly, since $l \parallel n$, ∠CBD and the 80-degree angle are alternate interior angles, which means $m\angle CBD = 80$ degrees. Now we have $m\angle ABC = m\angle ABD + m\angle CBD = 50 + 80 =$ **130** degrees.



138. The five new students increased the class by 3/2 – 2/2 = 1/2. Because five students equal half the original number of students, it follows that before the five students were added, Sally's class had 5 × 2 = **10** students.

139. Let *c* and *b* represent the costs of 1 cobble and 1 burrey, respectively. The first two sentences of the problem translate to the following algebraic equations, respectively: $2c + 3b = 19$ and $37 - 5c = 4b$. The second statement can be rewritten as $5c + 4b = 37$. Now we can add the two equations to get $(2c + 3b) + (5c + 4b) = 19 + 37 \rightarrow 7c + 7b = 56$. So 7 cobbles and 7 burreys cost 56 slugs. Dividing both sides of the equation by 7, we see that $1c + 1b = 8$. Thus, the cost of 1 cobble and 1 burrey is **8** slugs. If we wanted to verify this, we cold solve the system of equations and see that $b = 3$ slugs and $c = 5$ slugs, so $1c + 1b$ does, in fact, equal 8 slugs.

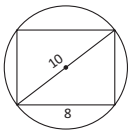
140. As shown, segment ON, drawn perpendicular to segment NP, is a radius of circle O, and, therefore, has length 16 cm. A radius of circle O and a radius of circle P, together, form segment OP, and OP = 16 + 4 = 20 cm. △ONP is a right triangle with a 16-cm leg and a hypotenuse of 20 cm. Using the Pythagorean Theorem, we can write $16^2 + NP^2 = 20^2 \rightarrow 256 + NP^2 = 400 \rightarrow NP^2 = 144$. Taking the square root of each side, we see that NP = **12** cm. You may also have recognized that 12-16-20 is a multiple of the 3-4-5 Pythagorean Triple.



Workout 5

141. The volume of the smaller cube is 10 × 10 × 10 = 1000 units^3. It follows, then, that the volume of the larger cube is 1000 + 728 = 1728 units^3. That means the edge length of the larger cube is $\sqrt[3]{1728}$= **12** units.

142. A rectangle is inscribed in a circle, as shown. The rectangle's diagonal, which divides it into two congruent right triangles, is also a diameter of the circle. This circle, with radius 5 cm, has diameter 10 cm. That means both of the right triangles have a hypotenuse of length 10 cm and a leg of length 8 cm. You may recognize that the side lengths of the right triangles are a multiple of the Pythagorean Triple 3-4-5 and, therefore, conclude that the other leg has length 6 cm. If not, the Pythagorean Theorem can be used to determine the length of the triangle's shorter leg, *w*. We have $8^2 + w^2 = 10^2 \rightarrow 64 + w^2 = 100 \rightarrow w^2 = 36 \rightarrow w = 6$ cm. Therefore, the rectangle has area 6 × 8 = **48** cm^2.



143. Because each person will get 1/3 of the original \$234 million, that's 234 ÷ 3 = \$78 million before taxes. We are told that they each will receive \$53 million after taxes, which means that, for each of them, 78 – 53 = \$25 million will be deducted for taxes. Thus, each winner will pay about 25 ÷ 78 × 100% ≈ **32**% in taxes.

144. To calculate a reduction of 20%, we will multiply by 0.8. To calculate an increase of 20%, we will multiply by 1.2. After six price changes, the item will cost 0.8 × 1.2 × 0.8 × 1.2 × 0.8 × 1.2 = 0.884736 ≈ 88.5% of its original price. Thus, *a* = **88.5**.

145. In a right triangle, one angle measures 90 degrees, so it follows that the sum of the other two angles is 180 − 90 = 90 degrees. The value of the quotient 90 ÷ 8 is **11.25** degrees.

146. The area of the rectangular pizza is 18 × 24 = 432 in^2. The area of the large round pizza is $\pi \times 12^2 = 144\pi$ in^2. For the same amount of money, with the large round pizza you get 144π **–** 432 ≈ **20** in^2 more pizza.

147. Because the cost of air conditioning varies directly with the average temperature, we can set up a proportion to solve for the unknown cost c as follows: $32.40/81 = c/96 \rightarrow 81c = 96 \times 32.40 \rightarrow c = 96 \times 32.40 \div 81 \rightarrow c = 38.40$. Kate's air conditioning bill for that particular week in August will be \$**38.40**.

148. Columbus ran 40 + 70 + 40 + 70 = 220 feet. The length of the diagonal that Pythagoras ran is $\sqrt{(70^2 + 40^2)} = \sqrt{(4900 + 1600)} = \sqrt{6500} = 10\sqrt{65}$ feet. Pythagoras ran $2 \times 10\sqrt{65} = 20\sqrt{65}$ feet. Columbus ran $220 - 20\sqrt{65} \approx$ **58.8** feet farther than Pythagoras.

149. We will compute a "weighted average" as follows: [(20 × 0) + (20 × 1) + (30 × 2) + (20 × 3) + (8 × 4) + (2 × 5)] ÷ 100 = 182/100 = 1.82 ≈ 2. So, on average, a couple getting married today can be expected to have **2** children.

150. If we let n be the number of nickels, then $5n$ is the value of those nickels in cents. Using d for the number of dimes and q for the number of quarters, we can write the following equation for the collection of coins: $5n + 10d + 25q = 530$. If we find an expression for n in terms of q and an expression for d in terms of q, we can substitute those expressions into the previous equation and solve for q. We are told that the number of dimes is 2 more than the number of nickels, so we have $d = n + 2$. Also, the number of quarters is four more than the number of dimes, so $q = d + 4$. We can rewrite this equation as $d = q - 4$, and we have these two equations: $d = n + 2$ and $d = q - 4$. We can set these two equations equal to one another to get an equation for n in terms of q. We have $n + 2 = q - 4 \rightarrow n = q - 6$. Now in the equation $5n + 10d + 25q = 530$, we can replace n and d with $q - 6$ and $q - 4$, respectively. Doing so yields $5(q - 6) + 10(q - 4) + 25q = 530 \rightarrow 5q - 30 + 10q - 40 + 25q = 530 \rightarrow 40q - 70 = 530 \rightarrow 40q = 600 \rightarrow q = 15$. The collection of coins has **15** quarters.

Warm-Up 11

151. If each of the seven numbers is doubled, the sum of the seven numbers will be doubled. Therefore, the mean of the seven numbers also will be doubled. The new mean will be **18**.

152. To find the decimal equivalent of 1/7, we divide 1 by 7. We get the following 6 repeating digits after the decimal point: $1/7 = 0.\overline{142857}$. Dividing 2013 by 6, we get 335 with a remainder of 3. This means that there will be 335 complete sets of the 6 digits, and the 2013th digit will be the 3rd of the 6 repeating digits, which is **2**.

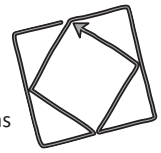
153. When z = 1, 2, 3 or 4, we have 19/1 = 19 ≥ 1; 19/2 = 9 ½ ≥ 2; 19/3 = 6 ⅓ ≥ 3; 19/4 = 4 ¾ ≥ 4. But for $z > 4$, it is always true that $19/z < z$. Therefore, since $19/z \geq z$ for 4 of the 19 positive integers less than 20, the probability that $19/z \geq z$ is **4/19**.

154. First, to find the value of a, consider the proportion $3/4 = a/36$. Cross-multiplying and solving, we see that $4a = 3 \times 36 \rightarrow 4a = 108 \rightarrow a = 27$. Next, to find the value of b, we'll use the proportion $3/4 = 36/b$. Cross-multiplying and solving, we see that $3b = 4 \times 36 \rightarrow 3b = 144 \rightarrow b = 48$. Thus, the value of $a + b$ is 27 + 48 = **75**.

155. Since Pat's share was \$2.25, which represents 2.25/10.80 = 5/24 of the pizza's total cost, it follows that her part was 5/24 of the pizza's total area. Therefore, the central angle of Pat's fractional part of the pizza was (5/24) × 360 = 5 × 15 = **75** degrees.

156. We know that 20 blocks is equivalent to 1 mile, so 60 blocks is 60 ÷ 20 = 3 miles. Traveling at an average rate of 3 mi/h, a pedestrian takes 1 hour, or 60 minutes, to travel 60 blocks. At a rate of 30 mi/h, the subway train takes 3 ÷ 30 = 1/10 hour = 6 minutes to travel 60 blocks. Compared to the subway train, it takes a pedestrian 60 − 12 = **54** minutes more to travel 60 blocks.

157. There are 9 choose 2 = 9 × 8 ÷ 2 = 36 ways to choose two different positive integers less than 10. Pairing 1 with a perfect square would result in a perfect square product. That gives us 1 × 4 = 4 and 1 × 9 = 9. We can also get 4 × 9 = 36 and 2 × 8 = 16. That's 4 out of 36, so the probability is 4/36 = **1/9**. Perhaps you think we should count the products 1 × 4 and 4 × 1 separately. In this case, there are a total of 72 ways to choose two different positive integers less than 10 in a particular order. The 8 ordered pairs that have a product that is a perfect square are 1 × 4, 4 × 1, 1 × 9, 9 × 1, 4 × 9, 9 × 4, 2 × 8 and 8 × 2. Therefore, the probability is 8/72 = **1/9**.



158. We know that the side length of the outer square is 12 inches. Each side of the inner square is the hypotenuse of a 45-45-90 right triangle, with legs of length 12 ÷ 2 = 6 inches. We can use the Pythagorean Theorem to determine the length of the hypotenuse. Or you may recall that for a 45-45-90 right triangle with legs of length a, the length of the hypotenuse is $a\sqrt{2}$. That means the inner square has sides of length $6\sqrt{2}$ inches. Therefore, the shortest length of yarn needed to create the figure using a single piece of yarn, as shown, is $(4 \times 12) + (4 \times 6\sqrt{2})$ = **48 + 24√2** inches.

159. Since $r = d/t$, rate (or speed) and time are inversely proportional. That means Rich would have had to drive at 5/3 his current speed to cover the same distance in 3/5 the time. If this could be accomplished by increasing his speed 20 mi/h, then 20 mi/h represents (5/3) − 1 = 2/3 of his current speed. It follows, then, that Rich's current speed is (3/2) × 20 = **30** mi/h.

160. Since the sum of the angle measures in △XYZ is 180 degrees, we can write the equation $x + 12 + 3x + 2x = 180 \rightarrow 6x + 12 = 180$. Solving for x yields $6x = 168 \rightarrow x = 28$. We can now find the supplement of ∠XZY, which is $180 - 3x = 180 - 3 \times 28 = 96$ degrees. Since △WXZ is isosceles, the remaining 180 – 96 = 84 degrees is divided equally between ∠W and ∠WXZ. Therefore, the measure of ∠W is 84 ÷ 2 = **42** degrees.

Warm-Up 12

161. Recall that $1000 = 10^3$. We can set up the proportion $10^3/0.02 = 10^8/x$. Cross-multiplying and solving, we have $10^3x = 0.02 \times 10^8 \rightarrow x = (0.02 \times 10^8)/10^3 \rightarrow x = 0.02 \times 10^5 \rightarrow x = 2000$. (Many students will solve this problem by simply moving the decimal point of 0.02 km^2 five places to the right.) This means the amount of soil that 10^8 cubic meters of pine mulch can fertilize is **2000** km^2.

162. Using the laws of exponents, 9^c can be rewritten as follows: $9^c = (3^2)^c = 3^{2c}$. Similarly, $27^{c-1} = (3^3)^{c-1} = 3^{3(c-1)} = 3^{3c-3}$. So we have $3^{2c} = 3^{3c-3}$. Now that each expression is a power of 3, we can conclude that $2c = 3c - 3 \rightarrow c =$ **3**.

163. Continuing each sequence for a few more terms, we have 2, 4, 8, 16, 32, 64, … for the first sequence and 10, 20, 30, 40, 50, 60, … for the second sequence. We quickly see that the first time that the *n*th term of the first sequence is greater than the *n*th term of the second sequence occurs when $n =$ **6**.

164. Mrs. Treble can choose any of 20 pieces first, followed by any of the remaining 19 pieces second, then any of 18 third and any of 17 fourth. If the order of selections mattered, there would be $20 \times 19 \times 18 \times 17 = 116{,}280$ ways this could happen. But the problem states only that Mrs. Treble will *select* 4 pieces of music, with no mention of the order in which they will be played. In addition, the use of the word *combinations* tells us that order is not important. Since every 4-piece combination can be ordered in $4! = 4 \times 3 \times 2 \times 1 = 24$ ways, we need to divide 116,280 by 24. Doing so, we see there are $116{,}280 \div 24 =$ **4845** combinations.

165. Since 3, 4 and 5 are relatively prime, any number that is a multiple of all three numbers must be a multiple of their product, which is $3 \times 4 \times 5 = 60$. Since $500 \div 60 = 8\frac{1}{3}$, there are **8** integers less than 500 that are positive and multples of all three integers 3, 4 and 5, namely 60, 120, 180, ..., 480.

166. From 2:45 to 3:15 p.m., the minute hand sweeps through an arc of 180 degrees, or 0.5 times around. From 3:15 to 7:15 p.m., it sweeps 4 times around. That's a total of 4.5 times around. The circumference of a circle with a radius of 4 inches is $2 \times 4 \times \pi = 8\pi$ inches. The tip of the minute hand must have traveled $4.5 \times 8\pi =$ **36π** inches.

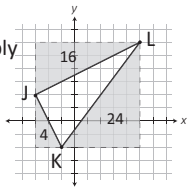
167. Since Elias, Frederico and Gina must be on different teams, we'll start by creating three teams and assigning Elias, Frederico and Gina each to a different one of those three teams. Since Frederico must be with Hsin-Hsin or Illiana, there will not be room on his team for the pair of Chico and Danzig, who must stay together. We also cannot put Chico and Danzig with Gina, because Danzig cannot be on a team with Gina as captain, and neither Chico nor Danzig can be captain. That means Chico and Danzig must be placed with Elias, and **Elias** must be the third captain. The following teams satisfy all conditions:

Team 1 - Elias (Captain), Chico, Danzig
Team 2 - Barry (Captain), Frederico, Hsin-Hsin
Team 3 - Alana (Captain), Gina, Illiana

168. Each square has area $4 \times 4 = 16$ ft^2. Each circle in Figure 1 has a radius of 1 ft and an area of π ft^2. So the total area of the 4 circles in Figure 1 is $4 \times \pi = 4\pi$ ft^2. The shaded area in Figure 1, then, is $16 - 4\pi$ ft^2. Each circle in Figure 2 has a radius of 1/2 foot and an area of $\pi/4$ ft^2. So the total area of the 16 circles in Figure 2 is $16 \times \pi/4 = 4\pi$ ft^2. The shaded area in Figure 2 is $16 - 4\pi$ ft^2. The absolute difference in the shaded areas in Figure 1 and Figure 2 is **0** ft^2.

169. Squaring both sides of the equation, we get $x^4 + 2 + 1/x^4 = 9$. When we subtract 2 from each side of the equation, we see that $x^4 + 1/x^4 =$ **7**.

170. Construct the 8-unit by 8-unit square that circumscribes △JKL, as shown. Notice that the area of the non-shaded triangle is simply the area of the square, $8 \times 8 = 64$ $units^2$, minus the areas of the three shaded right triangles. The areas of the three right triangles are $(1/2) \times 8 \times 4 = 16$ $units^2$, $(1/2) \times 6 \times 8 = 24$ $units^2$ and $(1/2) \times 2 \times 4 = 4$ $units^2$. Therefore, the area of △JKL is $64 - 16 - 24 - 4 =$ **20** $units^2$.



Workout 6

171. If Juwan wants his price to be 60% more than he paid, his price will be 160% of the price he paid to the manufacturer. Juwan should multiply the manufacturer's price by **1.6**, which is the decimal equivalent of 160%.

172. A cube exists in three-dimensional space, so the edge length of the new cube will be 150% of the old edge length in all three dimensions. The new volume will be $1.5^3 = 1.5 \times 1.5 \times 1.5 = 3.375$ times as great as the old volume. That's 337.5% of the old volume, which is an increase of $337.5 - 100 =$ **237.5**%.

173. With a radius of 7 inches, the pizza has a circumference of 14π inches. Since 60 degrees is 1/6 of 360 degrees, the curved side of the slice is 1/6 of the pizza's circumference, which is $(1/6) \times 14\pi = (7/3)\pi$ inches. The perimeter of the pizza slice is $7 + 7 + (7/3)\pi \approx$ **21.3** inches.

174. Triangle ABC is a 5-12-13 right triangle. This is a well-known Pythagorean Triple, in which these integer values satisfy the Pythagorean Theorem ($5^2 + 12^2 = 13^2$). Given that the coordinates of point B are (–4, 0), we can subtract 5 from the *x*-coordinate to find that point A is at (–9, 0). Now we have a triangle, similar to △ABC, that has vertices (–9, 0), (0, 0) and (0, *k*). The distance from point A to the origin is 9 units, and the distance from the origin to (0, *k*) is *k* units. We can now set up the proportion $9/5 = k/12$. Cross-multiplying and solving, we see that $5k = 108 \rightarrow k =$ **21.6**.

175. Since we can't find the roots of the trinomial by factoring, we could use the quadratic formula to find m and n. But there is a nice shortcut we can use to find the sum of the reciprocals of the roots. In general, the sum of the reciprocals of two numbers equals the sum of the two numbers divided by their product. Algebraically, we have $1/m + 1/n = n/(mn) + m/(mn) = (m + n)/(mn)$. It also is helpful to recall that for a quadratic trinomial in the form $x^2 + bx + c$, $-b$ equals the sum of the trinomial's roots and c is their product. Therefore, for the trinomial $x^2 - 7x + 5$, we know that $m + n = 7$, $mn = 5$ and $1/m + 1/n = (m + n)/(mn) =$ **7/5**.

176. There are 25 choices for the first number, 24 choices for the second number and 23 choices for the third number. That's 25 × 24 × 23 = 13,800 ways in which three different integers from 1 through 25 can be chosen. But to win this lottery, the order in which the three numbers are chosen doesn't matter. Each group of three numbers can be chosen in 3! = 6 different orders. Dividing, we see that there are 13,800 ÷ 6 = 2300 unique combinations of three different numbers that can be chosen for this lottery. Because Liam purchased 500 lottery tickets, each one with a unique three-number combination, his probability of winning is 500/2300 = 5/23 ≈ **0.22**.

177. We will use b to represent the price of a belt and s to represent the price of a scarf. Jayden's purchase can be written algebraically as $2b + 4s = 59.70$, and Katrina's purchase can be written as $3b + 5s = 80.60$. This time let's try using elimination, rather than substitution, to solve this system of equations. Since we ultimately want to find the cost of a belt, let's eliminate s from both equations. If we multiply both sides of Jayden's equation by 5, we get $5 \times (2b + 4s) = 5 \times 59.70 \rightarrow 10b + 20s = 298.50$. Then if we multiply both sides of Katrina's equation by 4, we get $4 \times (3b + 5s) = 4 \times 80.60 \rightarrow 12b + 20s = 322.40$. Now we can subtract Jayden's equation from Katrina's equation and solve for b. We have $(12b + 20s) - (10b + 20s) = 322.40 - 298.50 \rightarrow 12b + 20s - 10b - 20s = 23.90 \rightarrow 2b = 23.90 \rightarrow b = 11.95$. The cost of one belt is \$**11.95**.

178. To get 15 pieces of wood, 14 cuts must be made. Since each cut eliminates 1/8 inch of wood as sawdust, a total of 14 × (1/8) = 14/8 = 1.75 inches of wood will become sawdust. There will be 15 pieces of wood, each 2 inches in length. That will require 15 × 2 = 30 inches of wood. Therefore, the length of the original piece of wood must be 1.75 + 30 = **31.75** inches.

179. We are looking for a sum of consecutive numbers $1 + 2 + 3 + \ldots + n$ that equals (or is close to) 190. The sum of the positive integers from 1 to n is the nth triangular number. A formula for the nth triangular number is $n(n + 1)/2$. Setting this equal to 190, we get $n(n + 1)/2 = 190 \rightarrow n(n + 1) = 380$. Technically this is a quadratic equation and there are algebraic methods to solve it. On the other hand, if we consider that the left side of the equation shows a product of two consecutive numbers, we might realize that 19 × 20 = 380, so $n = 19$. So, in the last group there will be **19** penguins.

180. We are told that the circles in Figure 1 and Figure 2 both have a radius of 1 cm, which means that both circles have an area of $\pi \times 1^2 = \pi$ cm^2. Since the circle in Figure 1 is tangent to all four sides of the square, the diameter of the circle is the same length as each side of that square. So, the square in Figure 1 has side length 1 + 1 = 2 cm, and its area is 2 × 2 = 4 cm^2. It follows that the shaded area is $4 - \pi$ cm^2. Since the circle in Figure 2 intersects all four vertices of the inscribed square, the diameter of the circle is the same length as the square's diagonal. So, the square in Figure 2 has diagonal length 2 cm. In general, the diagonal of a square has length $s\sqrt{2}$, where s is the side length of the square. In this case we have $s\sqrt{2} = 2 \rightarrow s = 2/\sqrt{2} \rightarrow s = \sqrt{2}$. So, the square in Figure 2 has side length $\sqrt{2}$, and its area is $\sqrt{2} \times \sqrt{2} = 2$ cm^2. It follows, then, that the shaded area in Figure 2 is $\pi - 2$ cm^2. Subtracting, we see that the absolute difference between the shaded areas in Figures 1 and 2 is $|(\pi - 2) - (4 - \pi)| = \pi - 2 - 4 + \pi = 2\pi - 6 \approx$ **0.28** cm^2.

Warm-Up 13

181. With replacement, there are 10 × 10 = 100 different ordered pairs of tags that Sid can select from the bag. He can select two blue tags as follows: (blue 9, blue 9), (blue 10, blue 10), (blue 9, blue 10), (blue 10, blue 9). These are the only four ordered pairs of tags Sid can select that do not contain at least one red tag. Therefore, the number of ordered pairs of tags Sid can select that contain at least one red tag is 100 − 4 = **96** ordered pairs.

182. Let's say the original height is h and the original width is w. Then the area of the original rectangle is hw. The new height is $(1/2)h$, and the new width is $(1/4)w$. The area of the new rectangle is $(1/2)h \times (1/4)w = (1/8)hw$. The ratio of the area of the new rectangle to the area of the old rectangle is $[(1/8)hw]/(hw) =$ **1/8**.

183. We are told that the points $(1, a)$, $(2, b)$ and $(4, 18)$ are on the graph of the linear equation. That means the slope of the line is constant between the different pairs of points. Using the points $(1, a)$ and $(2, b)$, we can write the slope as $(b - a)/(2 - 1) = b - a$. Using the points $(1, a)$ and $(4, 18)$, we can write the slope as $(18 - a)/(4 - 1) = (18 - a)/3$. Setting these expressions equal to each other, we have $b - a = (18 - a)/3 \rightarrow 3b - 3a = 18 - a \rightarrow 3b - 2a = 18$. Dividing both sides of the equation by 2 yields $(3/2)b - a =$ **9**.

Here's another solution with a different approach. This is an example of a problem with some freedom of choice. We can choose an arbitrary value for a or b and then find the value of the other. The value of the expression $(3/2)b - a$ will be invariant. For example, suppose $a = 0$. Then the slope of the line from (1, 0) to (4, 18) is (18 – 0)/(4 – 1) = 18/3 = 6. This slope means that the y-value increases by 6 as the x-value increases by 1, so the point $(2, b)$ must be (2, 6) and $b = 6$. Now we can evaluate $(3/2)b - a$, which is (3/2) × 6 – 0 = 9. Suppose instead that we make $a = 18$. Then the line from (1, 18) to (4, 18) has a slope of zero (0). This means that the point (2, 18) is also on the line, so $b = 18$ also. Now the value of $(3/2)b - a$ is (3/2) × 18 – 18 = 27 – 18 = 9. We can prove this more generally with the letters a and b. The slope of the line from $(1, a)$ to (4, 18) is $(18 - a)/(4 - 1) = (18 - a)/3$. The value of b is exactly this amount greater than a, so $b = a + (18 - a)/3$, which simplifies as follows: $3a/3 + (18 - a)/3 = (2a + 18)/3$. Now we evaluate $(3/2)b - a$, using this value of b. We get $(3/2) \times (2a + 18)/3 - a = a + 9 - a =$ **9**. Often it helps to try some specific numbers before attempting a general solution to a problem.

184. The range of the girls' guesses was 360 – 300 = 60. The fact that the biggest error was 49 tells us that the actual number of marbles is between 300 and 360. In fact, we should notice that 11 + 49 = 60. So, either Sue's guess was off by 11 marbles and Donna's guess was off by 49 marbles, or vice versa. Therefore, the number of marbles in the jar is either 300 + 11 = 311 marbles, or 300 + 49 = 349 marbles. If there are 311 marbles in the jar, then Kim's guess of 325 would be off by 325 − 311 = 14 marbles. However, since we are told that Kim's guess is off by 24 marbles, the number of marbles in the jar must be **349** marbles. One final check, 349 − 325 = 24, and we have verified our answer.

185. The largest sphere that can fit in the cylinder will have a diameter of 8 inches. We will use the radius of 4 inches to determine the volume of the sphere with the formula $V = (4/3)\pi r^3$. We have $(4/3)\pi(4^3) = (256/3)\pi$. Since $(256\pi)/3 = \pi x/3$, it follows that x = **256**.

186. Let's say that each junior associate received x dollars. Then, together, the junior associates received a total of $4x$ dollars. Each full-time associate received $2x$ dollars. So, together, the five full-time associates received a total of $5 \times 2x = 10x$ dollars. The sales manager received $3 \times 2x = 6x$ dollars. The total amount divided among the employees was $4x + 10x + 6x = 20x$ dollars. So we have $20x = 1000 \rightarrow x = \$50$. Since each junior associate received \$50, it follows that the sales manager received $6 \times 50 = \$$**300**.

187. The expression $x^2 + 2xy + y^2$ is a perfect square trinomial and can be rewritten as $(x + y)^2$. So $(x + y)^2 = 81$, and since $x > 0$ and $y > 0$, it follows that $x + y = 9$. Therefore, the average of x and y is $9/2$ = **4.5**.

188. Using the order of operations, we evaluate all the exponents and then do multiplication, then addition and subtraction to get $1 - 5 \times 2 + 10 \times 2^2 - 10 \times 2^3 + 5 \times 2^4 - 2^5 = 1 - 5 \times 2 + 10 \times 4 - 10 \times 8 + 5 \times 16 - 32 = 1 - 10 + 40 - 80 + 80 - 32 =$ **–1**. Those familiar with binomial expansion will recognize the coefficients 1, 5, 10, 10, 5, 1 as the coefficients when a 5th degree binomial $(a + b)^5$ is expanded. Notice that when $a = 1$ and $b = -2$, we have $1a^5b^0 + 5a^4b^1 + 10a^3b^2 + 10a^2b^3 + 5a^1b^4 + 1a^0b^5 = 1(1^5)(-2)^0 + 5(1^4)(-2)^1 + 10(1^3)(-2)^2 + 10(1^2)(-2)^3 + 5(1^1)(-2)^4 + 1(1^0)(-2)^5 = 1 - 5(2) + 10(2^2) - 10(2^3) + 5(2^4) - 2^5$. We can easily determine the value of this expression by evaluating $(1 - 2)^5 = (-1)^5 =$ **–1**.

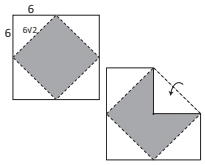
189. The number of bricks at each level is the product of the level number and the number one more than the level number. In other words, there are $n(n + 1)$ bricks on level n. We need to find the sum of the numbers of bricks on all levels of a structure with 12 levels. The values are shown in the table below, with a final answer of **728** bricks.

Level	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
#Added	*1×2 =* 2	*2×3 =* 6	*3×4 =* 12	*4×5 =* 20	*5×6 =* 30	*6×7 =* 42	*7×8 =* 56	*8×9 =* 72	*9×10 =* 90	*10×11 =* 110	*11×12 =* 132	*12×13 =* 156
Total	2	8	20	40	70	112	168	240	330	440	572	728

190. Since the cube has volume 27 in^3, it follows that it has edges with length $\sqrt[3]{27} = 3$ inches. Notice that each side of $\triangle$ABC is a diagonal of a face of the cube. Therefore, $\triangle$ABC is an equilateral triangle with sides of length $3\sqrt{2}$ inches. We can use the formula for the area of an equilateral triangle, $(s^2\sqrt{3})/4$, to determine the area of $\triangle$ABC. We have $[(3\sqrt{2})^2(\sqrt{3})]/4 \rightarrow (18\sqrt{3})/4 \rightarrow (9\sqrt{3})/2$ in^2. Therefore, $\triangle$ABC has area **$(9\sqrt{3})/2$** in^2.

Warm-Up 14

191. Let's start by evaluating the first few factorials in the sum. Doing so yields 1! = 1, 2! = 2, 3! = 6, 4! = 24, 5! = 120, 6! = 720. We notice that beginning with 5!, the units digit of the remaining addends will be zero. Therefore, to determine the units digit of the sum, we need only consider the first four terms. The sum of their units digits is 1 + 2 + 6 + 4 = 13, which means the units digit of the entire sum will be **3**.



192. The region of the square where points are closer to the center than they are to any vertex is bounded by a square with vertices that are the midpoints of each side of the original square, as shown. This square has side length $6\sqrt{2}$ units, so the area of the bounded region is $(6\sqrt{2})^2$ = **72** units^2. This is exactly half the area of the original square. By folding so that all of the vertices of the original square meet at the square's center, it's easy to see that the unshaded region exactly covers the shaded region, so they must have the same area.

193. Let's rewrite the product of the integers from 1 to 10 using only prime numbers: $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 = 2 \times 3 \times (2 \times 2) \times 5 \times (2 \times 3) \times 7 \times (2 \times 2 \times 2) \times (3 \times 3) \times (2 \times 5)$. We see that there are 8 factors of 2, so $a = 8$. There are 4 factors of 3, so $b = 4$. Therefore, the value of $a + b$ is 8 + 4 = **12**.

194. We can use the formula $d = rt \rightarrow t = d/r$ to determine Ava's rate. The amount of time it took Ava to go 150 km at a rate of r km/h was $150/r$ hours. The amount of time it took Lizzy to go 180 km at a rate of $r + 2$ km/h (2 km/h faster than Ava) was $180/(r + 2)$ hours. Since it took Ava and Lizzy the same amount of time, we have $150/r = 180/(r + 2)$. Cross-multiplying and solving for r, we get $180r = 150(r + 2) \rightarrow 180r = 150r + 300 \rightarrow 30r = 300 \rightarrow r = 10$. Therefore, Ava's rate was **10** km/h.

195. A prime number is positve and is only divisible by 1 and itself, so we can't multiply any primes or composites together to create this product. We'll need 1 and –1 to be two of the first three terms of the sequence. Initially, we might consider −1, 0, 1 to be the first three terms of this sequene. However, we are told that the product of the three terms is a prime number, and the product $-1 \times 0 \times 1 = 0$. Since the difference between −1 and 1 is 2, the third number must be 2 greater than 1 or 2 less than −1. The first three terms of the sequence are either –1, 1, 3 with a product of –3 or –3, –1, 1 with a product of 3. Since 3 is prime, and –3 is not, the sequence begins with −3, −1, 1 or 1, −1, −3, depending on whether the sequence is increasing or decreasing. In either case, the sum of the first three terms is –3 + (–1) + 1 = **–3**.

196. The area of the triangle is currently $(1/2) \times 3 \times 6 = 9$ ft^2, and the product of the base and the height is $3 \times 6 = 18$ ft^2. If the area is increased to 35 ft^2, the product of the base and height will be 70. We need to add the same number to both 3 and 6 and get a product of 70. In other words, we need to find a number k such that $(3 + k) \times (6 + k) = 70$. If we just try $k = 1, 2, 3, 4$, we find that when $k = 4$, we get $7 \times 10 = 70$. The sum of the new base and height is 7 + 10 = **17** feet. Those familiar with solving quadratic equations by factoring might multiply the binomials in the equation $(3 + k) \times (6 + k) = 70$ to get $k^2 + 9k + 18 = 70 \rightarrow k^2 + 9k - 52 = 0$. Factoring, we have $(k + 13)(k - 4) = 0 \rightarrow k = -13$ or $k = 4$. In this case, $k = 4$.

197. Let's start with the sum 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1. Then we can combine addends into larger powers of 2 as follows:

1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1	2 + 2 + 1 + 1 + 1 + 1 + 1	2 + 2 + 2 + 2 + 1	4 + 2 + 1 + 1 + 1	4 + 4 + 1
2 + 1 + 1 + 1 + 1 + 1 + 1 + 1	2 + 2 + 2 + 1 + 1 + 1	4 + 1 + 1 + 1 + 1 + 1	4 + 2 + 2 + 1	8 + 1

That's a total of **10** lists.

198. The sum of Anthony's scores on his five physics tests is 5 × 85 = 425 points. Since the median is 87 and the unique mode is 92, we know that the five numbers must be ___, ___, 87, 92, 92. The test scores known at this point have a sum of 87 + 92 + 92 = 271. That means the remaining two test scores have a sum of 425 − 271 = 154. If we want to make one of the two scores as low as possible, then we need to make the other score as high as possible. In this case, it can't be 87 because we are told that 92 is the only mode. So we go to the greatest integer less than 87, which is 86. Therefore, the lowest possible test score Anthony could have for one of his tests is 154 − 86 = **68**.

199. Let S represent the number of marbles Sara has and J represent the number of marbles Jo has. The first statement leads to the equation $S + 3 = 2(J - 3)$, which simplifies to $S + 3 = 2J - 6 \rightarrow S = 2J - 9$. The second statement leads to the equation $2(S - 2) = J + 2$, which simplifies to $2S - 4 = J + 2 \rightarrow 2S - 6 = J$. We can now substitute $2S - 6$ for J in the first equation to get $S = 2(2S - 6) - 9 \rightarrow S = 4S - 12 - 9 \rightarrow S = 4S - 21 \rightarrow 3S = 21 \rightarrow S = 7$. So Sara has **7** marbles.

200. Since 4 of 12 candies have vanilla crème filling, the probability is 4/12 that the first candy Angel picks has vanilla filling. The probability that her second pick also has vanilla filling is 3/11. The probability that these two events happen in succession is 4/12 × 3/11 = **1/11**.

Workout 7

201. Because 4 people did not contribute, 20 – 4 = 16 people must share the cost. If they each pay an extra \$1.50, the amount the 4 people did not pay must have been 16 × \$1.50 = \$24. The contribution of each of the 4 people would have been \$24 ÷ 4 = \$6. Based on contributions of \$6 each from all 20 people, the total needed for the gift must be 20 × \$6 = \$**120**.

202. We'll use three of the laws of exponents to simplify the expression: (1) $x^m x^n = x^{m+n}$, (2) $x^m/x^n = x^{m-n}$ and (3) $(x^m)^n = x^{mn}$. We have $\frac{(a^2b^{-3}c)^2(a^{-1}b^3c^3)^{-3}}{(a^{-1}b^3c^{-2})^2(a^4bc^{-3})^4} = \frac{(a^4b^{-6}c^2)(a^3b^{-9}c^{-9})}{(a^{-2}b^6c^{-4})(a^{16}b^4c^{-12})} = \frac{a^7b^{-15}c^{-7}}{a^{14}b^{10}c^{-16}} = a^{-7}b^{-25}c^9$. So $x = -7$, $y = -25$ and $z = 9$, and $x + y + z = -7 + (-25) + 9 =$ **–23**.

203. First we subtract, 48 − 25 = 23, to see that Archer earned \$23 for the miles driven. To earn \$23 at \$0.08 per mile, he drove 23 ÷ 0.08 = **287.5** miles.

204. There are 6 sets of 10 seconds in 1 minute, so Albert would have skipped 128 × 6 = 768 times in 1 minute at the same rate. There are 60 minutes in an hour, so he would have skipped 768 × 60 = **46,080** times in 1 hour at that rate.

205. Since Andrew and Beth each have 10 numbers from which to choose, there are 10 × 10 = 100 pairs of numbers they could choose. If Andrew chooses 1, any number Beth chooses will be relatively prime to 1. That's 10 pairs of relatively prime numbers. If Andrew chooses 2, there are 5 numbers (1, 3, 5, 7, 9) Beth could choose that are relatively prime to 2. We can continue this process through the scenario when Andrew chooses 10. The table shows how many numbers from 1 through 10 are relatively prime to each of the numbers 1 through 10. That's a total of 10 + 5 + 7 + 5 + 8 + 3 + 9 + 5 + 7 + 4 = 63 pairs of relatively prime numbers among the integers from 1 through 10. Therefore, the probability that the numbers chosen by Andrew and Beth will be relatively prime is **63/100**.

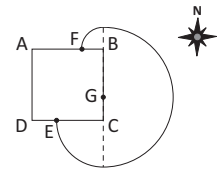
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
10	5	7	5	8	3	9	5	7	4

206. Because the number of bacteria increases by 50% every 3 hours, after 3 hours, the number of bacteria is 1.5 times what it was 3 hours ago. Initially, the number of bacteria is 960. After 3 hours, the number of bacteria would have increased to 1.5 × 960 = 1440. After 6 hours, it would have increased to 1.5 × 1440 = 2160 bacteria. After 9 hours, it would have increased to 1.5 × 2160 = 3240 bacteria. Finally, after **12** hours, the number of bacteria would have increased to 1.5 × 3240 = 4860 bacteria.

207. The volume of clay is 10 × 10 × 10 = 1000 cm^3. With radius 2 cm, each sphere has volume $(4/3) \times \pi \times 2^3 = (4/3) \times \pi \times 8 = (32/3)\pi$ cm^3. Dividing, $1000 \div (32/3)\pi \approx 29.84$. This means there is enough clay to make **29** spheres of radius 2 cm, but not quite enough to make a 30th sphere.

208. There are 60 seconds per minute, 60 minutes per hour, 24 hours per day and 7 days per week. So in 6 weeks, there are 6 × 60 × 60 × 24 × 7 seconds. We are looking for n! equivalent to this amount. We know that 24 = 4!, so $n > 4$. The 6 and 7 in the product pretty much guarantee that $n > 7$. But we'll still rewrite the expression 6 × 60 × 60 × 7 as a product of primes and see how many consecutive integers greater than 4 are in the product. Since 6 = 2 × 3, 7 is prime and 60 = 2 × 2 × 3 × 5, we have 6 × 60 × 60 × 7 = 2 × 2 × 2 × 2 × 2 × 3 × 3 × 3 × 5 × 5 × 7. In the expression below, the primes have been regrouped to show that 6 × 60 × 60 × 24 × 7 = 10!, and n = **10**. Of course we could take a more direct approach and multiply 6 × 60 × 60 × 24 × 7 to get 3,628,000. Then we'd need to determine which n! is closest to this value.

$$\underbrace{2\times 2}\times 5\times\underbrace{2\times 3}\times 7\times\underbrace{2\times 2\times 2}\times\underbrace{3\times 3}\times\underbrace{2\times 5}$$
$$1\times 2\times 3\times 4\times 5\times 6\times 7\times 8\times 9\times 10$$



209. The figure shows how the goat can cover the area of one-half of a circle with a radius of 3 m to the east of the building. Then to the north of the building, it can cover the area of one-quarter of a circle with radius of 1 m. Finally, the goat can cover the area of one-quarter of a circle with a radius of 2 m. The total area that can be covered by the goat is $[(1/2) \times \pi \times 3^2] + [(1/4) \times \pi \times 2^2] + [(1/4) \times \pi \times 1^2] = 9\pi/2 + \pi + \pi/4 = (9/2 + 1 + 1/4) \times \pi = (23/4) \times \pi \approx$ **18** m^2.

210. A single-digit number has one or two prime factors (otherwise, it is 1 or 0, and 1 is relatively prime to every integer, while 0 is not relatively prime to any integer). Suppose the first digit of our number is pq, the product of two primes p and q. Then the second digit of our number has to share one of these factors, say p, so that it is not relatively prime to pq. Suppose the second digit is just the prime p. Then there is no possibility for the third digit, which must share a prime factor with p but not with pq. This means the second digit must be pr, the prime p times a different prime factor r. Now the third digit can be rs, bringing in another prime s, so that it shares a prime factor with pr but not with pq. This logic might continue, leading to a fourth digit st, which is again the product of two primes, except that we quickly run out of small prime factors, and these products of two distinct primes cannot be single-digit numbers. In fact, the only single-digit number that is a product of two distinct primes is 6 = 2 × 3. This means that we must place 6 in the middle and place single-digit numbers on either side that have either the common factor of 2 or the common factor of 3. The options are 263, 362, 364, 463, 368, 863, 269, 962, 469, 964, 869 and 968. The greatest is **968**.

Warm-Up 15

211. An increase in water depth of 1 foot (from 34 feet to 35 feet) took 12 – 8 = 4 hours. So, we would expect an increase in water depth of 2 feet of water (from 35 feet to 37 feet) to take twice as long, or 2 × 4 = 8 hours. The total elapsed time would be 12 + 8 = **20** hours.

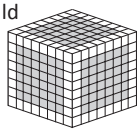
212. If we work this problem backwards we find that the closing price on Wednesday was 72,000 ÷ 10 = \$7200, the closing price on Tuesday was 7200 ÷ 3 = \$2400, and the closing price on Monday was 2400 ÷ 2 = \$**1200**.

213. We know each lateral face of the pyramid is an isosceles triangle with two sides of length 50 yards and a base of 60 yards. If we draw a line on a lateral face from the apex of the pyramid and perpendicular to the base edge of the pyramid, we create two right triangles. Now each triangle has a shorter leg of 30 yards and a hypotenuse of 50 yards. Notice that the length of the longer leg of this triangle has length 40 yards, since its side lengths are a multiple of the 3-4-5 Pythagorean Triple. We are asked to determine the total lateral surface area, in square feet. Before substituting these values into the area formula, let's convert our yards to feet: 60 yards is 60 × 3 = 180 feet, and 40 yards is 40 × 3 = 120 feet. Now each of the four lateral faces has area (1/2) × 180 × 120 = 10,800 ft^2. Thus, the total surface area of all four lateral faces is 4 × 10,800 = **43,200** ft^2.

214. There are five lines in the star. There are four ways to place markers on three of the four points on each line. Therefore, there are 5 × 4 = **20** arrangements with which a player can win the game.

215. Using the laws of exponents, we can simplify the expression to get $2^k = (64^8)/(32^{-3}) = [(2^6)^8]/[(2^5)^{-3}] = (2^{48})/(2^{-15}) = 2^{48} \times 2^{15} = 2^{63}$. Thus, k = **63**.

216. Since the cube can be cut into n^3 unit cubes, the volume of the cube is n^3 cubic units, and the edge length of the cube is $\sqrt[3]{(n^3)} = n$ units. So, in finding the value of n, we are actually trying to determing the cube's edge length. The unit cubes that are painted on only one face would be located on each face of the original cube, away from its edges. If we divide 216 by 6, we see that there must be 216 ÷ 6 = 36 of these cubes, covering an area of 6 units by 6 units, on each face. That would make the actual dimensions of each face of the cube 8 units by 8 units. Therefore, the value of n is **8**.



217. A line tangent to a circle must be perpendicular to the radius that intersects the point of tangency. This radius has endpoints (1, 4) and (3,7), and its slope is (7 – 4)/(3 – 1) = 3/2. The slope of the tangent line, which is perpendicular to this radius, must be the negative reciprocal of 3/2, which is **–2/3**.

218. When the output of a function is then used as an input, the process is called "iteration" and some very interesting mathematics can result. In this case, we want the 13th iteration of the input 13. Beginning with $f(13)$, we get $f(13) = 14 \rightarrow f(14) = 15 \rightarrow f(15) = 16 \rightarrow f(16) = 4 \rightarrow f(4) = -8 \rightarrow f(-8) = 64 \rightarrow f(64) = 8 \rightarrow f(8) = -6 \rightarrow f(-6) = 36 \rightarrow f(36) = 6 \rightarrow f(6) = -7 \rightarrow f(-7) = 49 \rightarrow f(49) =$ **7**.

219. One might conclude that the maximum number of days between ides would occur when there is an ides on the 13th of a 31-day month, followed by an ides on the 15th of the next month. However, this never occurs. Each of the months before March, May, July and October (months in which ides occur on the 15th) is a short month, with 28, 29 or 30 days. The best we can do is a 30-day month in which ides occur on the 13th followed by a month in which ides occur on the 15th. From the 14th to the 30th is 17 days, and from the 1st to the 14th of the following month is 14 days. That's 17 + 14 = **31** days. This exact scenario occurs between the ides in April and May, June and July, and September and October.

220. For each triangle, we know the length of the shorter leg is a = 9 units, so let b and c represent the lengths of the longer leg and hypotenuse, respectively. Substituting into the Pythagorean Theorem, we get $9^2 + b^2 = c^2 \rightarrow 9^2 = c^2 - b^2 \rightarrow 81 = (c - b)(c + b)$. We are looking for two integers, $c - b$ and $c + b$, whose product is 81. The factor pairs of 81 are 3 × 27, 9 × 9 and 1 × 81. We can exclude the pair 9 × 9 since $c - b \neq c + b$. Adding the equations $c - b = 3$ and $c + b = 27$, we get $2c = 30$. So c = 15 and b = 12, giving us the 9-12-15 Pythagorean Triple. Next, adding $c - b = 1$ and $c + b = 81$, we get $2c = 82$. So c = 41 and b = 40, giving us the 9-40-41 Pythagorean Triple. For these two right triangles, the sum of their longer legs is 12 + 40 = **52** units.

Warm-Up 16

221. It happens that a prime denominator leads to a special kind of repeating decimal pattern in which the digits of the second half of the pattern are the 9s complement of the corresponding digits in the first half of the pattern, whenever the number of digits in the pattern is even. (This is known as Midy's Theorem.) The m in the first half corresponds to a 1 in the second half, so m must be 9 – 1 = **8**. We could also just divide a 1 by 17 to confirm that m = 8.

222. Triangle PQR is a 30-60-90 triangle. Serious Mathletes should know well the ratios of the sides of 30-60-90 triangles. If we say that the length of the hypotenuse PR is 1 unit, then the length of the short leg PQ is 1/2 unit. (We know this because two 30-60-90 triangles placed side by side can make an equilateral triangle.) The length of the long leg QR is (√3)/2 units. (We can confirm this with the Pythagorean Theorem as follows: $(1/2)^2 + ((\sqrt{3})/2)^2 = 1/4 + 3/4 = 1$.) We know $m\angle PCQ$ = 90 degrees, and we'll let $m\angle PQC = a$, which means $m\angle CPQ = 90 - a$. Notice that the terminal side for angles PQC and RQD form a line. Since $m\angle PQC = a$, and $m\angle PQR$ = 90 degrees, it follows that $m\angle DQR = 90 - a$ and $m\angle DRQ = a$. So, triangles PQC and QRD are similar, and PQ and QR are corresponding sides. The ratio of the areas of similar triangles is proportional to the square of the ratio of their sides. Therefore, the ratio of the areas of triangles PQC and QRD is $(1/2)^2:((\sqrt{3})/2)^2 = (1/4):(3/4) =$ **1/3**.

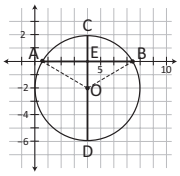
223. Seeing $n^2 - n$ as simply the difference between the square of a number and the number itself, some students will recognize that $8^2 - 8 = 64 - 8 = 56$ works, and they will conclude that n = 8. Alternatively, some students may factor out an n as follows: $n^2 - n = n(n - 1)$. The expression on the right shows the product of two consecutive numbers with the larger number first. Again, some students will realize that 8 × 7 works, so n = 8. Other students will write the quadratic equation $n^2 - n = 56 \rightarrow n^2 - n - 56 = 0$ and then find two numbers with a product of −56 and a difference of 1. Doing so likely will lead to the factored quadratic $(n - 8)(n + 7) = 0$, with solutions n = 8 or –7. The number of teams can't be negative, so there must be **8** teams in the league.

224. Odds are part-to-part ratios and probabilities are part-to-whole ratios. Since the odds against the Patriots playing are 4:1, the probability that they will not play is 4/5, and the probability that they will play is 1/5. Since the odds against the Texans playing are 7:1, the probability that they will not play is 7/8 and the probability that they will play is 1/8. Because the two events are independent, the probability that these two teams will meet in the championsip game is (1/5) × (1/8) = **1/40**.

225. Many Mathletes will have learned how to use the prime factorization of a number to determine how many factors the number has. The prime factorization of 144 is $2^4 \times 3^2$. We increase each exponent by 1 and multiply 5 × 3 to determine that there are 15 positive factors of 144. These factors can be made into 7 pairs of 2 different numbers with a product of 144 and one pair that is a repeat factor, namely (12, 12). Although the commutative property tells us that the product *xy* is equal to the product *yx*, the ordered pair (*x*, *y*) is not the same as the ordered pair (*y*, *x*). So, each of the 7 pairs with 2 different factors of 144 can be written in two different ways, which means there are 2 × 7 + 1 = 15 ordered pairs of positive integers (*x*, *y*) that satisfy *xy* = 144. And don't forget the 15 ordered pairs in which both values are negative. That brings the total to **30** ordered pairs in all.

226. Since one person must row the boat back across the river, only two people can be ferried across the river on each trip until the final trip, when all three people can stay on the other side. If we subtract these last three people from the nine people in the group, we get six people who must be ferried across two at a time. That's 3 round-trips across the river, for a total of 6 times across. Add to that the final trip, with the last three people, and we see that the boat must cross the river at least 6 + 1 = 7 times. Each time across the river, the boat travels 20 yards. So the minimum distance the boat must travel to get nine people across the river is 20 × 7 = **140** yards.

227. Let E be the point (4, 0), where diameter CD intersects the *x*-axis. Point A is 4 units from center O, because the radius is 4 units. Point E is 2 units from center O. Because the length of the hypotenuse of right triangle AEO is twice the length of the shorter leg, we recognize that it is a 30-60-90 right triangle. The measure of ∠AOE is 60 degrees, and by symmetry the measure of ∠BOE is also 60 degrees. Thus, the measure of central angle AOB is 60 + 60 = 120 degrees. The central angle measure is the same as the degree measure of arc ACB. The degree measure of arc ADB, then, is 360 − 120 = 240 degrees, and the ratio of the arc lengths is 120/240 = **1/2**.



228. Michael had to draw either a T and then a Y or a Y and then a T. In the bag described, there are 3 Ts and 1 Y out of 56 tiles, so the probability of getting a T and a Y when 2 tiles are randomly selected from the bag of tiles is 3/56 × 1/55 + 1/56 × 3/55 = 6/3080 = **3/1540**.

229. Using only the 5-gallon and 20-gallon jugs, we can measure 5, 10, 15, 20 or 25 gallons of water. The sum of these values is **75** gallons.

230. We'll begin on June 30th and work backward to determine the number of cars in Noah's sales quota for June. On June 30th, Noah reached his monthly sales quota when he sold 1 more than half of the number of cars he still needed to sell to reach his quota. That means 1 car was half of the number of cars Noah needed to sell to reach his quota. So, on June 30th, Noah must have started the day with 2 cars left to sell in June, and after he sold 2 cars he had 0 cars left to sell. On June 29th, Noah started the day with (2 + 1) × 2 = 3 × 2 = 6 cars left to sell in June, and he ended the day after having sold 1 more than half that, or (6 ÷ 2) + 1 = 3 + 1 = 4 cars. On June 28th, Noah started the day with (6 + 1) × 2 = 7 × 2 = 14 cars left to sell in June, and he ended the day after having sold 1 more than half that, or (14 ÷ 2) + 1 = 7 + 1 = 8 cars. So, to recap, on June 28th, 29th and 30th, Noah sold 8 cars, 4 cars and 2 cars, respectively. So, each day, Noah sold half as many cars as he sold the previous day. The table shows the number of cars Noah sold each day, beginning with June 30th and working back to June 25th. So, Noah's sales quota for June was 2 + 4 + 8 + 16 + 32 + 64 = **126** cars.

Day of June	30th	29th	28th	27th	26th	25th
Number of Cars Sold	2	4	8	16	32	64

Workout 8

231. The median of an arithmetic sequence is the same as the mean, so the sum of the six numbers must be 20.75 × 6 = **124.5**. We could go a step further and find all six numbers. If we use *d* to represent the constant difference between terms, the six terms are 12, 12 + *d*, 12 + 2*d*, 12 + 3*d*, 12 + 4*d* and 12 + 5*d*. Their sum is 12 × 6 + 15*d*, which has to equal 124.5. This means 15*d* = 124.5 – 72 = 52.5. Dividing 52.5 by 15, we find that *d* = 3.5. The numbers must be 12, 15.5, 19, 22.5, 26, and 29.5.

232. Narrowing down 51 contestants to a first-, second- and third-place finalist can occur in 51 × 50 × 49 = **124,950** ways.

233. We can think of the times to run each mile as a geometric sequence with 26 terms, with the first term being 5 and with the common ratio 1.01. There is a formula we can use to determine the sum of these 26 terms: $S_n = \frac{a_1(1 - r^n)}{1 - r}$, where S_n is the sum of the first *n* terms, a_1 is the first term and *r* is the common ratio. In this case we have *a* = 5, *n* = 26 and *r* = 1.01. Substituting, we have $S_{26} = \frac{5(1 - 1.01^{26})}{1 - 1.01} \rightarrow S_{26} = \frac{5(1 - 1.01^{26})}{-0.01} \rightarrow$ $S_{26} = 500(1.01^{26} - 1) \rightarrow S_{26} \approx$ **148** minutes.

234. First, let's consider combinations of coins that include one coin worth 32¢. There are 7 such combinations.

32 + 16 + 2 | 32 + 8 + 8 + 2 | 32 + 8 + 4 + 4 + 2 | 32 + 8 + 4 + 2 + 2 + 1 + 1
32 + 16 + 1 + 1 | 32 + 8 + 8 + 1 + 1 | 32 + 8 + 4 + 4 + 1 + 1

Next, let's consider combinations of Bina's coins that include two coins worth 16¢ each. There are 5 such combinations.

16 + 16 + 8 + 8 + 2 | 16 + 16 + 8 + 4 + 4 + 2 | 16 + 16 + 8 + 4 + 2 + 2 + 1 + 1
16 + 16 + 8 + 8 + 1 + 1 | 16 + 16 + 8 + 4 + 4 + 1 + 1

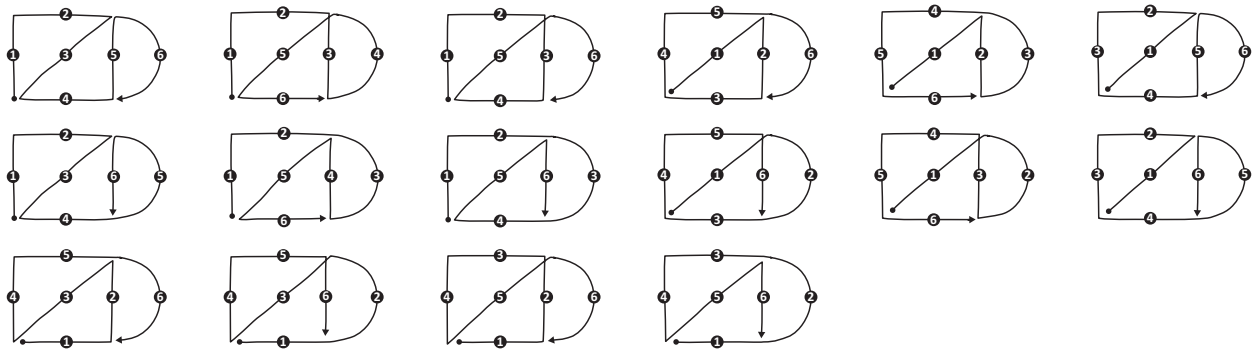
So, there are 7 + 5 = **12** combinations of Bina's coins that have a combined value of 50¢.

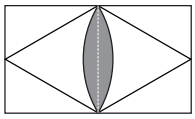
235. The perimeter of the original photo is 2 × (8 + 10) = 2 × 18 = 36 inches. The area of the original photo is 8 × 10 = 80 in^2. After John reduces the size of the photo, the new perimeter will be 27/36 = 3/4 that of the original photo. In fact, each of the new photo's linear measurements will be 3/4 that of the original. The area of the new photo will be $(3/4)^2$ = 9/16 that of the original photo. So, the area of the new photo will be 80 × 9/16 = **45** in^2.

236. If we let *w* represent the width of the each rectangle, then the length is 3*w* + 1 and the perimeter is 2 × (*w* + 3*w* + 1) = 8*w* + 2. The perimeter is less than or equal to 150 cm. We can write the inequality 8*w* + 2 ≤ 150. Solving for *w*, we have 8*w* ≤ 148 → *w* ≤ 18.5 cm. Thus, the maximum width is **18.5** cm.

237. To calculate with percents, we convert them to decimals (or to fractions) and the word *of* translates to multiplication, so 2% of 5% of 3*x* is 0.02 × 0.05 × 3*x* = 0.003*x*. This is three thousandths of *x*, or **0.3**% of *x*.

238. The figure can be drawn in one continuous pen stroke only if it begins at D and ends at C, or vice versa. They are the only two points that have an odd number of paths in and out. At the starting point, there is one more path departing than arriving, while at the ending point there is one less path departing than arriving. This explains why the number of paths in and out of these points must be odd. If a point is not the beginning or end of our stroke, the path must depart that point the same number of times it arrives at the point. Shown here are the 16 continuous pen strokes starting at D and ending at C. But since we also can make these 16 continues pen strokes by starting at C and ending at D, that's another 16 ways, for a total of **32** ways.





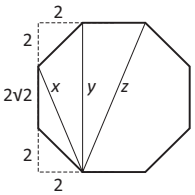
239. If we draw a vertical line to cut the shaded region in half, we can see that each half is the difference between the area of 1/6 of a circle and the area of an equilateral triangle with a side length equal to the radius. The area of a 60-degree sector of a circle with radius 2 units is (1/6) × π × 2^2 = 4π/6 = 2π/3 units^2. The area of an equilateral triangle with a base of 2 units and height of √3 units is (1/2) × 2 × √3 = √3 units^2. The difference is 2π/3 – √3 units^2. This is only half the shaded area, so we need to double it to (4π/3) – 2√3, which is approximately **0.72** units^2.

240. The probability that at least two people pick the same integer is 1 – *P*(all four people pick different numbers). The first person can pick any number. The second person has a 9/10 chance of picking a different number, the third person has an 8/10 chance of picking a different number and the fourth person has a 7/10 chance of picking a different number. The probability that all four pick different numbers is 1 × 9/10 × 8/10 × 7/10 = 504/1000. The probability that at least two people pick the same number is 1 – 0.504 = 0.496 or **49.6**%.

Warm-Up 17

241. We let *n* represent the number of students with one or more younger siblings. Then we can set up the proportion 6/5 = *n*/180. Cross-multiplying and solving for *n*, we see that 5*n* = 6 × 180 → 5*n* = 1080 → *n* = **216** students at the school have one or more younger siblings. Another way to interpret the given ratio is that the number of students who have one or more younger siblings is 6/5 of the number of students who do not. So, (6/5) × 180 = **216** students have one or more younger siblings.

242. An octagon has 8 vertices, and each vertex shares a diagonal with each of 5 other vertices (excludes the vertex itself and the two adjacent vertices). Although there are 8 × 5 ÷ 2 = 20 diagonals in an octagon, there are only three possible lengths for these diagonals, as shown. We'll let *x*, *y* and *z* represent these three lengths, in inches. There are 8 diagonals of length *x* inches, 8 diagonals of length *y* inches and 4 diagonals of length *z* inches. We are asked to find the median length, which, in this case, is the average between the 10th and 11th lengths in an ordered list of the 20 lengths. Since *x* < *y* < *z*, were we to actually write out the lengths of the 20 diagonals in ascending order, we would observe that the 10th length is *y* inches, as is the 11th length. So, we need to determine the value of *y*. All we know is the side length of the octagon, 2√2 inches. Let's imagine the octagon is inscribed in a square, as shown. Then each of the four corners of the square, located on the exterior of the octagon, is a 45-45-90 right triangle with hypotenuse of length 2√2 inches. It follows, then, that the triangle's legs have length 2 inches. Thus, the inscribing square has side length 2 + 2√2 + 2 = 4 + 2√2 inches. Since the diagonal of length *y* inches is the same length as a side of the square, we now know that *y* = **4 + 2√2** inches, which is the median length of all the diagonals.



243. To stamp the integers 1 through 19, Kevin and Evan will make 12 impressions of the digit 1 and 2 impressions of each of the digits 2 through 9. To stamp the integers from 20 throgh 29, they will make an impression of the digits 0 through 9 for the units digit of each integer, and they will make an additional 10 impressions of the 2 for the tens digit of each integer. If they continue in this manner, after they stamp the integer 99, they will have exhausted all 20 impressions for each stamp except the stamp for the digit 0. Therefore, the last number Kevin and Evan can stamp is **99**.

244. There are 4! = 4 × 3 × 2 × 1 = 24 ways to place the four operation symbols in the four boxes. Since the numbers cannot be reordered and no grouping symbols are permitted, we will attempt to minimize the absolute value by placing the multiplication sign in the space between the 1 and 2. Also, we should try the division sign last so that we divide 4 by 5. That leaves two possibilities for placing the addition and subtraction signs into the expression. Our best option to minimize the absolute value is inserting the operation symbols to create the expression 1 × 2 – 3 + 4 ÷ 5 = –0.2, which has an absolute value of **0.2**. We have included a table which shows that none of the other possible 23 values is less than 0.2.

1 + 2 – 3 × 4 ÷ 5 = 0.6	1 – 2 + 3 × 4 ÷ 5 = 1.4	1 × 2 + 3 – 4 ÷ 5 = 4.2	1 ÷ 2 + 3 – 4 × 5 = –16.5
1 + 2 – 3 ÷ 4 × 5 = –0.75	1 – 2 + 3 ÷ 4 × 5 = 2.75	1 × 2 + 3 ÷ 4 – 5 = –2.25	1 ÷ 2 + 3 × 4 – 5 = 7.5
1 + 2 × 3 – 4 ÷ 5 = 6.2	1 – 2 × 3 + 4 ÷ 5 = –4.2	$\boxed{1 \times 2 - 3 + 4 \div 5 = -0.2}$	1 ÷ 2 – 3 + 4 × 5 = 17.5
1 + 2 × 3 ÷ 4 – 5 = –2.5	1 – 2 × 3 ÷ 4 + 5 = 4.5	1 × 2 – 3 ÷ 4 + 5 = 6.25	1 ÷ 2 – 3 × 4 + 5 = –6.5
1 + 2 ÷ 3 – 4 × 5 = –18.33	1 – 2 ÷ 3 + 4 × 5 = 20.33	1 × 2 ÷ 3 + 4 – 5 = –0.33	1 ÷ 2 × 3 + 4 – 5 = 0.5
1 + 2 ÷ 3 × 4 – 5 = –1.33	1 – 2 ÷ 3 × 4 + 5 = 3.33	1 × 2 ÷ 3 – 4 + 5 = 1.67	1 ÷ 2 × 3 – 4 + 5 = 2.5

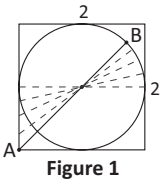


Figure 1

245. As Figure 1 shows, the distance from the circle's center to the circle remains r = 1 unit. However, the distances from the center of the circle to the square increase as we approach the corner of the square. Thus, the greatest distance is AB. By drawing two radii that intersect the square at the points of tangency on two adjacent sides of the circumscribing square, as shown in Figure 2, a small unit square is created. With sides of length 1 unit, the diagonal of this unit square has length √2 units. The greatest distance between a point on the circle and a point on the square is **1 + √2** units.

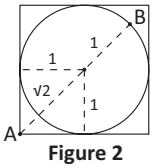
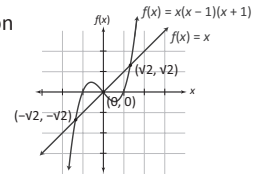


Figure 2

246. We want to find values of x that satisfy the equation $x(x - 1)(x + 1) = x$. In general, dividing both sides of an equation by x should be avoided. A contradiction or a false solution could result since the value of x could, in fact, be 0. To play it safe, we'll expand the left side of the equation and then subtract x from both sides. Doing so yields $x(x - 1)(x + 1) = x \rightarrow x(x^2 - 1) = x \rightarrow x^3 - x = x \rightarrow x^3 - 2x = 0 \rightarrow x(x^2 - 2) = 0$. So, $x = 0$ or $x^2 - 2 = 0 \rightarrow x^2 = 2 \rightarrow x = \pm\sqrt{2}$. Since we were asked to consider only nonzero real numbers for x, we will disregard the solution $x = 0$ and determine the value of the product $(\sqrt{2}) \times (-\sqrt{2}) = \mathbf{-2}$. The graph shows the intersection of the cubic function $f(x) = x(x - 1)(x + 1)$ and the line $f(x) = x$.



247. We will let p, q and r represent the side lengths of the triangle, with $p < q < r$. We know that $p:q:r$ = 3:4:6 and $p + q + r$ = 26 inches. Let's suppose p = 3 inches, q = 4 inches and r = 6 inches. Then the perimeter would be 3 + 4 + 6 = 13 inches, which is half of 26 inches. If we double each of the three side lengths, we have (3 × 2) + (4 × 2) + (6 × 2) = 6 + 8 + 12 = 26 inches. Therefore, the length of the longest side is **12** inches.

248. It appears that Marlow walks twice as fast as the moving sidewalk moves, since it takes her 90/180 = 1/2 of the time to travel the same distance walking alongside the moving sidewalk that it does standing on the moving sidewalk. If she walks on the moving sidewalk, she would be traveling at her walking speed plus the speed of the moving sidewalk, which is half her walking rate. She would be going 3/2 of her normal walking speed, so it would take 2/3 the time, or 2/3 × 90 = **60** seconds.

249. We will let f, t and w represent the number of five-, ten- and twenty-dollar bills, respectively. From the information provided, we can write several equations. First, since Sam received \$440, we have $5f + 10t + 20w = 440$. Next, since he received 30 bills, it follows that $f + t + w = 30$. And finally we have $4f = w$ because Sam received four times as many twenty-dollar bills as five-dollar bills. Substituting $4f$ for w in the first equation yields $5f + 10t + 20(4f) = 440 \rightarrow 5f + 10t + 80f = 440 \rightarrow 85f + 10t = 440$. Similarly, substituting $4f$ for w in the second equation yields $f + t + 4f = 30 \rightarrow 5f + t = 30$. If we multiply each term in the equation $5f + t = 30$ by 10, we get $50f + 10t = 300$. Now we can subtract the two equations to get $(85f + 10t) - (50f + 10t) = 440 - 300 \rightarrow 35f = 140 \rightarrow f = 4$ five-dollar bills. If Sam received 4 five-dollar bills, it follows that $5 \times 4 + t = 30 \rightarrow 20 + t = 30 \rightarrow t = 10$. So, Sam received **10** ten-dollar bills.

250. We only need to find one three-letter word to rule out a number. The number cannot be 222, since this can be used to spell *CAB*. It cannot be 223, since this can spell *BAD*. It cannot be 224, since this number can be used to spell *BAG*. When we get to **225**, however, there are no words among the 3 × 3 × 3 = 27 possible three-letter selections, which follow: *AAJ*, *AAK*, *AAL*, *ABJ*, *ABK*, *ABL*, *ACJ*, *ACK*, *ACL*, *BAJ*, *BAK*, *BAL*, *BBJ*, *BBK*, *BBL*, *BCJ*, *BCK*, *BCL*, *CAJ*, *CAK*, *CAL*, *CBJ*, *CBK*, *CBL*, *CCJ*, *CCK*, *CCL*.

Warm-Up 18

251. There are 12 girls and 10 boys in Mrs. Rodriguez's class. Looking at the two graphs, the only way a girl and a boy can be chosen with the same number of letters in their names is if those chosen have first names with 5, 9, 10 or 11 letters. There are no students in Mrs.Rodriguez's class that have first names with only 1 letter or with 3 letters. While one or more girls in the class have a first name with 2, 4, 8 or 12 letters, no boys do. And though some boys in the class have a first name with 6 or 7 letters, no girls do. The table shows the probabilities of choosing a girl with N letters in her name, choosing a boy with N letters in his name, and choosing a girl and a boy, both with N letters in their names, where N is 5, 9, 10 or 11. The probability of the girl and boy who are chosen having the same number of letters in their names is 1/120 + 1/120 + 4/120 + 2/120 = 8/120 = **1/15**.

	N			
	5	**9**	**10**	**11**
P(Girl & *N* letters)	1/12	1/12	2/12	2/12
P(Boy & *N* letters)	1/10	1/10	2/10	1/10
P(Girl, Boy & *N* letters)	1/120	1/120	4/120	2/120

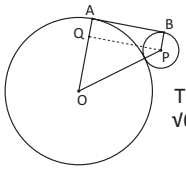
252. The domain of a function is the set of input values for which the function produces an output value. Typically it is described as the set of all x for which $f(x)$ exists. The given function has $\sqrt{(12 - x)}$ in the numerator and $4 - x^2$ in the denominator, and we are asked to determine for how many positive integers $f(x)$ exists. In the numerator, we want to avoid getting a negative number underneath the radical. To do that, we must restrict the domain to values that satisfy $x \leq 12$. We also want to avoid getting a denominator of zero. That means x cannot equal 2 or –2. Thus, the **11** positive integers in the domain of $\sqrt{(12 - x)}/(4 - x^2)$ are 1, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12.

253. The possible even sums greater than 7 are 8, 10 and 12. If (a, b) represents an ordered pair of dice numbers with these sums, the 9 possible rolls are: (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (4, 6), (5, 5), (6, 4) and (6, 6). Both dice are even in 6 out of 9 of these, so the probability is 6/9 = **2/3**.

254. The surface of a cylinder consists of two circular bases and a rectangular wrap around the middle. The area of the bases is $2 \times \pi \times r^2 = 2\pi r^2$, and the area of the rectangle is the height of the cylinder times the circumference of the base, which is $h \times 2\pi r$. In this case, the height is equal to the radius, so the rectangle's area is $2\pi r^2$. The surface area is $2\pi r^2 + 2\pi r^2 = 4\pi r^2$ units^2. Since the surface area is 100π units^2, we have $4\pi r^2 = 100\pi$, which means $r^2 = 25 \rightarrow r = 5$ units. The volume of the cylinder is the area of the base times the height, so we get $\pi \times 5^2 \times 5 =$ **125π** units^3.

255. In the quadratic equation $x^2 + nx + 24 = 0$, the value of n is the sum of two numbers whose product is 24. The 4 possibilities involving positive integers are 1 + 24 = 25, 2 + 12 = 14, 3 + 8 = 11 and 4 + 6 = 10. Therefore, the probability that a number n chosen at random from the set will lead to integer solutions is **4/25**.

256. Silas's clock runs 5 quarters of an hour in 4 quarters of an hour, so the 6 hours that his clock shows from 10:00 p.m. to 4:00 a.m. was really only 6 × 4/5 = 24/5 = 4 4/5 hours. The actual time at the moment the clock stopped was 2:48 a.m. Silas woke up 4 hours later, which was **6:48** a.m.



257. We will draw segments OA, OP and BP. We will also draw a segment PQ, which is parallel to segment AB and 2 cm below it. Since tangent line AB is perpendicular to radius OA, and segments QP and AB are parallel, we know that △OQP is a right triangle. The length OQ is 8 – 2 = 6 cm, and OP is 8 + 2 = 10 cm. Using the Pythagorean Theorem, we find that QP is $\sqrt{(10^2 - 6^2)} = \sqrt{(100 - 36)} = \sqrt{64} = 8$ cm. Since segment PB is perpendicular to segments AB and QP, segment AB is the same length as segment QP, so it's **8** cm.

258. There are ${}_6P_3 = 6 \times 5 \times 4 = 120$ ways to pick three of the integers when order matters. If we pick the 1 first, the increasing arithmetic sequences are 1, 3, 5 or 1, 5, 9. If we pick the 3 first, we can get 3, 5, 7 or 3, 7, 11. If we pick the 5 first, we can get 5, 7, 9. If we pick the 7 first, we can get 7, 9, 11. Each of these 6 increasing sequences can be reversed to make a decreasing sequence, so there are 12 possible arithmetic sequences. The probability is thus 12/120 = **1/10**.

259. The difference between consecutive terms must always be the same, so $6 - p = (2p - 3) - 6$. Solving this for p, we get $6 - p = 2p - 9$, then $15 = 3p$ and finally $p = 5$. Plugging this value in, we find that the first three terms of the sequence are simply 5, 6 and 7. The common difference is 1. Thus, the tenth term must be 5 + 9 = **14**.

260. We might notice first that the digit represented by R must be small, either 1 or 2, since multiplying this four-digit number by 4 does not produce a five-digit number. But no multiple of 4 can end in 1, so R = 2. Now we notice that 4 × S ends in a 2, so S is either 3 or 8. It can't be 3, since 4 times the 2 in the thousands place is at least 8000. This means that S = 8 and that A must be small so that nothing carries from the hundreds to the thousands place. Now we consider the T in the tens place of RATS. This will be multiplied by 4 and then 3 will be added (from the carry) to produce the A in STAR, so A must be odd. Since A also has to be small, it can only be 1. Since the digit 2 has been taken, we find that T must equal 7 so that 4 × 7 + 3 leaves a 1 in the tens place. Everything now works, and the multiplication problem is 2178 × 4 = 8712. The sum S + T + A + R is 8 + 7 + 1 + 2 = **18**.

Workout 9

261. Notice that when the result of $m - 34$ is subtracted from the result of $263 - n$, the result is 104. So we have $(263 - n) - (m - 34) = 104$. When we simplify this equation, we get $263 - n - m + 34 = 104 \rightarrow 297 - n - m = 104 \rightarrow n + m =$ **193**.

262. We know that 1024 is 2^{10}. It can also be written as $(2^2)^5 = 4^5$ or as $(2^5)^2 = 32^2$. Since we must satisfy $a < b < c$, we must have $a = 1$, $b = 3$ and $c = 5$, so we get $(1 + 3)^5 = 1024$. The value of b is **3**.

263. If we substitute 200 for d into the formula, we get $t = 0.45\sqrt{200} \approx 6.36396 \approx$ **6.4** seconds.

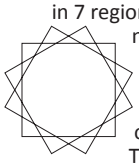
264. The two semicircles of radius 2 units can be put together to form a full circle with an area of $\pi \times 2^2 = 4\pi$ units^2. The shaded area in the middle would be 2 × 4 = 8 units^2, but two semicircles of radius 1 have been removed, for a loss of π units^2. The total area is $4\pi + 8 - \pi \approx$ **17.42** units^2.

265. To find the surface area of the cone, we first need to find the *slant height*, s, with the help of the Pythagorean Theorem as follows: $s = \sqrt{(5^2 + 1.5^2)} = \sqrt{(25 + 2.25)} = \sqrt{27.25}$ inches. The surface area of the cone is πrs, not including the circular base. The ice cream cone is covered with $\pi \times 1.5 \times \sqrt{27.25} \approx$ **24.6** in^2 of chocolate.

266. The probability of winning on the draw from Bag A is 1/10. If a red ball is drawn from Bag B, it is returned to Bag B, before another ball is drawn. This makes it a bit tricky to directly calculate the probability of winning with Bag B. It's best if we consider that the probability of winning with Bag B equals $1 - P(\text{not winning in 20 draws from Bag B}) = 1 - (99/100)^{20}$. Therefore, the absolute difference between the probabilities of winning with Bags A and B is $|(1/10) - (1 - (99/100)^{20})| = |(-9/10) + (99/100)^{20}| \approx$ **0.082**.

267. We could substitute the 11 values of x into the expression $x^2 - 2x + 1$ to find our answer. But if we recognize that $x^2 - 2x + 1 = (x - 1)^2$, our calculations are much easier. Substituting into either expression, we find that the **7** elements in the set are 0, 1, 4, 9, 16, 25 and 36.

268. In the figure that accompanies the problem, notice that there are 6 distinct points of intersection among the three squares, which results in 7 regions. Futher investigation with these squares will show that n points of intersection result in $n + 1$ regions. So, we need to maximize the number of distinct points of intersection among the squares. To do that, we need to make sure that each square intersects each side of the other two squares, unlike the example provided with the problem. By rotating the squares, we can actually have each square intersect each of the other two squares at two different points per side. It is not possible for a square to intersect either of the other squares at more than two points on a side. So if Square 1 intersects Square 2 at most twice per side, that's 2 × 4 = 8 distinct points of intersection. The same is true for intersections between Square 2 and Square 3 and intersections between Square 1 and Square 3. That's a total of 8 + 8 + 8 = 24 distinct points of intersection, and 24 + 1 = **25** regions, as shown in the figure.



269. Figure 1 shows the sums for the 36 outcomes when rolling two standard, fair dice. The 11 different sums have been circled to illustrate the distribution of the sums. If we number the faces of one die 0, 1, 2, 3, 4 and 5, and number the faces of the other die 2, 3, 4, 5, 6 and 7, we can achieve the same probability distribution for the sums, as shown in Figure 2. The sum of the integers on the faces of one of the nonstandard dice is $a = 0 + 1 + 2 + 3 + 4 + 5 = 15$. The sum of the integers on the faces of the other nonstandard die is $b = 2 + 3 + 4 + 5 + 6 + 7 = 27$. Thus, $a \times b = 15 \times 27 =$ **405**.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Figure 1

+	0	1	2	3	4	5
2	2	3	4	5	6	7
3	3	4	5	6	7	8
4	4	5	6	7	8	9
5	5	6	7	8	9	10
6	6	7	8	9	10	11
7	7	8	9	10	11	12

Figure 2

270. Using only the digits 0, 1, 2 and 4 to create a positive, four-digit integer, there are 3 choices for the thousands digit (1, 2 and 4 only). Then 3 choices remain for the hundreds digit. For the tens digit, there are 2 digits left to choose from, which leaves only one option for the units digit. Using the Fundamental Counting Principle, we see that there are $3 \times 3 \times 2 \times 1 = 18$ different positive, four-digit integers containing just the digits 0, 1, 2 and 4. They are included here in an organized list:

1024	2014	4012
1042	2041	4021
1204	2104	4102
1240	2140	4120
1402	2401	4201
1420	2410	4210

These 18 integers have a sum of 45,108 and an arithmetic mean of $45{,}108 \div 18 =$ **2506**.

Surface Area & Volume Stretch

271. We are given the measurements $l = 34$ inches, $h = 30$ inches and $s = 32$ inches, where l, h and s represent the pyramid's slant height, the pyramid's height and the side length of the base. Since $s = 32$ inches, it follows that $B = 32 \times 32 = 1024$ in^2 and $P = 32 \times 4 = 128$ inches. So, we have $SA = B + (1/2)Pl = 1024 + (1/2) \times 128 \times 34 = 1024 + 2176 =$ **3200** in^2, and $V = (1/3)Bh = (1/3) \times 1024 \times 30 =$ **10,240** in^3.

272. Since the base of the prism is a 4-cm by 5-cm rectangle, it has area $B = 4 \times 5 = 20$ cm^2 and perimeter $P = 2 \times (4 + 5) = 2 \times 9 = 18$ cm. With $h = 10$ cm, we have $SA = 2B + Ph = 2 \times 20 + 18 \times 10 = 40 + 180 =$ **220** cm^2, and $V = Bh = 20 \times 10 =$ **200** cm^3.

273. For this cylinder, $h = 9$ m and $r = 5$ m. So, $SA = 2\pi r^2 + 2\pi rh = 2\pi \times (5^2 + 5 \times 9) = 2\pi \times (25 + 45) = 2\pi \times 70 =$ **140π** m^2, and $V = \pi r^2 h = \pi \times 5^2 \times 9 = \pi \times 25 \times 9 =$ **225π** m^3.

274. For this cone, $h = 12$ cm and $r = 9$ cm. Before we can determine the cone's surface area, we must first find its slant height. The altitude, radius and slant height of the cone form a right triangle in which the values of h, r and l are the lengths of the long leg, short leg and hypotenuse, respectively. We can use the Pythagorean Theorem to determine l. We have $l^2 = 12^2 + 9^2 \rightarrow l^2 = 144 + 81 \rightarrow l^2 = 225 \rightarrow l = \sqrt{225} \rightarrow l = 15$ cm. So, $SA = \pi r^2 + \pi rl = \pi \times (9^2 + 9 \times 15) = \pi \times (81 + 135) =$ **216π** cm^2, and $V = (1/3)\pi r^2 h = (1/3)\pi \times 9^2 \times 12 = (1/3)\pi \times 81 \times 12 =$ **324π** cm^3.

275. With $r = 6$ ft, we have $SA = 4\pi r^2 = 4\pi \times 6^2 = 4\pi \times 36 =$ **144π** ft^2, and $V = (4/3)\pi r^3 = (4/3)\pi \times 6^3 = (4/3)\pi \times 216 =$ **288π** ft^3.

276. The surface area of the pyramid is the sum of its base area and its lateral surface area. In the case of this square pyramid, the area of the base is the square of the side length, or s^2, and its lateral surface area is $(1/2)Pl = (1/2) \times 4s \times 10 = 20s$. Since we are told that the surface area of the pyramid is 96 cm^2, we have $s^2 + 20s = 96 \rightarrow s^2 + 20s - 96 = 0$. This quadratic equation can be solved by factoring to get $(s + 24)(s - 4) = 0 \rightarrow s + 24 = 0$ or $s - 4 = 0$. Since the length cannot be negative, $s =$ **4** cm.

277. We are told that this cylinder with height 10 cm has a volume of 250π cm^3. That means $\pi \times r^2 \times 10 = 250\pi \rightarrow 10r^2 = 250 \rightarrow r^2 = 25 \rightarrow r = 5$ cm. The surface area of the cylinder is $2\pi r^2 + 2\pi rh = 2\pi \times (5^2 + 5 \times 10) = 2\pi \times 75 =$ **150π** cm^2.

278 a. For this pyramid, $B = 64$ cm^2 and $h = 12$ cm. The volume of the pyramid, then, is $V = (1/3)Bh = (1/3) \times 64 \times 12 = 256$ cm^3. If a plane perpendicular to its height slices through the pyramid 3 cm from its apex, the portion of the pyramid above this plane is a smaller pyramid with $h = 3$ cm. We need to detemine the area of this smaller pyramid's base. One very important theorem pertaining to similar figures tells us that if two similar figures have corresponding lengths in the ratio $a:b$, then the ratio of their areas is $a^2:b^2$, and the ratio of their volumes is $a^3:b^3$. Since the heights of the smaller pyramid and the original pyramid are in the ratio $3/12 = 1/4$, it follows that the areas of their bases are in the ratio $1^2/4^2 = 1/16$. Therefore, the smaller pyramid's base has an area that is 1/16 that of the original pyramid, or $1/16 \times 64 = 4$ cm^2. So, the smaller pyramid has volume $V = (1/3)Bh = (1/3) \times 4 \times 3 = 4$ cm^3. Subtracting the volume of the smaller pyramid from the volume of the original pyramid gives us the volume of the frustum. Thus, the volume of the frustum is $256 - 4 =$ **252** cm^3.

b. We can find the surface area of the frustum by subtracting the surface area of the lateral faces of the smaller pyramid from the surface area of the orginal pyramid, and then adding to the result the area of the smaller pyramid's base (which is the same as the area of the top base of the frustum). Before we can determine the surface area of the original pyramid, we need to find its perimeter and slant height. With $B = 64$ cm^2, it follows that the pyramid's base has side length $\sqrt{64} = 8$ cm and perimeter $8 \times 4 = 32$ cm. To find the slant height, we will use the right triangle created by the pyramid's slant height, height and the segment drawn on the base from the center of the base perpendicular to the pyramid's base edge. Using the Pythagorean Theorem, we have $l^2 = 4^2 + 12^2 \rightarrow l^2 = 16 + 144 \rightarrow l^2 = 160 \rightarrow l = \sqrt{160} \rightarrow l = 4\sqrt{10}$ cm. So, for the original pyramid, $SA = B + (1/2)Pl = 64 + (1/2) \times 32 \times 4\sqrt{10} = 64 + 64\sqrt{10}$ cm^2. Again, using the ratio 1/16, we know the smaller pyramid has surface area $(1/16) \times (64 + 64\sqrt{10}) = 4 + 4\sqrt{10}$ cm^2. Notice that 4 cm^2 is the area of the smaller pyramid's base and $4\sqrt{10}$ cm^2 is the combined surface area of the four lateral faces. So from the surface area of the original pyramid, we need to subtract $4\sqrt{10}$ cm^2 and add 4 cm^2 to get the surface area of the frustum. Doing so, we see that the surface area of the frustum is $64 + 64\sqrt{10} - 4\sqrt{10} + 4 =$ **68 + 60√10** cm^2.

279 a. If we find the volume of the original cone and subtract the volume of the smaller cone above the plane that divides the original cone, we'll have the volume of the resulting frustum. With a radius of 18 inches and a height of 24 inches, the original cone has volume $V = (1/3)\pi r^2 h = (1/3)\pi \times 18^2 \times 24 = 2592\pi$ in^3. Since the ratio of the height of the smaller cone to the height of the original cone is 1:2, it follows that the ratio of their volumes is $1^3:2^3 = 1:8$. Therefore, the volume of the smaller cone is $(1/8) \times 2592\pi = 324\pi$ in^3, and the volume of the frustum is $2592\pi - 324\pi =$ **2268π** in^3.

b. The lateral height of the original cone is equivalent to the length of the hypotenuse of a right triangle with legs 18 inches and 24 inches. This right triangle has side lengths that are 6 times the 3-4-5 Pythagorean Triple, namely 18-24-30. So $l = 30$ inches. Recall that the surface area of a cone is the sum of the area of its base and its lateral surface area. For the original cone, we have $\pi r^2 + \pi r l = (\pi \times 18^2) + (\pi \times 18 \times 30) = 324\pi + 540\pi = 864\pi$ in^2. From this we can subtract the lateral surface area of the smaller cone, then add to it the area of the smaller cone's base (area of the top base of the frustum). Again, since the ratio of the radius of the smaller cone to that of the original cone is 1:2, it follows that the ratio of their areas is $1^2:2^2 = 1:4$. Therefore, the smaller cone has base area $(1/4) \times 324\pi = 81\pi$ in^2 and lateral surface area $(1/4) \times 540\pi = 135\pi$ in^2. The surface area of the frustum is $864\pi - 135\pi + 81\pi =$ **810π** in^2.

280. Since the original cone had a diameter of 18 m, its radius was 9 m. We are also told that it had height 9 m, and since it was sliced 3 m from the base, the smaller cone that was removed had height 6 m. The ratio of the heights of the smaller cone and the original cone is 6:9 = 2:3. The volume of the original cone was $(1/3)\pi r^2 h = (1/3)\pi \times 9^2 \times 9 = 243\pi$ m^3. The ratio of the volumes of the smaller cone and the original cone is $2^3:3^3 = 8:27$. Therefore, the smaller cone had volume $(8/27) \times 243\pi = 72\pi$ m^3. So, again to get the volume of the frustum, we simply subtract the volume of the smaller cone from the volume of the original cone to get $243\pi - 72\pi =$ **171π** m^3.

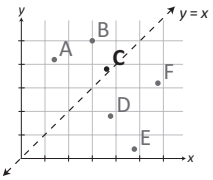
Data & Statistics Stretch

281. The results from the 20 survey respondents have been organized in a line plot. In the line plot, each X represents one survey response. For example, by counting the Xs, we quickly discover that of those pet owners who responded, 4 indicated that they own two pets and 1 reported owning five pets. The median of these 20 responses is the average of the 10th and 11th responses when all are arranged in ascending or descending order. In this case, we can see from the line plot that both the 10th and 11th responses are three pets. Thus, the median is 3. It is quite easy to determine the mode from the line plot, because the response with the most Xs is 3, which means 3 is the mode. Since the median and mode both have a value of 3, it follows that their mean also is **3** pets.

282. We need to determine the mean and median of the 20 roller coaster speeds in the stem-and-leaf plot. Using a calculator, add the 20 values and get a sum of 2378. Therefore, the mean of the roller coaster speeds is $2378 \div 20 = 118.9$ mi/h. As in the previous problem, with 20 values, the median is the average of the two middle values. In this case, the 10th and 11th values are 119 and 120, and their average is 119.5. Thus, the absolute difference between the mean and median is $|118.9 - 119.5| = |-0.6| =$ **0.6** mi/h.

If you don't have a calculator handy, there is a way to make finding the sum of the 20 numbers less time consuming. Instead of adding 90 + 92 + 93 + 95 +..., we can regroup to make the arithmetic a little easier to manage. For each stem, we can multiply its value by the number of leaves, and then add the sum of the leaves. For example, the stem 9, which represents 90, has 5 leaves with a sum of 0 + 2 + 3 + 5 + 6 = 16. So, we have (90 × 5) + 16 = 450 + 16 = 466. Once we have the total associated with each stem, we then can add those to get the sum of the data. While there is still arithmetic to do, this method requires fewer calculations so there are fewer opportunities to make an error.

283. According to the graph, the driver's speed was not constant throughout the hour from 10:30 to 11:30. During that time, the driver's speed decreased from 50 mi/h to 40 mi/h. The driver's speed decreased at a constant rate of change from 10:30 to 11:00. So, the average speed from 10:30 to 11:00 was (50 + 40)/2 = 45 mi/h. Then from 11:00 to 11:30 the driver's speed remained constant at 40 mi/h. From 10:30 to 11:30, the driver's average speed was (45 + 40)/2 = **42.5** mi/h.



284. To determine which point on the graph represents the highest-quality conversation, we need to determine for which point (x, y), the ratio x/y is closest to 1. Since this graph has no scale and we don't know the exact coordinates of any of the points representing coversations A through F, we'll see which point is closest to the line $y = x$. As shown in the figure, point **C** represents the conversation with the highest quality.

285. The mean of the five values equals the sum of the five values, S, divided by 5. Algebraically, we have $S/5 = 27 \rightarrow S = 27 \times 5 \rightarrow S =$ **135**.

286. Again, we'll let S represent the sum of the five numbers. When each of the five numbers is doubled, the sum is also doubled. So, we have $2S/5 = 60 \rightarrow 2S = 300 \rightarrow S = 150$. The sum of the five original numbers was 150, and their mean was 150/5 = **30**. It makes sense that the mean that results when each number is doubled is twice the mean of the original five numbers since $2S/5 = 2(S/5)$.

287. Let S be the sum of the original 20 numbers. We can write $S/20 = 37 \rightarrow S = 740$. When two numbers, a and b, are removed, we have $[S - (a + b)]/18 = 38 \rightarrow S - (a + b) = 684 \rightarrow a + b = S - 684$. We know that $S = 740$, so it follows that $a + b = 740 - 684 = 56$. The average of the two numbers that were removed is $(a + b)/2 = 56/2 =$ **28**.

288. We will use the mean, median and unique mode to draw additional conclusions about these six numbers. Let a, b, c, d, e and f represent the six numbers, in ascending order. Since the sum of the six numbers is $a + b + c + d + e + f = S$, and the mean is 8, we can write $S/6 = 8 \rightarrow S = 48$. We now know that the six numbers have a sum of 48. Since we have a group of six numbers, and there is no middle number, the median is the mean of the two middle numbers. In this case the median, which is 7, is the mean of c and d. We can write $(c + d)/2 = 7 \rightarrow c + d = 14$. So, the sum of the two middle numbers is 14. Lastly, we know that 3 is the unique mode, which means at least two of our six numbers must be 3. Now our goal is to maximize the range of our six numbers, $f - a$. Since all six numbers are positive integers, the least possible value of one of these numbers is 1.

We'll let $a = 1$. Since at least two of the six numbers must be 3, we'll let $b = c = 3$. Since $c + d = 3 + d = 14$, it follows that the value of d must be 11. The sum of the four numbers we've identified is $1 + 3 + 3 + 11 = 18$. That means $e + f = 48 - 18 = 30$. Since we want the value of f to be as large as possible, we'll let the value of e be as small as possible, but not equal to d since 3 is the unique mode and only appears twice. Thus, $e = 12$ and $f = 30 - 12 = 18$. These six numbers 1, 3, 3, 11, 12, 18 have a mean, median and unique mode of 8, 7 and 3, respectively. The maximum range for six positive numbers with these characteristics is $18 - 1 =$ **17**.

289. We are told that the mean of the three terms is 10. In an arithmetic sequence with an odd number of terms, the middle term is both the mean and median of the sequence. If we let d represent the common difference between terms in this sequence, the three terms can be written as $10 - d$, 10, $10 + d$. Since the mean of their squares is 394, we have $[(10 - d)^2 + 10^2 + (10 + d)^2]/3 = 394 \rightarrow (10 - d)^2 + 10^2 + (10 + d)^2 = 1182 \rightarrow 100 - 20d + d^2 + 100 + 100 + 20d + d^2 = 1182 \rightarrow 2d^2 + 300 = 1182 \rightarrow 2d^2 = 882 \rightarrow d^2 = 441 \rightarrow d = \pm 21$. Whether we use $d = -21$ or $d = 21$, the largest of the three original terms is $10 - (-21) = 10 + 21 =$ **31**.

290. Let the six different positive integers be, in ascending order, a, b, c, d, e, f, with a sum of 66. The mean of these six numbers is $66/6 = 11$ and is one of the six integers. Since all six numbers are different, we would expect the 11 to be somewhere near the median of the six numbers, maybe c or d. We want the value of f to be as large as possible, which means the other numbers need to be as small as possible. The least possible value is 1. So, let $a = 1$. We also know the range, $f - 1$, is in the set of six numbers, so $e = f - 1$. Now if we let $d = 11$ and make the values of b and c as small as possible, namely $b = 2$ and $c = 3$, we'll have the set $\{1, 2, 3, 11, f - 1, f\}$. It follows that $1 + 2 + 3 + 11 + f - 1 + f = 66 \rightarrow 17 + 2f - 1 = 66 \rightarrow 2f = 50 \rightarrow f = 25$, and $e = f - 1 = 25 - 1 = 24$. There are a few sets with six different positive integers that have a sum of 66 and that include both the mean and range among the six numbers. The sets {5, 10, 11, 12, 13, 15} and {1, 8, 9, 11, 18, 19} are among those sets, but the set {1, 2, 3, 11, 24, 25} is the set containing the greatest possible number, **25**.

Geometric Proportions Stretch

291. Recall that if two similar figures have corresponding lengths in the ratio $a:b$, then the ratio of their areas is $a^2:b^2$, and the ratio of their volumes is $a^3:b^3$. We know the ratio of the lengths of corresponding edges of the smaller prism to the larger prism is 1:2, so the ratio of their volumes is $1^3:2^3 = 1:8$. That means the volume of the smaller prism is 1/8 of the volume of the larger prism. Therefore, the number of smaller prisms with a combined volume equal to the volume of the larger prism is **8** smaller prisms.

292. Let's number the prisms so we can reference them more easily. From the previous problem, the larger prism will be prism #1, and the smaller prism #2. Prism #2 is divided into prism #3, which is 3/4 of prism #2, and prism #4, which is 1/4 of prism #2. So, the volume of prism #4, V_4, is 1/4 of the volume of prism #2, V_2. In other words, $(1/4)V_2 = V_4 \rightarrow V_2 = 4V_4$. In the previous problem, we found that $V_2 = (1/8)V_1$. Substituting, we have $4V_4 = (1/8)V_1 \rightarrow V_4 = [(1/8)/4]V_1 \rightarrow V_4 = (1/32)V_1$. Thus, the smallest prism has a volume that is **1/32** of the volume of the largest prism. Note that prism #1 and prism #4 are not similar.

293. As a fraction, the ratio of the areas of the small and large circles is 1/9. To determine the ratio of their radii, which is a measure of length, we take the square root of the ratio of their areas. Therefore, the ratio of the radius of the small circle to the radius of the large circle is $\sqrt{1}/\sqrt{9} =$ **1/3**.

294. Since the ratio of the radii of the small and large circles is 1/3, it follows that all other corresponding measures of length have the same ratio. We can set up the proportion $1/3 = c/3.12$, where c represents the length of the S-curve in the small circle. Cross-multiplying and solving, we get $3c = 3.12 \rightarrow c =$ **1.04** cm.

295. Since the two pyramids are similar, we can write the ratio of their corresponding lengths, comparing the new pyramid and the old pyramid, as $30/20 = 3/2$. The ratio of their volumes, then, is $3^3/2^3 =$ **27/8**.

296. From the previous problem, we know that the ratio of the volume of the new pyramid, V_N, to that of the original pyramid, V_O, is $V_N/V_O = 27/8 \rightarrow V_N = (27/8)V_O$. Since the new pyramid's volume is 27/8 of the original pyramid's volume, the difference in their volumes is $(27/8)V_O - V_O = (19/8)V_O$. So, we need to determine the time, t, in years, that it will take to add that volume to the original pyramid, considering it took 16 years to construct the original pyramid. We can set up the proportion $V_O/16 = [(19/8)V_O]/t$. Cross-multiplying and solving for t, we see that $V_O \times t = 16 \times (19/8) \times V_O \rightarrow t = 2 \times 19 =$ **38** years is the time it will take for construction of the larger pyramid to be completed.

297. The sphere has a diameter of 2 inches, so its radius is 1 inch. The volume of the sphere is $(4/3)\pi r^3 = (4/3)\pi \times 1^3 = (4/3)\pi \approx$ **4.19** in^3.

298. The ratio of the diameter of the original sphere to that of the hollow sphere is $2/10 = 1/5$. So, the ratio of their surface areas is $1^2/5^2 =$ **1/25**.

299. We already determined that the volume of the glass is $(4/3)\pi$ in^3. We need to determine the volume of the air in the finished hollow sphere. The ratio of the volume of the original sphere to that of the finished hollow sphere is $1^3/5^3 = 1/125$. Therefore, the volume of the finished hollow sphere is $(4/3)\pi \times 125 = (500/3)\pi$ in^3. Since this is the combined volume of glass and air in the finished hollow sphere, it follows that the volume of the air in the sphere is $(500/3)\pi - (4/3)\pi = (496/3)\pi$ in^3. Therefore, in the finished hollow sphere, the ratio of the volume of glass to the volume of enclosed air is $[(4/3)\pi]/[(496/3)\pi] = 4/496 =$ **1/124**.

300. Each of the triangular faces of the sculpture has area $4617 \div 20 = 230.85$ cm^2. So, since each triangular face on the model has area 2.85 cm^2, the ratio of the area of a face on the model to the area of a face on the actual sculpture is $2.85/230.85 = 1/81$. Thus, the ratio of corresponding lengths on the model and sculpture is $\sqrt{1}/\sqrt{81} = 1/9$, and the ratio of their volumes is $1^3/9^3 = 1/729$. So, we can set up the proportion $34/V = 1/729$, where V represents the volume of the actual sculpture. Cross-multiplying and solving for V, we get $V = 729 \times 34 =$ **24,786** cm^3. Note: The area of each face of a model icosahedron with volume 34 cm^3 is actually closer to 2.50 cm^2. The area 2.85 cm^2 was used to simplify calculations while still reinforcing the geometric proportions concepts.

PROBLEM INDEX

It is difficult to categorize many of the problems in the *MATHCOUNTS School Handbook*. It is very common for a MATHCOUNTS problem to straddle multiple categories and cover several concepts. This index is intended to be a helpful resource, but since each problem has been placed in exactly one category and mapped to exactly one Common Core State Standard (CCSS), the index is not perfect. In this index, the code **9 (3) 7.SP.3** refers to problem 9 with difficulty rating 3 mapped to CCSS 7.SP.3. For an explanation of the difficulty ratings refer to page 54. For an explanation of the CCSS codes refer to page 81.

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54	(3)	N-RN.1
62	(3)	7.RP.2
64	(3)	6.EE.9
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129	(4)	SMP
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86	(3)	6.RP.3
94	(3)	6.RP.3
98	(4)	6.NS.1
121	(3)	6.RP.3
135	(3)	7.G.1
141	(4)	6.G.2
143	(3)	7.RP.3
147	(3)	6.RP.3
153	(3)	7.EE.4
155	(4)	6.RP.3
161	(3)	6.EE.7
182	(2)	6.G.1
194	(4)	6.EE.7
211	(3)	6.RP.3
241	(2)	6.RP.3
247	(3)	6.RP.1
248	(5)	6.RP.3
293	(4)	G-SRT.5
296	(4)	7.RP.1
298	(4)	7.G.6
300	(6)	7.RP.1

Solid Geometry

23	(5)	8.G.7
30	(3)	7.G.6
34	(4)	7.G.6
120	(4)	7.G.6
185	(4)	G-GMD.3
207	(3)	8.G.9
213	(4)	8.G.9
254	(4)	8.G.9
265	(4)	8.G.9
271	(3)	7.G.6
272	(3)	7.G.6
273	(3)	7.G.6
274	(3)	7.G.6
275	(3)	7.G.6
276	(5)	7.G.6
277	(5)	7.G.6
278	(5)	7.G.6
279	(5)	7.G.6
280	(5)	7.G.6
291	(4)	8.G.9
292	(4)	8.G.9
295	(4)	6.RP.3
297	(3)	G-GMD.3
299	(5)	G-GMD.3

Percents & Fractions

17	(4)	7.RP.3
24	(3)	7.RP.3
35	(4)	6.RP.1
46	(3)	6.RP.3
65	(3)	7.RP.3
78	(3)	7.RP.3
79	(3)	7.NS.2
85	(3)	7.RP.3
100	(4)	7.NS.3
111	(3)	7.RP.3
115	(3)	7.RP.3
117	(3)	6.RP.3
122	(2)	6.RP.3
131	(5)	6.RP.3
144	(4)	7.RP.3
152	(3)	7.NS.2
171	(2)	7.RP.3
178	(3)	7.NS.2
204	(2)	7.RP.3
237	(3)	7.RP.3

Coordinate Geometry

41	(3)	8.G.3
72	(4)	F-IF.2
76	(4)	8.EE.6
106	(3)	G-C.2
128	(5)	8.G.8
170	(5)	3.MD.7
172	(4)	7.RP.3
173	(4)	G-C.2
183	(4)	8.EE.6
217	(5)	8.F.3

Plane Geometry

48	(4)	8.G.7
69	(5)	7.G.5
83	(4)	7.RP.3
87	(3)	G-C.2
92	(4)	7.G.4
95	(3)	SMP
137	(4)	8.G.5
145	(3)	G-CO.10
146	(3)	7.G.6
168	(4)	7.G.4
174	(5)	G-SRT.5
192	(4)	G-SRT.6
209	(5)	7.G.4
220	(6)	8.G.7
227	(5)	8.G.8
239	(7)	G-C.2
242	(5)	G-SRT.6
257	(6)	SMP
294	(3)	G-SRT.5

Logic

37	(4)	SMP
58	(4)	7.NS.3
67	(4)	SMP
89	(3)	SMP
105	(2)	SMP
110	(4)	SMP
167	(5)	SMP
184	(3)	SMP
216	(5)	6.G.2
226	(3)	SMP
250	(4)	SMP
256	(5)	SMP
268	(5)	SMP

Sequences, Series & Patterns

8	(4)	G-CO.10
40	(4)	F-LE.1
163	(3)	F-BF.2
179	(3)	F-BF.2
189	(4)	F-LE.2
195	(4)	F-BF.2
206	(4)	F-LE.1
231	(5)	6.SP.5
233	(5)	F-BF.2
259	(4)	F-BF.2
289	(6)	6.SP.5

Measurement

1	(1)	2.MD.2
2	(2)	6.EE.7
19	(3)	G-SRT.5
27	(4)	8.G.7
28	(4)	6.G.1
50	(2)	6.G.1
55	(5)	7.G.4
60	(4)	7.G.6
88	(4)	7.G.5
90	(4)	7.G.6
104	(4)	8.EE.8
108	(5)	SMP
118	(3)	6.RP.3
123	(5)	3.MD.7
125	(5)	G-SRT.5
130	(4)	6.RP.3
140	(5)	7.G.6
142	(4)	8.G.7
148	(3)	8.G.7
158	(4)	SMP
160	(5)	G-CO.10
166	(5)	7.G.4
180	(5)	7.G.4
190	(4)	7.G.6
222	(5)	G-SRT.6
235	(4)	4.MD.3
236	(4)	6.EE.5
245	(5)	G-SRT.6
264	(4)	7.G.4