## Warm-Up 1

1. $\qquad$
2. $\qquad$ calories

Rocky's family recipe for macaroni and cheese makes 4 servings of 310 calories each. Rocky decided to make $1 \frac{1}{2}$ times the amount in the recipe. How many calories are in Rocky's batch of macaroni and cheese?
3. $\qquad$ inches


Alberto's dad is 6 feet 3 inches tall, and his mother is 5 feet 7 inches tall. One method used to predict a young child's adult height is to take the average of the mother's height and the father's height. Using this method, what is Alberto's expected adult height, in inches?
A cake with a regular hexagonal top is sliced along each of its 9 diagonals. What percent of the resulting 24 pieces are triangles?

4. $\qquad$ If the dot pattern shown here is continued, how many dots will there be in Figure 5?

Figure 1 Figure 2

5. $\$$ $\qquad$ The toll for a major highway is 8 cents for every 5 miles traveled. What is the toll, in dollars, for a trip of 115 miles on this highway?
6. $\qquad$ \% A candle 25 cm tall burns at the rate of 5 cm per hour. What percent of the original candle is left after it has burned for 2 hours?


If a fair coin is flipped 17 times, what is the probability that the number of heads will equal the number of tails?
7. $\qquad$
8. $\qquad$ Two squares, each with an area of 25 units $^{2}$, are placed side-by-side to form a rectangle. What is the perimeter of the rectangle?
9. $\qquad$ The ratio of girls to boys in the seventh grade at Hypatia Middle School is 3:2. There are 134 boys in the seventh grade. What is the total number of students in the seventh grade at Hypatia Middle School?
10. $\$$ $\qquad$ When Johanna and Klara ate at their favorite restaurant, the subtotal was $\$ 26.40$. A $7 \%$ tax and an $18 \%$ tip were added to the bill, both applied to the subtotal. What was the total cost, including tax and tip?

## Warm-Up 2

11. $\qquad$ Half of a third of $x$ equals a fourth of $y$ plus a fifth of $y$. If $x=27$, what is the value of $y$ ?
12. $\qquad$ What is the positive difference between the range and the interquartile range of the data set represented by this box-and-whisker plot?

13. $\qquad$ If $x$ and $y$ are positive integers such that $x^{y}=8$, what is the maximum possible value of $x+y$ ?
14. $\qquad$ jars
15. $\qquad$ in $^{2}$

Isosceles triangle XYZ is inscribed in circle Q , as shown. If diameter XZ is 2 inches, what is the area of $\Delta X Y Z$ ?

16. $\qquad$ A state creates license plates that each contain two letters followed by three digits. The first letter must be a vowel $(A, E, I, O, U$ ), and duplicate letters and digits are allowed. How many different license plates are possible?

IS 030
17. $\qquad$ \%

What percent of the first 50 positive integers contain no odd digits?
18. $\qquad$


What percent of the grid shown here is not shaded?
19. $\qquad$ For what positive integer $n$ is $2 n+3 n+4 n=n^{n}$ ?
20. $\qquad$ At Hall of Oats, yogurt-covered raisins sell for $\$ 3.99$ per pound. How much will $33 \frac{1}{3}$ pounds of yogurt-covered raisins cost?
21. \$ Using his \$50 gift card, Xi bought 5 apps for $\$ 1.99$ each and a new set of headphones for $\$ 10$. After these purchases, what was the remaining balance on his $\$ 50$ gift card?

22. Tweets

If it takes 24 seconds to write a Tweet and 8 seconds to send it, what is the greatest number of Tweets that can be written and sent in 6 minutes?
23. $\qquad$ A car averages 20 miles per gallon of gas in city driving and 30 miles per gallon in highway driving. At these rates, how many gallons of gas will the car use on a 300 -mile trip if $\frac{4}{5}$ of the trip distance is highway driving and the rest is city driving?
24. $\qquad$ The sum of the lengths of the edges of a cube is 24 inches. What is the volume of the cube?
25. $\qquad$ If Molly can choose from 5 kinds of fruit, 3 salads and 4 beverages for her lunch, how many different combinations of a fruit, a salad and a beverage can she make?
26. $\qquad$ The Kola Superdeep Borehole in Russia was drilled to a depth of about 40,200 feet. Given that there are 5280 feet per mile, about how deep was the hole, in miles? Express your answer as a decimal to the nearest tenth.
27. $\qquad$ If $x \odot y$ is defined as $x y+(x-y)$, what is the value of $4 \odot 2$ ?
28. $\qquad$
 The fuel tank in Alexia's car holds 13.4 gallons of gas. How many gallons of gas does she have when her tank is one-quarter full? Express your answer as a decimal to the nearest hundredth.
29. $\qquad$ \% The Dr. Seuss story The Cat in the Hat contains 236 distinct words, 1 of which has three syllables and 14 of which have two syllables. The rest of the words have only one syllable. What percent of the words have only one syllable? Express your answer to the nearest tenth.
30. $\qquad$ In a triangle with angles measuring $a, b$ and $c$ degrees, the mean of $b$ and $c$ is $a$. What is the value of $a$ ?

## Warm-Up 3

31. $\qquad$ The numerator of a fraction is one-half the denominator. If the numerator is increased by 2 and the denominator is decreased by 2 , the value of the fraction is $\frac{2}{3}$. What is the numerator of the original fraction?
32. $\qquad$ What is the product of the digits of 7 !?
33. $\qquad$ In the $x y$-plane, lines $a$ and $b$ intersect at point $(5,-2)$, and lines $b$ and $c$ intersect at point $(-3,3)$. What is the slope of line $b$ ? Express your answer as a common fraction.
34. $\qquad$ units ${ }^{2}$

All of the angles in the figure shown are right angles. What is the total area of the figure?
35. $\qquad$ The figure shows equilateral triangle AED inside square $A B C D$. Segment $A C$ is a diagonal of the square. What is the measure of $\angle E F C$ ?

36. $\qquad$ The heights of the five starters of a college basketball team are $6^{\prime} 6^{\prime \prime}, 6^{\prime} 7^{\prime \prime}, 6^{\prime} 9^{\prime \prime}, 6^{\prime} 11^{\prime \prime}$ and $7^{\prime}$. What is the mean height of these players, in inches?
37. $\qquad$ There are some frogs and some lily pads at a pond. If lily pads with frogs on them have two frogs each, then there is one lily pad with no frogs on it. If each lily pad has exactly one frog on it, then there is a frog with no lily pad. How many frogs are at the pond?

38. $\qquad$


To make punch for her upcoming party, Mary uses a recipe that calls for $\frac{1}{2}$ cup of fruit juice per serving. If she has 3 gallons of fruit juice, what is the greatest number of servings of punch she can make? [Note: 1 gallon = 16 cups]
39. $\qquad$ Jim has $\frac{1}{3}$ as many goldfish as Hannah. Hannah has 5 times as many goldfish as Ping. If Ping has 18 goldfish, how many goldfish does Jim have?
40. $\qquad$ What is the product of 9 and $1 . \overline{3}$ ?

## Warm-Up 4

41. $\qquad$

What is the ratio of the area of $\triangle A B D$ to the area of parallelogram MNOP, shown here? Express your answer as a common fraction.

42. $\qquad$ What is the value of $2015^{2}-2013^{2}$ ?
43. $\qquad$ What is the difference between the following sums?

| 12,345 | 54,321 |
| ---: | ---: |
| 12,340 | 4,321 |
| 12,300 | 321 |
| 12,000 | 21 |
| $+10,000$ |  |

44. 




In the figure, MP is $20 \%$ less than MN . If $\mathrm{PN}=4$ units and $\overline{\mathrm{MO}}$ is three times as long as $\overline{\mathrm{PN}}$, what is the length of $\overline{\mathrm{OP}}$ ?
45. $\qquad$ Four regular hexagons of side length 1 unit are placed together as shown. How many paths of length 7 units are there from point A to point $B$ along edges of the hexagons?

46. $\qquad$ The top base has been removed from a right rectangular prism as shown. The result is an open box that measures 6 inches by 5 inches by 2 inches. Each exterior face of the box is painted red, and each interior face of the box is painted blue. What is the total area of the box's painted surfaces?

47. $\qquad$ What is the value of $\frac{a b^{2} c+5}{x^{0} y}$ if $a=3, b=-2, c=10, x=-7$ and $y=5$ ?

A polyabolo is a polygon formed by joining congruent isosceles right triangles in such a way that each triangle shares a side with at least one other triangle. Three distinct polyaboloes can be formed from two triangles, as shown below. If rotations and reflections are not counted separately, how many distinct polyaboloes can be formed from three triangles?

49. $\qquad$ McCall has \$14.00 in change, and he has only nickels, dimes and quarters. If McCall has the same number of each type of coin, how many dimes does McCall have?
50. $\qquad$ days

Lorna observed that her brother wore green on 9 of the last 21 days. Assuming the same rate, on how many days would Lorna expect him to wear green in the next five weeks?

## Workout 2

51. $\qquad$ What is the sum of the first 150 odd positive integers?
52. $\$$ $\qquad$
 Lani started a cleaning service. During her first month in business, Lani spent $\$ 380$ on supplies and drove 800 miles at an average cost of $\$ 0.30$ per mile. In addition, her business phone and other expenses were $\$ 198$. That month Lani completed 60 jobs, earning $\$ 50$ per job. What was Lani's profit during her first month in business?
53. $\qquad$ If $2^{x}=7$, what is the value of $2^{2 x}$ ?
54. 



How many of the permutations using one or more letters from the word TEXAS are also permutations using the letters in


TENNESSEE? Two such permutations to include are ST and TS.
55. $\qquad$ /s

If Tom travels 150 miles in 1 hour 45 minutes, what is his speed in feet per second? Express your answer as a decimal to the nearest tenth.
56.

57.


How many positive integers each have one less digit than their squares?
58. $\qquad$ If $f(x)=2 x^{2}+8$, what is the positive value of $x$ for which $f(x)=136$ ?
59.


What is the number of red balloons on a float containing 25 balloons, if $56 \%$ of the balloons are not red?
60. $\qquad$
In 1990, the ten most popular names for baby boys in Gladwell were given to 3375 babies, representing about $18.70 \%$ of all baby boys born in Gladwell in 1990. In 2000, the ten most popular names for baby boys in Gladwell were given to 2115 babies, representing about $14.60 \%$ of all baby boys born in Gladwell in 2000. How many more baby boys were born in Gladwell in 1990 than in 2000? Express your answer to the nearest hundred.


At how many minutes after noon do the hour hand and minute hand of an analog clock first meet again? Express your answer to the nearest whole number.

## Warm-Up 5

61. $\qquad$


A burger restaurant advertises that there are 96 possible ways to order your burger, assuming you have it on a white, wheat or sourdough bun. How many ways can you order a burger on a sourdough bun?
62. $\qquad$ Working together, 2 groomers can brush 8 horses in 3 hours. How many hours would it take 3 groomers to brush 12 horses at this rate?

In a regular hexagon, what is the ratio of the length of the shortest diagonal to the length of the longest diagonal? Express your answer as a common fraction in simplest radical form.
64. $\qquad$ Liz is a student in Ms. Xu's class. Liz says to her classmates, "Of all the pairs of students Ms. Xu can choose as class leaders, I am included in one-tenth of those pairs." How many students are in Ms. Xu's class?
65. $\qquad$ If $y=-x$ and $y \neq 0$, what is the value of $\frac{x^{2013}}{y^{2013}}$ ?
66. $\qquad$ During Bill's three-hour meeting, the word global was used, on average, once every five minutes during the first two hours. If the word global was used 54 times throughout the meeting, then what was the average number of minutes between uses in the third hour?
67. $\qquad$ When a dot is placed in the figure shown, all cells along the dot's horizontal row and two diagonals are shaded. For instance, when the sample dot is placed in the figure, 13 cells are shaded. What is the minimum number of dots that must be placed so that all cells are shaded?

68. $\qquad$ 1

Given the points $A(-2,1)$ and $B(3,4)$, what are the coordinates of point $C$ in the fourth quadrant such that $m \angle C A B=90$ degrees and $A B=A C$ ? Express your answer as an ordered pair.
69. $\qquad$ The measure of $\angle A$ is 32 degrees. What is the positive difference between the degree measures of the complement and the supplement of $\angle A$ ?
70. $\qquad$ cm

In the figure shown, point $G$ is the midpoint of $\overline{\mathrm{FH}}, \overline{\mathrm{JH}} \perp \overline{\mathrm{FH}}$ and the lengths of $\overline{\mathrm{JH}}$ and $\overline{\mathrm{FG}}$ are 8 cm and 3 cm , respectively. If $\angle E G F \cong \angle J F H$ and $\angle \mathrm{FJH} \cong \angle \mathrm{FEG}$, what is the perimeter of $\triangle \mathrm{EFG}$ ?


## Warm-Up 6

71. $\qquad$ The competition scores for eight students from Descartes Middle School are listed below. What is the positive difference between the median and the range of these scores?
$12,28,17,8,25,19,10,22$
72. $\qquad$ What is the sum of the exponents when $\frac{\left(3 a^{2} b^{3}\right)\left(a b^{2}\right)}{3 a b}$ is written in simplest form?
73. $\qquad$ What is the length of the diagonal of a rectangle with side lengths of 5 feet and 3 feet? Express your answer in simplest radical form.
74. $\qquad$ What is the slope of the line that intersects the $x$-axis at $x=91$ and intersects the $y$-axis at $y=7$ ? Express your answer as a common fraction.
75. $\qquad$ What is the units digit of $2013^{2013}$ ?
76. $\qquad$ If Russell rolls two standard dice once, what is the probability that the sum of the two numbers rolled is not a prime number? Express your answer as a common fraction.
77. $\qquad$ units ${ }^{2}$

What is the area of the shaded region?

78. $\qquad$

Chris and Sandy ran a half-mile race. Sandy ran the race at a steady pace of $\frac{1}{6}$ mile per minute, and Chris ran at a steady pace of $\frac{1}{5}$ mile per minute. How many seconds after Chris finished the race did Sandy cross the finish line?
79. $\qquad$ A regular polyhedron has 8 vertices and 12 edges. How many faces does it have?
80. $\qquad$ $\mathrm{in}^{2}$

What is the area, in square inches, of the largest triangle that can fit in a 3-inch by 4-inch rectangle?

## Workout 3

81. $\qquad$
inches
The area of the isosceles trapezoid shown is $84 \mathrm{in}^{2}$. What is its perimeter? Express your answer as a decimal to the nearest tenth.

82. $\qquad$ \% What single percent discount is equivalent to two successive discounts of $15 \%$ and $10 \%$ ? Express your answer to the nearest tenth.
83. $\qquad$ In a certain word game the vowels $A, E, I, O$ and $U$ are worth 5 points each. There are two of
 each of these vowels in the game set. The remaining letters of the alphabet are worth 2 points each, and there is only one of each. If Molly chooses two letters at random and without replacement, what is the probability that the letters have a total value of 7 points? Express your answer as a common fraction.
84. $\qquad$ \% The rate of inflation is given by the following formula: $\frac{b-a}{a} \times 100$, where $a$ represents the previous year's consumer price index (CPI) and $b$ represents the current year's CPI. If the CPI for 2011 was 224.94, and the 2010 CPI was 218.06, what was the rate of inflation from 2010 to 2011? Express your answer as a decimal to the nearest hundredth.
85. $\qquad$

86. $\qquad$ \% Only 64\% of the students in Ms. Kreeger's class passed both of the two most recent tests. On the most recent test, $80 \%$ of the students passed. What percent of students who passed the most recent test also passed the previous test?
87. $\qquad$ If $f(x)=2 x-3$ and $g(x)=\frac{x-3}{2}$, what is $f(g(3))$ ?
88. $\qquad$


How many minutes faster will Jacob complete a 100-mile drive traveling at a rate of 65 miles per hour than if he traveled at a rate of 55 miles per hour? Express your answer to the nearest whole number.
89. $\qquad$ A train traveling at 45 miles per hour enters a tunnel that is 1 mile long. The length of the train is $\frac{1}{8}$ mile. How many minutes after the front of the train enters the tunnel does the back of the train exit the tunnel? Express your answer as a decimal to the nearest tenth.
90. $\qquad$ What is the sum of the digits of $2015^{2}$ ?

## Warm-Up 7

91. $\qquad$ people

John's four brothers each have names that begin with the letter J, but none of the other members of his family has a name that begins with J. If a person in John's family is randomly selected, there is a $25 \%$ chance that the person's name will start with J. How many people are in John's family?
92. $\qquad$ The sum of two integers is 7 , and the sum of their squares is 25 . What is the product of these two integers?
93. $\qquad$ When Roger hits the BAKE button on his oven, the temperature display shows + + +. The first time he hits the TEMP button, the display changes to $350^{\circ}$. Each time Roger hits the TEMP button thereafter, the displayed temperature increases by $5^{\circ}$. After pressing the BAKE button, how many times does Roger need to hit the TEMP button to reach $425^{\circ}$ ?
94. $\qquad$ A piggy bank contains a certain number of coins, of which 53 are dimes and 19 are nickels. The remainder of the coins in the bank are quarters. If the probability of randomly selecting a quarter from the bank is $\frac{1}{4}$, how many quarters does the bank contain?

95. $\qquad$ What is the area, in terms of $a$ and $b$, of a rectangle with a length and width of $3 a^{2} b$ units and $2 a b^{3}$ units, respectively?
96. $\qquad$ The perimeter of a isosceles right triangle is 80 cm . What is the length of the hypotenuse? Express your answer in simplest radical form.
97. triangles How many triangles of any size are contained in this figure?

98. $\qquad$ If $a, b$ and $c$ are positive integers such that $a+b=9$ and $a c-2 b c=0$, what is the value of $a$ ?
99. segment

Points A through $H$ are distributed along a line as shown. Seven segments are created with adjacent points as endpoints. On which of the seven segments should a new point $X$ be placed so that the sum of the distances from point $X$ to each point $A$ through $H$ is as small as possible?

100. $\qquad$ In a set with $n$ elements, where $n$ is a positive integer, what fraction of the subsets contain an even number of elements? Express your answer as a common fraction.

## Warm-Up 8

101. $\qquad$ There exist pairs of integers, $x$ and $n$, for which $x^{n}=\left(2^{5}\right)\left(4^{4}\right)\left(8^{\frac{8}{3}}\right)\left(16^{\frac{3}{4}}\right)$. What is the greatest possible value of $n$ among these pairs?
102. $\qquad$ If $\frac{1}{x}+\frac{1}{y}=\frac{1}{2}$ and $\frac{1}{x}-\frac{1}{y}=\frac{1}{4}$, what is the value of $\frac{1}{x^{2}}-\frac{1}{y^{2}}$ ? Express your answer as a common
fraction.
103. $\qquad$ If $n$ is a randomly chosen positive integer less than 2013, what is the probability that the sum $1^{n}+2^{n}+3^{n}$ is divisible by 3 ? Express your answer as a common fraction.
104. $\qquad$ In $\triangle A B C, A C=12$ units and $B C=7$ units. If the area of $\triangle A B C$ is 42 units ${ }^{2}$, what is the degree measure of $\angle C$ ?
105. $\qquad$ ways Each face of a cube is colored either red or blue. In how many distinct ways can the cube be colored? Two colored cubes are distinct if one cannot be rotated to look like the other.
106. $\qquad$ A five-person committee has to meet at one of five possible times. Each member has a conflict at exactly one of the five times, and the conflicts are random and independent of each other. What is the probability that there
 is a time when all five people can meet? Express your answer as a common fraction.
107. $\qquad$ Five distinct odd integers have a mean of 35 and a range of 22 . What is the smallest possible value of the least of these five integers?
108. 



After deducting his $10 \%$ commission, Jun sent $\$ 27$ to the newspaper dealer for whom he delivers papers. If each newspaper sells for 20 cents, how many papers did Jun deliver?
109. $\qquad$ What is the value of $r$ for which $(r-5)^{2}=(r+2)^{2}$ ? Express your answer as a decimal to the nearest tenth.
110. $\$$ $\qquad$ A popular brand of Brazilian coffee costs $\$ 20$ per pound, and a particular brand of Colombian coffee costs $\$ 16$ per pound. If you mix 15 pounds of Brazilian coffee with 5 pounds of Colombian coffee, how many dollars does one pound of the mixture cost?

# Workout 4 

111. $\qquad$
degrees


What is the value of $x+y$ in the figure shown?
112. \$ $\qquad$ Sherry bought a sport utility vehicle. She received a discount of $18 \%$ off the manufacturer's list price but then had to pay $8 \%$ state sales tax. If she paid $\$ 17,712$ for the car after the discount and tax were applied, how much was the manufacturer's list price?
113. $\qquad$ units a decimal to the nearest tenth.

114. $\qquad$ amounts
115. $\qquad$ A regular octagon is inscribed in a square, as shown. If the sides of the octagon are 1 cm in length, what is the length of a side of the square? Express your answer as a decimal to the nearest tenth.

116. $\qquad$ A box of fewer than 100 but more than 12 cookies can be shared equally among 4,10 or 12 people with no cookies left over. How many cookies are in the box?
117. $\qquad$ The denominator of a positive common fraction is 3 more than its numerator. If $\frac{5}{28}$ is added to this fraction, the result is the same as the positive difference between the reciprocal of the original fraction and 1 . What is this common fraction?
118. $\qquad$ What is the smallest possible value of $a^{\left(b^{c}\right)}$ where $a, b$ and $c$ are distinct integers chosen from the set $\{2,3,4\}$ ?
119. $\qquad$ How many different products are possible when two one-digit positive integers are multiplied?
120. $\qquad$ For every two used paperback books Clarissa buys at the regular price, she gets a third book for a nickel. If Clarissa spent $\$ 4.65$ for nine paperback books, what is the regular price of a used paperback book, in cents?

## Warm-Up 9

121. $\qquad$ A bag contains three quarters, two dimes and a nickel. If two coins are randomly drawn without replacement, what is the probability that both coins are the same denomination? Express your answer as a common fraction.
122. $\qquad$


If the area of the $C$ in this logo is 58 units $^{2}$, what is the area of the $M$ ?
123. $\qquad$ There were 42 eighth graders who voted to go to Washington, D.C. for a class trip. This represents $\frac{2}{9}$ of the students in eighth grade. How many eighth graders did not vote for the class trip to Washington, D.C.?
124. $\qquad$ The mean of five different integers is 30 . If the smallest integer is 7, what is the greatest possible value of any of the integers?
125. $\qquad$ If $f(x)=g(x)+2$, and $g(x)=\frac{1}{2} f(x)$, what is the value of $f(2013)$ ?

## PARKONGB

126. $\qquad$


There are 20 cars in Lot $A$, and there are 20 more cars in Lot B than there are in Lot $C$. If there are a combined total of 100 cars parked in all three lots, how many cars are in Lot C?
127. $\qquad$ If $w=x+y+z$, what is the arithmetic mean of $w, x, y$ and $z$ in terms of $w$ ? Express your answer as a common fraction.
128. $\qquad$ What is the degree measure of an angle whose supplement is three times as large as its complement?
129. $\qquad$ The three concentric circles shown have center N and diameters of 16 cm , 12 cm and 10 cm . Points N, P and Q are collinear. What is the distance from point $P$ to point $Q$ ?

130. $\qquad$ If $x$ and $y$ are real numbers such that $x^{2}=y^{2}$ and $x \neq y$, what is the value of $x^{2}+2(x+y)-y^{2}+8$ ?

## Warm-Up 10

131. 

## degrees

132. $\qquad$ Julius found three sheets of paper torn from a book, each with page numbers on both sides. If three of these page numbers were 1,82 and 93 , what is the sum of the page numbers on the other sides of the three sheets?
133. $\qquad$


When a circle is inscribed in a square of side length 2 units, the four points of intersection between the circle and square are the vertices of a smaller square. What is the positive difference between the area of the circle and the area of the smaller square? Express your answer in terms of $\pi$.
134. $\qquad$ Five people are arranged in a line. In how many ways can they be arranged in a different order so that each person is standing beside at least one person he or she originally stood beside?
135. $\qquad$ Forty-eight candies are divided into two piles. The candies in the first pile are placed six to a bag, and the candies in the second pile are placed three to a bag. If a total of nine bags are used, how many candies are in the larger of the two piles?
136. $\qquad$ Maria is exactly 10 years older than Abe. Four years ago, Maria was twice as old as Abe was then. What is Maria's age now?

In the graph shown, if $f(-2)=p$ and $f(p)=r$, what is the value of $r+f(-1)$ ?

138. $\qquad$ For $f(x)=2 x+2$, the domain is $\{0,1,2, \ldots, 9,10\}$. How many integers are in both the domain and the range of $f$ ?

The sum of three consecutive integers is 24 . What is the product of the three integers?
140. $\qquad$ cm

The length of each edge of a cube is 10 cm , and point $K$ is placed at the center of a face of the cube. A line is drawn through the cube, as shown, from point $K$ to point J , a vertex of the cube on the opposite face. What is the length of $\overline{\mathrm{KJ}}$ ? Express your answer in simplest radical form.

141. $\qquad$ A man who initially weighed 220 pounds completed a diet-and-exercise program. After the 12 -week program, his body fat percentage had dropped from $30 \%$ to $20 \%$, and his weight had dropped to 200 pounds. If every part of his body that is not fat or muscle has a constant weight of 120 pounds, how many pounds of muscle did he gain during the program?

142. $\qquad$ A piece of paper is 0.1 mm thick. It is folded in half and then cut along the fold line. The cut pieces then are stacked one on top of the other. This folding and cutting process is completed a total of 10 times. If all of the pieces are stacked on top of each other, how many centimeters tall will the stack be? Express your answer as a decimal to the nearest hundredth.
143. $\qquad$ The number 34 is removed from a set of five numbers with a mean of 20 . What is the mean of the remaining four numbers? Express your answer as a decimal to the nearest tenth.
144. $\qquad$ The population of Big City has always doubled every 5 years. If the current population is 25,600 people, what was Big City's population 20 years ago?
145. $\$$ $\qquad$ Kara bought 1 package of cookies and 4 popsicles for $\$ 3.59$. Cam bought 5 packages of cookies and 2 popsicles for $\$ 12.55$. How much will it cost to buy both a package of cookies and a popsicle?
146. $\qquad$


A sphere has a radius of 4 inches. What is the surface area of the smallest cube that could circumscribe the sphere?
147. $\qquad$ What is the positive difference between the mean and the median of the squares of the first 10 positive integers?
148. $\qquad$ \% A department store advertises that it has reduced its prices by $10 \%$, then reduced the lower prices by $20 \%$, then by $30 \%$, then by $40 \%$. What single percent discount would yield the same final price? Express your answer to the nearest hundredth.
149. numbers

Each Dance-A-Thon contestant has a three-digit ID number that is divisible by 8 . If the tens and units digits cannot be the same, what is the maximum number of contestants' numbers that can have 4 as the hundreds digit?
150. $\qquad$ A unit fraction is a fraction whose numerator is 1 and whose denominator is a natural number greater than 1. If three unit fractions with distinct single-digit denominators have a sum of $\frac{5}{8}$, what is the sum of the denominators?

# Warm-Up 11 

151. $\qquad$ cm The area of an equilateral triangle is $16 \sqrt{3} \mathrm{~cm}^{2}$. What is its perimeter?
152. $\qquad$ The ratio of the height of a parallelogram to its base is $3: 5$. If the area of the parallelogram is $135 \mathrm{~mm}^{2}$, what is the length of its base?
153. $\qquad$ Triangle EFG has side lengths $x-1, x+1$ and $x+3$. For what value of $x$ is $\Delta \mathrm{EFG}$ a right triangle?
154. $\qquad$ What is the sum of the coordinates of the $x$ - and $y$-intercepts of $3 x-2 y=15$ ? Express your answer as a mixed number.
155. $\qquad$ A larger cube is created from 64 white unit cubes. Two opposite faces of that larger cube are painted black, and the remaining four faces are painted red. The unit cubes then are placed in a bag. If one unit cube is drawn at random, what is the probability that it has two red faces and one black face? Express your answer as a common fraction.
156. $\qquad$ Megan rolls two standard dice, hoping for double sixes. Melanie flips five coins, hoping that all of them land heads. What is the probability of the more likely outcome? Express your answer as a common fraction.
157. $\qquad$
cookies
During the first four days of this week, Katie and her friends together ate an average of 8 cookies a day. If the cookies they ate on the fifth day are included, together they ate an average of 10 cookies a day for the five days. How many cookies did they eat on the fifth day?
158. $\qquad$ Joseph's books on animation are grouped into books about cartoon mice, cartoon rabbits and cartoon toys, in a ratio of 5:3:2, respectively. If Joseph has 21 cartoon rabbit books, how many books on animation does he have altogether?
159. $\qquad$ If $a$ is $\frac{4}{9}$ of $b$, and $c$ is $\frac{3}{4}$ of $a$, what fraction of $b$ is $c$ ? Express your answer as a common fraction.
160. $\qquad$ $\mathrm{m}^{2}$

A 6-meter by 8-meter rectangle overlaps a 7-meter by 9-meter rectangle so that they share two sides and a vertex as shown. In square meters, what is the total area of the rectangles not shaded?


## Warm-Up 12

161. $\qquad$

At Mercury Junior High 5\% of the students are taking both French and Latin. If 25\% of the students are taking French, what is the probability that a randomly chosen student taking French is also taking Latin? Express your answer as a common fraction.
162. $\qquad$ cm

A rectangular prism has a length of $3 x \mathrm{~cm}$, a width of $\frac{1}{3} y \mathrm{~cm}$ and a height of $x y \mathrm{~cm}$. Its volume is $144 \mathrm{~cm}^{3}$. If the height of the prism is twice its length, what is the length of the prism?
163. $\qquad$ Two concentric circles are each divided into 8 congruent sections, as shown. The area of the larger circle is 3 times the area of the smaller circle. The shaded region represents what portion of the entire figure? Express your answer as a common fraction.

164. $\qquad$ A playground has a length of $a$ yards $b$ feet $c$ inches. In terms of $a, b$ and $c$, how many feet long is the playground?
165. $\qquad$


The diagram shows the net for a right square pyramid. Each side of the base is 2 cm long. The length of each side of the isosceles triangular faces is $\sqrt{3} \mathrm{~cm}$. What is the volume of the pyramid? Express your answer as a common fraction.
166. $\qquad$ What is the value of $\frac{1+2+3+\cdots+2012}{1+2+3+\cdots+2013}$ ? Express your answer as a common fraction.
167. $\qquad$ What is the length of the shortest side of $\triangle A B C$ whose perimeter is 64 units, if the ratio $A B: B C$ is $4: 3$ and $A C$ is 20 less than the sum of the lengths of sides $A B$ and $B C$ ?
168. $\qquad$ As shown, a regular pentagon can be divided into triangles only by connecting vertices with non-overlapping diagonals in one way. In how many different ways can a regular hexagon be divided into triangles by connecting vertices with non-overlapping diagonals? (Rotations and reflections are not considered different.)

169. $\qquad$ To create his special blend of lemonade, Manny starts with a lemonade mix that is $20 \%$ lemon juice. Then he adds pure lemon juice to make a blend that is $25 \%$ lemon juice. How many gallons of pure lemon juice must he add to 30 gallons of the lemonade mix to make his special blend of lemonade?
170. Snacklies

Pink Snacklies come 3 to a pack, and green Snacklies come 5 to a pack. A basket of pink Snacklies contains 8 more packs than a basket of green Snacklies, although both baskets contain the same number of Snacklies. How many Snacklies are in each basket?

## Workout 6

171. $\qquad$
turns
Juwan played a game in which he could score either 11 or 16 points on each turn. Juwan scored exactly 175 points. How many turns did Juwan take?
172. $\qquad$ The stock exchange index went up 3.76\% on Monday, down 4.25\% on Tuesday, down 1.16\% on Wednesday, up 2.12 \% on Thursday and down $5.38 \%$ on Friday. What was the net percent that the stock exchange index was down for the week? Express your answer to the nearest hundredth.
173. $\qquad$ \%

A company's budget increased from $\$ 29$ million to $\$ 133$ million over a period of 10 years, growing by the same percent each year. What was the annual percent increase? Express your answer to the nearest hundredth.
174. $\qquad$ Jorge has a bag with 6 red marbles and 12 blue marbles. He randomly selects 4 marbles from the bag, one at a time without replacement. What is the probability that he selects 2 red marbles followed by 2 blue marbles? Express your answer as a common fraction.
175.


A 9-inch $\times 12$-inch rectangular picture is framed by a border of uniform width. The combined area of picture plus border is $180 \mathrm{in}^{2}$. In inches, what is the width of the border? Express your answer as a decimal to the nearest tenth.
176. $\qquad$ On average, a bushel of corn contains 72,800 kernels and weighs 56 pounds. There are 16 ounces per pound, and an average ear of corn contains 650 kernels. In ounces, how much do the kernels from one average ear of corn weigh?
177. $\qquad$ A set of six consecutive positive integers is divided into three groups of two numbers each. The sum of the numbers in each group is 31 . What is the least possible product of the two numbers within one of the groups?
178. $\qquad$ If $x+y=2$ and $x^{2}+y^{2}=34$, what is the value of $x^{3}+y^{3}$ ?
179. $\qquad$ A lottery ticket costs 50 cents and contains four distinct numbers from 1 to 20 , inclusive. How much money would a person need to spend to buy a lottery ticket for every possible combination of four numbers?
180. $\qquad$ teachers

At Gauss Middle School, the current student-to-teacher ratio is 17.5:1. The school currently has 560 students. Next year, the student-to-teacher ratio must be 19:1. If the school gains 10 students, how many fewer teachers will be needed next year?

## Warm-Up 13

181. $\qquad$ What is the value of the sum $1+4+7+10+\ldots+91+94$ ?
182. $\qquad$ What is the sum of the prime factors of 969 ?
183. $\qquad$ $\mathrm{ft}^{2}$ This drawing shows six identical squares joined at pairs of their vertices to form a regular hexagon. Each square has sides of length 4 feet. What is the total area of the shaded triangles? Express your answer in simplest radical form.

184. $\qquad$ A triangle has sides of integer lengths $20 \mathrm{~cm}, 13 \mathrm{~cm}$ and $x \mathrm{~cm}$, where the side of length $x \mathrm{~cm}$ is not the longest side. If the area of the triangle is $66 \mathrm{~cm}^{2}$, what is the value of $x$ ?
185. $\qquad$ A set contains all possible five-digit positive integers that can be made using each of the digits $1,3,5,7$ and 9 once. What is the median of that set?
186. $\qquad$


Two unit squares are chosen at random, without replacement, from the $4 \times 4$ grid shown. What is the probability that the squares do not share a side? Express your answer as a common fraction.
187. $\qquad$ Dean pays the exact amount for a $\$ 1.00$ hot dog, using 36 coins. What is the greatest number of nickels he can use?
188. $\qquad$ If $(4 x+7)^{2}=a x^{2}+b x+c$, what is the value of $a+b$ ?
189. $\qquad$ A yo-yo will regain $80 \%$ of its distance with each successive drop. How many drops will it take before it rises less than half its initial distance?
190. $\qquad$ $\mathrm{n}^{2}$ Sara outlines the head of a fox by drawing two congruent equilateral triangles along one side of a larger equilateral triangle as shown. The smaller triangles have sides of length 2 inches. What is the area of the entire figure? Express your answer in simplest radical form.


## Warm-Up 14

191. $\qquad$

In right triangle MNO shown here, $\mathrm{MO}=15$ units, $\mathrm{MP}=17$ units and $\mathrm{MN}=25$ units. What is the length of $\overline{N P}$ ?

192. $\qquad$ If the volume of one cube is 8 times the volume of another cube, then what is the ratio of the area of a face of the smaller cube to the area of a face of the larger cube? Express your answer as a common fraction.
193. $\qquad$ Parker's favorite ice cream shop has a sundae special on Sundays. He can create his own sundae by choosing one of 8 flavors of ice cream, one of 4 fruit toppings, one of 3 nut toppings and one of 2 kinds of whipped cream. How many additional unique sundaes could be made if Parker is allowed to skip one or more of the following: fruit topping, nut topping and whipped cream?
194. $\qquad$ In the figure, $A, B$ and $C$ are points on the number line with coordinates shown. If $A C=5 A B$, what is the value of $C$ ? Express your answer as a common fraction.

195. $\qquad$ The ratio of boys to girls in the math club is $4: 5$. If there are 3 more girls than boys in the math club, how many girls are in the math club?
196. $\qquad$ In the magic square shown, the three numbers in each of the rows, columns and diagonals have equal sums. What is the value of $d$ in this magic square?

| $a$ | $b$ | 13 |
| :---: | :---: | :---: |
| $c$ | $d$ | $e$ |
| $f$ | 4 | 25 |

197. $\qquad$ If $x$ is a real number and $x^{27}=27$, what is the value of $x^{9}$ ?
198. $\qquad$ The difference between two positive two-digit integers is 9 , and their sum is 99 . What is the larger of the two integers?
199. $\qquad$ What is the value of $\frac{81^{2}-18^{2}}{81-18}-\frac{72^{2}-27^{2}}{72-27}+\frac{63^{2}-36^{2}}{63-36}-\frac{54^{2}-45^{2}}{54-45}$ ?
200. $\qquad$ What four-digit palindrome has exactly one factor that is prime?

## Workout 7

201. $\qquad$ sides A regular polygon has interior angles between 128 degrees and 130 degrees. How many sides does the polygon have?
202. $\qquad$ cm
203. $\qquad$
laps
Selina can run around the track 3 times in 8 minutes. Marta can run around the same track 2 times in 5 minutes. If Selina and Marta begin at the same time and the same place, what is the combined number of laps the girls both will have run when they next reach the starting point at the same time?
204. $\qquad$ This year, 200 boys and 250 girls attended Edison Middle School. If the number of boys enrolled were to increase by $10 \%$, what is the maximum possible increase in the number of girls enrolled that would produce at most an $8 \%$ increase in the total student enrollment?
205. $\qquad$ A cube consists of 64 white unit cubes. All exterior faces of the cube except the bottom face are painted red. How many of the unit cubes have exactly two faces painted red?
206. $\qquad$ The four Mathletes ${ }^{\circledR}$ on the Descartes Middle School MATHCOUNTS team each calculated the mean number of cookies they brought for lunch on the days they brought bag lunches to school last month. Using the information in the chart, what was the mean number of cookies for all the bag lunches the four Mathletes brought last month? Express your answer as a decimal to the nearest hundredth.

| Name | Mean Number of Cookies <br> in Bag Lunch | Number of Bag Lunches Brought <br> to School Last Month |
| :---: | :---: | :---: |
| Amy | 2 | 16 |
| Jerrod | 4 | 12 |
| Miguel | 3 | 20 |
| Patty | 2 | 12 |

207. $\qquad$
 In isosceles triangle QRS, the length of base QR is $\frac{1}{5}$ the perimeter of the triangle. If the length of the altitude drawn from point $Q$ to side $S R$ is 52 mm , what is the perimeter of $\triangle Q R S$ ? Express your answer as a decimal to the nearest tenth.
208. $\qquad$ What is the least positive integer $n$ for which $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{n}>2$ ?


In the figure shown, each circle is tangent to two other circles and to two sides of the square and has a radius of 7 units. What is the area of the shaded region? Express your answer as a decimal to the nearest hundredth.
210. $\qquad$ What is the area of the largest possible quadrilateral with a perimeter of 24 cm ?

## Warm-Up 15

211. $\qquad$ One-half of the children who belong to a local club are over 14 years old. One-fourth of the remaining children are under 10 years old. What fraction of the club's members who are children are age 10 to 14 years old, inclusive? Express your answer as a common fraction.
212. $\qquad$ The $n$th triangular number is the sum of the first $n$ positive integers. Bill meant to add up the first ten triangular numbers but accidently missed two of them. If his total was 186, what is the positive difference between the two triangular numbers he missed?

Five positive integers have a mean of 8 . What is the greatest possible value for the median of the five integers?
214. $\qquad$ If a local bank has only $\$ 2$ bills and $\$ 5$ bills available, how many combinations of bills can a bank teller use to make exactly $\$ 100$ ?
215. $\qquad$
points
Twenty-five students take a quiz. If 10 points are added to each of the lowest five scores, by how many points will the mean of the quiz scores increase?

The equation of the graph shown is $f(x)=x^{2}-9$. What is the area of the triangle with vertices at the $x$-intercepts and vertex of the parabola?


What is the perimeter of right triangle $A B C$, shown here, if the area of square AEDC is 100 units $^{2}$ and the area of square BCFG is 64 units $^{2}$ ?
218. $\qquad$ Each week Laura's mom randomly assigns the seven daily chores to Laura and her six siblings so that each child gets one chore. What is the probability that two different children are chosen to wash dishes on Friday and Saturday? Express your answer as a common fraction.
219. $\qquad$ units ${ }^{2}$

A cube has a volume of $343 x^{3} y^{6}$ units ${ }^{3}$. What is the area of a base of the cube? Express your answer in terms of $x$ and $y$.

In the figure shown, every interior angle has a measure of 60 degrees. How many equilateral triangles of any size are there in the figure?


## Warm-Up 16

221. $\qquad$

For her son's birthday party, Teena prepared enough food to feed 20 adults or 32 children. If only 15 adults attend the party, how many children, in addition to those adults, can be fed with the food Teena prepared?
222. $\qquad$ The math team purchased 16 pies for a Pi Day celebration. Some of the pies cost $\$ 8.50$ each, and the rest cost $\$ 7.00$ each. If the total cost for the pies was $\$ 121.00$, how many of the pies cost $\$ 8.50$ each?
223. $\qquad$ If $(x+y)^{2}=100$ and $x y=20$, what is the value of $x^{2}+y^{2}$ ?
224. $\qquad$ $\mathrm{cm}^{2}$

Three circles are enclosed in rectangle ABCD as shown. If the area of each circle is $9 \pi \mathrm{~cm}^{2}$, what is the area of rectangle $A B C D$ ?

225. $\qquad$ The coach asks each of the 10 math team members to solve the 5 most difficult problems from the last competition. The newest team member, Robert, can solve only 3 of the problems. There is enough time to present the solution to 3 problems. If the coach assigns each of 3 randomly selected team members a different one of the 5 problems to present, what is the probability that Robert is selected and assigned to present the solution to a problem he is unable to solve? Express your answer as a common fraction.
226. $\qquad$ Two noncongruent right triangles have integer side lengths, and each triangle has a perimeter of 60 units. What is the positive difference between the areas of these two triangles?
227. $\qquad$


Cubes with 2-inch edges are stacked to form the figure shown. What is the total volume of the structure?
228. $\qquad$ m museum director is hanging paintings on a rectangular wall in a museum. The paintings measure $3 \mathrm{~m} \times 4 \mathrm{~m}, 2 \mathrm{~m} \times 7 \mathrm{~m}$ and $3 \mathrm{~m} \times 5 \mathrm{~m}$. Once hung, the paintings leave $76 \mathrm{~m}^{2}$ of the wall uncovered. If the dimensions of the wall are $w$ meters by $h$ meters, where $w$ and $h$ are positive integers, what is the smallest possible perimeter of the wall?

The sum of the five numbers in the set $\left\{2 x+3, x+11, \frac{1}{3}(4 x+5), 5 x-7, \frac{1}{2} x+20\right\}$ is 127 . What is the median of this set of numbers?

In $\triangle A B C$ shown here, $A E=6 \mathrm{~cm}, A C=10 \mathrm{~cm}$ and $\overline{D E} \| \overline{B C}$. If $\triangle A B C$ has an area of $250 \mathrm{~cm}^{2}$, what is the area of trapezoid DBCE?


## Workout 8

231. $\qquad$ Quintavius adds the numbers 1 through 6 while Grizabella adds the numbers 1 through $n$. If Grizabella's sum is 5 times Quintavius' sum, what is the value of $n$ ?
232. $\qquad$ ${ }^{\circ} \mathrm{C}$ The formula $T(h)=100-0.0005 h$, where $h$ represents the number of feet above sea level, is an accurate estimate for the temperature, $T$, in degrees Celsius, at which water will boil. However, it provides a reasonable estimate only for $0 \leq h \leq 20,000$. What is the difference between the least and greatest values for $T(h)$ when $h$ is within this range?
233. $\qquad$ A circle is centered at one vertex of a square of side length 1 unit, as shown. If the areas of the two shaded regions are equal, what is the radius of the circle? Express your answer as a decimal to the nearest hundredth.

234. $\qquad$ Jonathan's bicycle tires each have a diameter of 26 inches. If he rides his bicycle 100 feet in a straight line, how many complete revolutions will one of his tires make?
235. $\qquad$ A bag contains 4 blue and 3 yellow marbles. The marbles are removed from the bag one at a time without replacement. What is the probability that the fifth marble removed is yellow? Express your answer as a common fraction.
236. $\qquad$ The sequence $1,3,4,7, \ldots$ is continued by adding the two preceding numbers to get the next term. What is the sum of the first 10 terms of this sequence?
237. \$ $\qquad$ A certain model of car decreases in value at a rate of $10 \%$ per year. If Helen paid $\$ 25,000$ for this model five years ago, how much is it worth now?
238. $\qquad$ numbers

A shrinking number is a positive three-digit integer in which the hundreds digit is greater than the tens digit, and the tens digit is greater than the ones digit. In other words, for a three-digit number $A B C, A>B>C$. How many three-digit numbers are shrinking numbers?
239. $\qquad$ The raw scores on the physics group projects are 18, 29, 32, 35, 36, 49, 53, 64, 66. The teacher wants to rescale the scores using the linear formula $G=k R+c$, where $G$ is the final grade and $R$ is the raw score. She wants the highest score to scale to 100 and the median to scale to 80 . What is the value of the product $k c$ ? Express your answer as a common fraction.

Rebecca cut shapes out of construction paper to create a face. The nose and mouth were created using an equilateral triangle and a semicircle, as shown. What is the ratio of the area of the nose to the area of the mouth if the perimeters of the nose and the mouth are equal? Express your answer as a decimal to the nearest hundredth.


## Warm-Up 17

241. $\qquad$ If the diameter of circle $P$ is $\frac{3}{5}$ the diameter of circle $Q$, what is the ratio of the circumference of circle $Q$ to the circumference of circle $P$ ? Express your answer as a common fraction.
242. $\qquad$ What is the units digit of $7^{282}$ ?
243. $\qquad$ The Wu family wishes to have both their car and their RV with them during their weekend camping trip. Mrs. Wu leaves home in the RV 1.5 hours before Mr. Wu departs from home in the car. Mr. Wu drives 15 miles per hour faster than Mrs. Wu and arrives at the campsite 30 minutes later than she arrives. If Mrs. Wu drives $45 \mathrm{mi} / \mathrm{h}$, how many miles is it from their home to the campsite?
244. $\qquad$ Let $a \# b=\frac{a}{b}+\frac{b}{a}$. What is the value of $[(1$ \# 2) \# 3] $-[1$ \# (2 \# 3) $]$ ? Express your answer as a common fraction.
245. $\qquad$ Angle A is formed by a line tangent to a circle and a line through the center of the circle, as shown. If $m \angle A=20$ degrees, what is $m \angle B C A$ ?

246. $\qquad$ The $x$-value of an ordered pair is randomly selected from the set $\{2,4,6,8,10\}$. The $y$-value of the same ordered pair is randomly selected from the set $\{5,6,7,8,9,10\}$. What is the probability that the ordered pair is on the line $y=x+1$ ? Express your answer as a common fraction.
247. $\qquad$ What is the mean of the first 80 positive integers? Express your answer as a decimal to the nearest tenth.
248. $\qquad$ Michael rolls $n$ standard six-sided dice and reports that the sum of the numbers displayed is 17. How many values of $n$ are possible?
249. $\qquad$


The figure shows two cones with a common vertex and parallel bases. If the ratio of the base radii ZY and WV is 3 to 5 , what is the ratio of the volume of the smaller cone to the volume of the larger cone? Express your answer as a common fraction.
250. $\qquad$ m

Pentagon HOUSE comprises square OUSE and equilateral triangle OHE. The area of $\triangle O H E$ is $100 \sqrt{3} \mathrm{~m}^{2}$. What is the perimeter of pentagon HOUSE?


## Warm-Up 18

251. $\qquad$
252. $\qquad$
253. $\qquad$ $\mathrm{mi} / \mathrm{h}$

Dora is traveling to visit Diego at an average speed of $60 \mathrm{mi} / \mathrm{h}$. Dora realizes that if she drives at an average speed of $75 \mathrm{mi} / \mathrm{h}$, her travel time will be reduced by 6 minutes. How many miles per hour would Dora's travel need to average for her travel time to increase by 6 minutes?

For positive integers $x$ and $y,(x+y)^{2}=324$ and $x^{2}+y^{2}=194$. What is the positive difference between $x$ and $y$ ?
255. $\qquad$ What is the probability that a randomly selected real number $x$ between -10 and 10 satisfies $x^{2}+x>2$ ? Express your answer as a common fraction.
256. $\qquad$ The students at Pericles Junior High participated in three events in the L. O. Kwint speech contest. Two students participated in all three events, 4 students did extemp and debate, 5 did extemp and dramatic reading, 6 did debate and dramatic reading. If 12 students from PJH participated in each event, how many students from PJH participated in the contest?
257. $\qquad$ baskets

A farmer's market sells only baskets of apples and baskets of oranges, each for a fixed whole number of dollars. Abigail paid $\$ 23$ for five baskets of fruit. Benedict paid $\$ 24$ for four baskets of fruit. If Charles paid $\$ 20$, how many baskets of fruit did he buy?
258. $\qquad$


The figure shows cubes stacked to form three layers. The pattern is continued to create a solid structure with the rows in each successive layer extending out one cube past the layer above them. How many cubes are needed to create a 10-layer structure?
259. $\qquad$ In rectangle $A B C D$, point $E$ lies on $\overline{B C}$ so that $\frac{B E}{E C}=2$ and point $F$ lies on $\overline{D C}$ so that $\frac{C F}{F D}=2$. Segments $A E$ and $A C$ intersect $\overline{B F}$ at points $X$ and $Y$, respectively. The extended ratio $\mathrm{FY}: \mathrm{YX}: \mathrm{XB}=a: b: c$ so that $a, b$ and $c$ are relatively prime positive integers. What is the value of $a+b+c$ ?

260. ordered How many distinct ordered pairs of positive integers $x$ and $y$ satisfy $x+y=2013$ ?
261. $\qquad$
dots
This V-Dot pattern progresses as shown. As the pattern continues, how many dots will it take to make V-Dot 40?

262. $\qquad$ \% The base of the right square prism shown has sides of length 10 cm , and each lateral face has an area of $200 \mathrm{~cm}^{2}$. The prism is sliced into two pieces by plane $W X Y Z . \overline{W X}\|\overline{A B}, \overline{Y Z}\| \overline{\mathrm{GH}}$ and the ratio of $G Y$ to $X B$ is $2: 3$. If $m \angle Y X B=135$ degrees, what percent of the volume of the square prism is the volume of the smaller piece?

263. $\qquad$ cents/g

According to the U.S. Mint specifications below, what is the difference between the value-to-weight ratios of a quarter and a dime?

| Coin | Penny | Nickel | Dime | Quarter |
| :--- | :--- | :---: | :---: | :---: |
| Weight (g) | 2.500 | 5.000 | 2.268 | 5.670 |

264. $\qquad$ A boat travels 2 miles upstream in the same time that it would take the same boat to travel 3 miles downstream. If the rate of the stream's current is $5 \mathrm{mi} / \mathrm{h}$, how many miles per hour would the boat travel in still water?
265. $\qquad$


Right triangle H is formed by the sides of squares $\mathrm{J}, \mathrm{K}$ and L . The side length of square $L$ is 4.5 units, and the area of square $J$ is 81 units $^{2}$. Triangle M is formed by the sides of squares K and L . Right triangle $N$ shares a side of triangle $M$. $A$ side of $L$ and a side of triangle $N$ are collinear. What is the area of the quadrilateral formed by triangles $M$ and $N$ ? Express your answer as a decimal to the nearest hundredth.
266. $\qquad$ After Carlos accidentally spills water on his paper, he is left with the partial equation $1729^{2}-2 \times 1730^{2}+17 \square \square^{2}=34 \square \square$, where each $\square$ represents a smudged digit, not necessarily all the same. What is the sum of the four smudged digits?
267. $\qquad$ The four-digit integer 8 XYZ consists of four distinct digits and is divisible by 7,8 and 9 . What is the value of $X+Y-Z$ ?
268. $\qquad$ A sector of circle $P$ is enclosed by $\overline{P Q}$ and $\overline{P R}$, and $\overparen{Q R}$ has length 9 inches. If the sector's perimeter is 33 inches, what is the area of this sector?

269. $\qquad$ A $15 \times 20$ rectangle completely covers 300 unit squares on a rectangular grid. If a diagonal of the rectangle is drawn, through how many of the unit squares' interiors would it pass?
270. $\qquad$ paths A path in the $x y$-plane consists of steps of 1 unit in the positive $x$ or $y$ direction. How many paths from $(0,0)$ to $(4,4)$ do not pass through the point $(2,2)$ ?

## Functions Stretch

A function maps a set of input values (domain) to a set of output values (range) in such a way that each input value maps to exactly one output value. The following exercises explore various types of functions through the use of equations, tables and graphs.
271. $\qquad$ What is the value of $f(-2)$ if $f(x)=x^{2}+x+5$ ?
272. $\qquad$ What is the value of $x$ if $f(x)=4$ for $f(x)=3 x-14$ ?

The table shown represents the function $h(x)$.
273. $\qquad$ According to the table, what is the value of $h(2)$ ?
274. $\qquad$ What is the value of $x$ if $h(x)=58$ ?

| $\boldsymbol{x}$ | $\boldsymbol{h}(\boldsymbol{x})$ |
| :---: | :---: |
| -1 | 3 |
| 0 | 10 |
| 1 | 7 |
| 2 | 6 |
| 3 | 19 |
| 4 | 58 |

275. $\qquad$ Based on the graph of $f(x)=-x^{4}+5 x^{2}-3$, shown here, what is the value of $f(0)$ ?

276. $\qquad$ Based on the graph of $f(x)=x^{3}+6 x^{2}-13 x-42$, shown here, what is the sum of all $x$-values for which $f(x)=0$ ?

277. $\qquad$ A function that is defined by more than one equation, each with its own domain, is called a piecewise function. What is the value of $f(1)$ for the piecewise function defined and graphed as shown?

$$
f(x)=\left\{\begin{array}{rr}
x^{2}-2, & x<1 \\
x+1, & x \geq 1
\end{array}\right.
$$


278. $\qquad$ The composition of the function $f$ with the function $g$ is defined as $f(g(x))$. The domain of $f(g(x))$ is the set of all $x$-values such that $x$ is in the domain of $g$, and $g(x)$ is in the domain of $f$. If $f(x)=4 x-6$ and $g(x)=2 x+1$, what is the value of $f(g(2))$ ?
279. $\qquad$ The functions $f$ and $g$ are graphed below. If $g(5)=r$ and $f(r)=s$, what is the value of $s$ ?


280. $\qquad$ What are the values of $a$ and $b$ that complete the table, representing the linear function $k(x)$ ?

| $\boldsymbol{x}$ | $\boldsymbol{k}(\boldsymbol{x})$ |
| :---: | :---: |
| -3 | -5.5 |
| 0 | $b$ |
| 3 | 9.5 |
| $a$ | 17 |

281. $\qquad$ minutes

Working alone, Alice can paint a room in 1 hour. Bob can paint the room in 2 hours alone. How many minutes will it take them to paint the room together?
282. $\qquad$
minutes 30 minutes to learn the posture Working with an instructor doubles his rate of learning How many total minutes will it take James to learn the posture if his instructor arrives after he has studied from the book for 10 minutes?
283. $\qquad$
minutes
A hose could fill a small pool in 50 minutes if the pool did not leak. Alas, the pool leaks at a steady rate that can drain it completely in 300 minutes. How many minutes will it take the hose to fill the leaky pool?
284. $\qquad$ pm A vineyard's grapes can be harvested by 10 workers in 5 hours. If one worker starts the harvest at noon, and another worker joins the harvest each hour on the hour, at what time will the harvest be completed?
285. $\qquad$ Alfonso can write 100 practice problems for the math team in 20 hours, Beauregard can write the same number of problems in 30 hours, and Clyde can write that number of problems in 40 hours. Working together, at what time will the three of them finish writing 100 practice problems if Alfonso starts at noon, Beauregard joins him at 1 pm and Clyde joins them at 2 pm ?
286. $\qquad$ minutes

Vincent can process 50 orders in 2 hours working alone. When Leela is in the room, Vincent works at twice his normal speed. When Fry is in the room, Vincent works at half his normal speed. If Vincent works alone for 10 minutes, then with Leela for 10 minutes, then alone for 10 minutes, then with Fry for 10 minutes, then alone for 10 minutes, and this pattern continues (alone, with Leela, alone, with Fry), how many minutes will it take to process 50 orders?
287. $\qquad$ Larry and Curly are trying to fill a sandbox with sand. Working alone, Larry can fill an empty sandbox in 4 hours, and Curly can do the same job in 5 hours. Moe is trying to empty the sandbox. Working alone, Moe can empty a full sandbox in 6 hours. If the sandbox is half full at the time Larry and Curly begin filling the sandbox and Moe begins emptying it, how many minutes will it take for the sandbox to be filled? Express your answer to the nearest whole number.
288. $\qquad$ Danielle and Jennifer can do a job in 2 hours working together. Danielle could do it in 3 hours alone. How many hours would it take Jennifer to do the job alone?
289. $\qquad$ One hose can fill a pool twice as quickly as another, smaller hose. If the two hoses together can fill the pool in 6 hours, how many hours would it take the smaller hose alone to fill the pool?
290. housekeepers

The Hilbert Lodge has a housekeeping staff of ten. Working alone, one housekeeper can clean all of the rooms in the lodge in 4 hours. A different housekeeper can clean all of the rooms in 5 hours, and still another takes 6 hours to clean all the rooms, working alone. Working alone, each of the remaining seven housekeepers can clean all the rooms in 7, 8, $9,10,11,12$ and 13 hours, respectively. What is the minimum number of housekeepers needed to clean all of the rooms in Hilbert Lodge in exactly 2 hours?

Coordinate geometry is where algebra intersects geometry. The characteristics of equations are made visual by plotting ordered pairs on a Cartesian coordinate plane. The mathematician and philosopher René Descartes is given credit for creating the idea of using a coordinate plane.

Some of the problems that follow involve using the slope, $m=\frac{\Delta y}{\Delta x}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$, and a point, $\left(x_{1}, y_{1}\right)$, or points on a line to determine the equation of the line. The equation of a line can be written in the following forms:

$$
\begin{aligned}
\text { Standard (general) form } & \mathrm{A} x+\mathrm{By}=\mathrm{C} \text {, where } \mathrm{A}, \mathrm{~B} \text { and } \mathrm{C} \text { are integers } \\
\text { Slope-intercept form } & y=m x+b \text {, where } b \text { is the } y \text {-intercept } \\
\text { Point-slope form } & y-y_{1}=m\left(x-x_{1}\right) \\
\text { Intercept form } & \frac{x}{a}+\frac{y}{b}=1 \text {, where } a \text { and } b \text { are the } x \text { - and } y \text {-intercepts, respectively } \\
\text { Two-point form } & \frac{y-y_{1}}{y_{2}-y_{1}}=\frac{x-x_{1}}{x_{2}-x_{1}}
\end{aligned}
$$

The preceding forms may be of use when solving the following problems, but all equation answers should be in slope-intercept form. Any non-integer values should be expressed as common fractions unless otherwise indicated.
291. $\qquad$ 1

What is the midpoint of the line segment whose endpoints are $(-4,-2)$ and $(6,-5)$ ? Express your answer as an ordered pair.
292. $\qquad$ Two lines are parallel if they have the same slope. What is the equation of a line parallel to $y=2 x-5$ that passes through the point $(3,5)$ ?
293. $\qquad$ Two lines are perpendicular if the product of their slopes is -1 . In other words, their slopes are opposite inverses (or negative reciprocals). What is the equation of the line perpendicular to $y=\frac{2}{3} x-\frac{1}{3}$ at the point $(-4,-3)$ ?
294. $\qquad$ What is the equation of a line with $x$-intercept $(-3,0)$ and $y$-intercept $(0,6)$ ?
295. $\qquad$ A line has a slope of $\frac{1}{2}$ and a $y$-intercept of $(0,-3)$. That line intersects the line $2 x+3 y=-2$ at a point. What is the sum of the coordinates of the point?
296. $\qquad$ A circle can be reprsented by an equation in the form $(x-h)^{2}+(y-k)^{2}=r^{2}$, where $(h, k)$ are the coordinates of the center of the circle, and $r$ is the radius of the circle. The graph of $x^{2}+y^{2}=25$ is a circle with center $(0,0)$ and a radius of 5 units ( $r^{2}=25$, from the equation). The graph of the line $y=x-7$ intersects this circle at two points. What is the sum of the $x$-coordinates of these two points?
297. $\qquad$ The point $(-2,10)$ lies on the circle $(x-3)^{2}+(y+2)^{2}=169$. What is the equation of the line tangent to that circle at that point?
298. $\qquad$ The two circles, $x^{2}+y^{2}=25$ and $(x-7)^{2}+(y-7)^{2}=25$, have a common chord. What is the equation of the line containing that chord?
299. $\qquad$ units

Based on the Pythagorean Theorem, the distance $d$ between two points $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$ in the coordinate plane is $d=\sqrt{\left(y_{2}-y_{1}\right)^{2}+\left(x_{2}-x_{1}\right)^{2}}$. What is the length of the common chord in question 298? Express your answer in simplest radical form.
300. $\qquad$ Triangle $A B C$ has vertices $A(3,2), B(-2,1)$ and $C(6,-5)$. What is the equation of the line containing the altitude from vertex $A$ to side $B C$ ?

## ANSWERS

In addition to the answer, we have provided a difficulty rating for each problem. Our scale is 1-7, with 7 being the most difficult. These are only approximations, and how difficult a problem is for a particular student will vary. Below is a general guide to the ratings:

Difficulty 1/2/3-One concept; one- to two-step solution; appropriate for students just starting the middle school curriculum.
4/5 - One or two concepts; multistep solution; knowledge of some middle school topics is necessary. 6/7-Multiple and/or advanced concepts; multistep solution; knowledge of advanced middle school topics and/or problem-solving strategies is necessary.


[^0]|  | MarmaUo 5 |  |  |
| :---: | :---: | :---: | :---: |
| Answer | Difficulty |  |  |
| 61. 32 | (2) | 66. 2 | (3) |
| 62. 3 | (3) | 67. 3 | (4) |
| 63. $\sqrt{ } / 2$ | (5) | 68. $(1,-4)$ | (5) |
| 64. 20 | (5) | 69. 90 | (3) |
| 65. -1 | (3) | 70. 12 | (4) |

61. 32
62. 3
63. V3/2
64. -1

## Answer

71. 2
72. 6
73. V34
74. $-1 / 13$
75. 3

## Warm-Up 6

## Difficulty

(2)
(3)
(3)
(3)
(4)
80. 6
76. $7 / 12$
77. 8
78. 30
79. 6

## Workout 3

Answer
81. 38.4
82. 23.5
83. $14 / 31$
84. 3.16
85. 72

Difficulty
(4)
86. 80
(3)
(4)
(2)
(4)
90. 19
(3)
(3)
(3)
(4)
(2)
(3)
(3)
(3)
(4)
(3)

## Answer

91. 20
92. 12
93. 16
94. 24
95. $\quad 6 a^{3} b^{4}$

## Warm-Up 7

Difficulty
(2)
96. $80 \mathrm{~V} 2-80$
(5)
(3)
(2)
(3)
(3)
97. 35

## Answer

101. 24
102. $1 / 8$
103. $1 / 2$
104. 90
105. 10

## Warm-Up 8

## Difficulty

(3)
(4)
(5)
(4)
(3)

## Workout 4

## Answer

111. 90
112. 20,000 or 20,000.00
113. 9.6
114. 11
115. 2.4

Difficulty
(4)
(3)
(4)
(3)
(4)
(3)
(4)

Answer
121. $4 / 15$
122. 62
123. 147
124. 116
125. 4

## Warm-Up 9

## Difficulty

(4)
(3)
(3)
(3)
(5)
130. 8
(3)
(4)
(3)
128. 45
129. 3
126. 30
127. $w / 2$
(3)

Answer
151. 24
152. 15
153. 7
154. $-2 \frac{1}{2}$
155. $1 / 8$

## Warm-Up 11

Difficulty
(5)
156. $1 / 32$

Answer
131. 30
132. 177
133. $\pi-2$
134. 13
135. 42

Answer
141. 6
142. 10.24
143. 16.5
144. 1600
145. 2.69

Difficulty
(4)
(2)
(4)
(5)
(5)
5)
140. 5 V 6

## Workout 5

## Difficulty

(5)
(3)
(4)
(4)
(5)
150. 17
(4)
(4)
(4)
(4)
(5)
146. 384
147. 8
148. 69.76
149. 11

## Answer

171. 15
172. 5.12
173. 16.45
174. $11 / 204$
175. 1.5

## Answer

 161. $1 / 5$162. 6
163. $1 / 6$
164. $3 a+b+c / 12$
165. $4 / 3$

## Workout 6

Difficulty
(4)
(4)
(4)
(3)
(5)

Difficulty
(3)
176. 8
(3)
(5)
(4)
(4)

166. 1006/1007
(5)
(4)
(3)
(4)
(4)

| Answer | MarmiUp 3 |  |  |
| :---: | :---: | :---: | :---: |
|  | Difficulty |  |  |
| 181. 1520 | (3) | 186. $4 / 5$ | (4) |
| 182. 39 | (4) | 187. 16 | (3) |
| 183. $24 \sqrt{ } 3$ | (4) | 188. 72 | (3) |
| 184. 11 | (5) | 189. 4 | (3) |
| 185. 55,555 | (4) | 190. 6V3 | (4) |

Answer
211. 3/8
212. 22
213. 12
214. 11
215. 2

Warm-Up 14

Answer
191. 12
192. $1 / 4$
193. 288
194. 7/6
195. 15

Difficulty
(4)
(4)
(5)
(3)
(3)
200. 1331
196. 16
197. 3
198. 54
199. 0

## Workout 7

 Difficulty(4)
206. 2.73
(5)
(4)
(4)
(3)
210. 36
(4)
(5)
(4)
(5)
(4)

Answer
201. 7
202. 15.52
203. 31
204. 16
205. 20
(4)
(4)
(4)
(5)
(5)
)
)
Answer
221. 8
222. 6
223. 60
224. 108
225. $3 / 25$

Warm-Up 15
Difficulty
(4)
216. 27
217. 24
(4)
(4)
(5)
(5)

## Warm-Up 16

Difficulty
(3)
(4)
(4)
(4)
(5)
)
226. 30
(5)
(4)
(4)
(5)
(5)

## Workout 8

## Answer

231. 14
232. 10
233. 0.56
234. 14
235. 3/7

Difficulty
(4) 236. 319
(4) 237. $14,762.25$
(5) 238. 120
(4) 239. 112/3
(6)
240. 0.81
(3)
241. 5/3
242. 9
243. 180
244. $-116 / 195$
245. 35

## Answer <br> Warm-Up 17 <br> Difficulty

(3) 246. 1/10
(4)
(4)
(5)
(4)
(5)
250. 100
248. 15
249. $27 / 125$
(3)
(4)
(5)
(5)

## Functions Stretch

Answer
271. 7
272. 6
273. 6
274. 4
275. -3

Difficulty
(3)
276. -6
277. 2
278. 14
279. 4
(3)
)
280. $a=6, b=2$

## Work Stretch

## Answer

281. 40
282. 20
283. 60
284. 9:30
285. 10:00

Difficulty
(3)
(4)
(4)
(5)
(5)
260. 2012

## Workout 9

Answer
261. 3281
262. 45
263. 0
264. 25
265. 60.75

Difficulty
(5)
266. 16
(5)
(3)
(4)
(5)
(5)
(4)
(5)
(4)
(4)
)
)

## Coordinate Geometry Stretch

| Answer  <br> 291. $\left(1,-\frac{7}{2}\right)$ (3) | 296. 7 | (5) |  |
| :--- | :--- | :--- | :--- |
| 292. $y=2 x-1$ | (4) | 297. $y=\frac{5}{12} x+\frac{65}{6}$ | (5) |
| 293. $y=-\frac{3}{2} x-9$ | (4) | 298. $y=-x+7$ |  |
| 294. $y=2 x+6$ | (4) | 299. $\sqrt{ } 2$ |  |
| 295. 0 | (5) | 300. $y=\frac{4}{3} x-2$ |  |

## PROBLEM INDEX

It is difficult to categorize many of the problems in the MATHCOUNTS School Handbook. It is very common for a MATHCOUNTS problem to straddle multiple categories and cover several concepts. This index is intended to be a helpful resource, but since each problem has been placed in exactly one category and mapped to exactly one Common Core State Standard (CCSS), the index is not perfect. In this index, the code $\mathbf{1 0}$ (3) $7 . R P .3$ refers to problem 10 with difficulty rating 3 mapped to CCSS 7.RP.3. For an explanation of the difficulty ratings refer to page 61. For an explanation of the CCCS codes refer to page 87.



|  | 7 | (2) | 7.SP. 7 |
| :---: | :---: | :---: | :---: |
|  | 25 | (2) | 7.SP. 8 |
|  | 45 | (3) | SMP |
|  | 54 | (4) | S-CP. 9 |
|  | 61 | (2) | 7.SP. 8 |
|  | 64 | (5) | S-CP. 9 |
|  | 76 | (3) | 7.SP. 7 |
|  | 83 | (4) | 7.SP. 8 |
|  | 94 | (3) | SMP |
|  | 103 | (5) | SMP |
|  | 106 | (5) | 7.SP. 5 |
|  | 114 | (3) | SMP |
|  | 119 | (4) | SMP |
|  | 121 | (4) | 7.SP. 8 |
|  | 134 | (5) | SMP |
|  | 149 | (4) | SMP |
|  | 155 | (4) | 7.SP. 8 |
|  | 156 | (4) | 7.SP. 8 |
|  | 161 | (4) | 7.SP. 8 |
|  | 174 | (4) | 7.SP. 8 |
|  | 179 | (4) | S-CP. 9 |
|  | 186 | (4) | 7.SP. 8 |
|  | 193 | (5) | 7.SP. 8 |
|  | 205 | (3) | SMP |
|  | 218 | (4) | 7.SP. 5 |
|  | 225 | (5) | S-CP. 9 |
|  | 235 | (6) | 7.SP. 8 |
|  | 246 | (4) | 7.SP. 8 |
|  | 248 | (4) | SMP |
|  | 255 | (5) | A-REI. 4 |
|  | 270 | (4) | SMP |
|  | 3 | (2) | 4.0A. 2 |
|  | 12 | (3) | 6.SP. 4 |
|  | 36 | (2) | 6.SP. 5 |
|  | 71 | (2) | 6.SP. 5 |
|  | 107 | (5) | 6.SP. 5 |
|  | 124 | (3) | 7.SP. 1 |
|  | 127 | (4) | SMP |
|  | 143 | (4) | 6.SP. 5 |
|  | 147 | (4) | 6.SP. 5 |
|  | 157 | (4) | 6.RP. 3 |
|  | 206 | (4) | 7.SP. 1 |
|  | 213 | (5) | 6.SP. 5 |
|  | 215 | (6) | 6.SP. 5 |
|  | 229 | (5) | 6.SP. 5 |
|  | 239 | (5) | SMP |
|  | 247 | (3) | 6.SP. 5 |


[^0]:    * The plural form of the units is always provided in the answer blank, even if the answer appears to require the singular form of the units.

