

# Warm-Up 1

1. \_\_\_\_\_ What is the value of  $1 + 2 + 3 + \dots + 7 + 8 + 9 + 8 + 7 + \dots + 3 + 2 + 1$ , where all of the integers from 1 through 9 and then back down to 1 are added together?

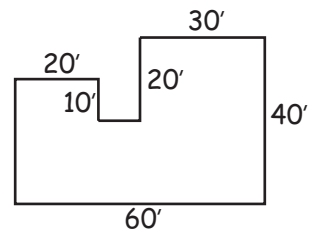
2. \_\_\_\_\_ What is the value of  $(2x + 5)^2$  when  $x = 3$ ?

3. \_\_\_\_\_ feet A toy car travels one foot in 10 seconds. At this rate, how far will the car travel in two minutes?

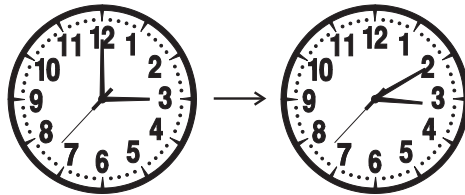


4. \_\_\_\_\_ Place one member of the set  $\{1, 2, 3, 4\}$  into each blank in the expression  $\_\_ (\_\_ + \_\_) + \_\_$ , such that each member of the set is used exactly once and the value of the expression is as small as possible. What is the value of the expression?

5. \_\_\_\_\_ sq ft The floor plan for a single-story house is shown. Each pair of consecutive sides is perpendicular. What is the area of the floor for the house whose dimensions are given?



6. \_\_\_\_\_ degrees



How many degrees does the minute hand of a clock move during a 10-minute period?


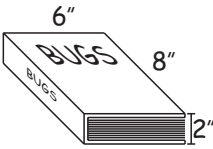
7. \_\_\_\_\_ cups A recipe uses two eggs, three cups of flour and some other ingredients. The recipe makes enough batter for six servings. How many cups of flour will be needed to make enough batter for 10 servings?

8. \$ \_\_\_\_\_ Aaron worked 10 hours per week for six weeks at his part-time office job, where he earned \$8 per hour. He also worked full-time at his regular job, earning \$400 per week. What were Aaron's total earnings for the six-week period?

9. \_\_\_\_\_ kg A science workbook is one centimeter thick and weighs 75 grams. How much will a one-meter-high stack of these workbooks weigh, in kilograms? Express your answer as a decimal to the nearest tenth.

10. \_\_\_\_\_ years Three years ago Mary was three times as old as her sister. Now Mary is twice as old as her sister. How old is Mary now?

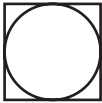
# Warm-Up 2

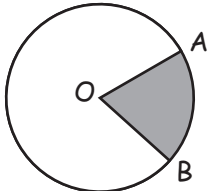
- <sup>pieces</sup> Grady distributed  $x$  pieces of candy evenly among nine Halloween bags such that every bag received the greatest possible number of whole pieces of candy, but some candy was left over. What is the greatest possible number of pieces that could have been left over?
- <sup>sq in</sup> Noah will mount a 5-inch by 5-inch photograph on an 8-inch by 10-inch mat board. How many square inches of mat board will be visible?  

- <sup>layers</sup> When a piece of paper is folded in half, there are two layers of paper. How many layers would there be if a piece of paper is folded in half a total of 10 times without ever unfolding it?
- <sup>\$</sup> Pi plates cost \$24 each. Shipping costs \$10 for orders under \$100 and \$15 for orders of \$100 or more. How much more does it cost to order and have delivered five Pi plates instead of four Pi plates?
- <sup>\$</sup> Charlene considers herself to be a great bargain shopper because she found a prom dress that cost her only \$22 before tax. The dress was on a rack labeled "50% off lowest marked price," and the lowest marked price was already a 75% reduction from the original price. What was the original price of the dress?
- <sup>inches</sup> Fisher wrote a book about bugs. He created the front cover, spine and back cover from one continuous piece of  $\frac{1}{4}$ -inch thick cardboard. The cardboard fits exactly over the pages, and the final dimensions of the book are 6 inches by 8 inches by 2 inches, as shown. If the entire cardboard piece (front, spine and back) were removed, what would be the sum of the length, width and thickness of the remainder of the book (the stack of paper pages)? Express your answer as a mixed number.  

- <sup>units</sup> Triangle ABC has a perimeter of 2007 units. The sides have lengths that are all integer values with  $AB < BC \leq AC$ . What is the smallest possible value of  $BC - AB$ ?
- For all positive integers  $n$ , the expression  $n!$  denotes the product of the first  $n$  positive integers. When  $5!$  is expressed as an integer, what is the ones digit?
- <sup>nurses</sup> A month ago the ratio of nurses to doctors on a hospital staff was 3:5. Since that time two additional nurses joined the staff, no nurses left and the number of doctors remained the same. The ratio of nurses to doctors on the hospital staff is now 4:5. How many nurses are now on the staff?
- <sup>feet</sup> Cooling a room requires 27 BTUs per square foot per hour. Jose buys an air conditioner with a power of 10,800 BTUs per hour. What is the width of the largest square room that the air conditioner can cool?

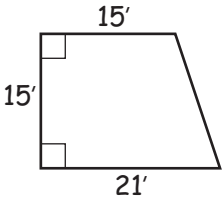
# Workout 1

## Quiz Scores

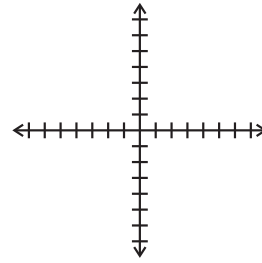
6	3 5 5
7	1 7 9 9 9 9
8	1 3 4 4 5 9 9
9	0 1 2 2 3 5 8 8

- \_\_\_\_\_ points The stem-and-leaf plot shows the scores on Mrs. Norris' last quiz for her third period class. (6|3 represents 63 points.) What was the mean score on this quiz? Express your answer to the nearest whole number.
- \_\_\_\_\_ \$ Tansy wanted to leave the waiter an 18% tip, and her bill was \$21.50. How much will the total cost of her dinner and tip be if she rounds the tip itself to the next higher dollar?
- \_\_\_\_\_ What is the largest possible value of  $x$  given that  $x^3 = 5x$ ? Express your answer in simplest radical form.
- \_\_\_\_\_ cu in The exterior dimensions of an empty cooler in the shape of a rectangular prism are 18 inches by 18 inches by 24 inches. For insulation, the top, the bottom and each of the four walls are two inches thick. What is the volume of the empty space inside the cooler?
- \_\_\_\_\_ sq cm  This square has an area of 49 sq cm. What is the area of the inscribed circle? Express your answer as a decimal to the nearest tenth.
- \_\_\_\_\_ \$ Joe's Store guarantees that it will refund what you paid for an item and give you an additional 10% if you find the item for less at a different store. Li paid \$137.80 for a DVD player at Joe's Store. Li then found an ad for the same DVD player at Mike's Store for less. How much money should Joe's Store give to Li?
- \_\_\_\_\_ degrees In the circle with center  $O$ , the shaded region is 20% of the area of the entire circle. What is the measure of angle  $AOB$ ?


- \_\_\_\_\_ degrees In a triangle with three distinct angle measures, the smallest angle measures  $30^\circ$ . The measures of the other two interior angles are each a whole number of degrees. What is the measure of the largest possible angle in the triangle?
- \_\_\_\_\_ \$ A roll of carpet is 15 feet wide. The carpet costs \$20 per square yard. A customer must purchase the carpet in a piece that is 15 feet in width, but the carpet can be cut to any length. Beth wants to use exactly one piece of carpet to cover a floor that is trapezoidal in shape, as shown, rather than putting multiple pieces together. However, she must also pay for any wasted carpet. What is the cost of the shortest length of carpet that she can use?


- \_\_\_\_\_ What is the value of  $x$  such that  $(x, 0)$  is a solution of the equation  $y = 3x - 4$ ? Express your answer as a common fraction.

# Warm-Up 3



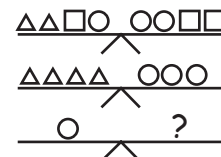
1. sq units What is the area of the circle that is centered at the origin and is tangent to the line  $y = 7$ ? Express your answer in terms of  $\pi$ .
2. \_\_\_\_\_ When the expression  $3^{444} + 4^{333}$  is written as an integer, what is the units digit?
3. \_\_\_\_\_ The sum of three numbers is 98. The ratio of the first to the second is 2:3, and the ratio of the second to the third is 5:8. What is the value of the second number?
4. \_\_\_\_\_ What is the value of  $-(1^{2008}) + (-1)^{2007}$ ?
5. cups A cider recipe created by the Xpress Co. combines nine cups of applesauce with one cup of water. The mixture is brought to a boil and then simmered until the volume is reduced by 15%. If a cider batch begins with 36 cups of applesauce, how much cider will there be after the appropriate amount of water is added and the simmering process is complete?

6. \_\_\_\_\_ A total of 180 marbles (gray, white and black) are placed in a line. The first five are gray, followed by four white, followed by three black, followed by five gray, followed by four white, followed by three black, ... . If this pattern continues, what is the color of the 158<sup>th</sup> marble in this line?



7. \_\_\_\_\_ What is the value of  $n$  for which  $(3!)(5!)(7!) = n!$ ?
8. inches A regular hexagon is inscribed in a circle of radius four inches. What is the perimeter of the hexagon?
9. \_\_\_\_\_ If a standard six-sided die is rolled twice, what is the probability that the result of the second roll is not less than the result of the first roll? Express your answer as a common fraction.

10. squares If only squares may be used, how many squares must be placed on the right side of the third scale so that all three scales are balanced? (The distance of the objects from the centers of these scales is not relevant.)

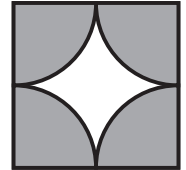


# Warm-Up 4

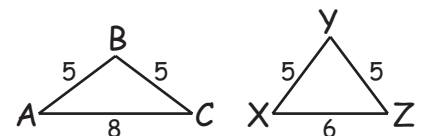
1.            \$ The shipping cost of a dozen lemons is directly proportional to how many miles they are shipped. A dozen lemons shipped the 200 miles to Taraville cost \$5 to ship. How much will it cost to ship a dozen lemons 1000 miles?



2.            sq cm Four congruent quarter-circles are drawn inside a square of side length 4 centimeters, as shown. What is the area of the shaded portion of the square region? Express your answer in terms of  $\pi$ .



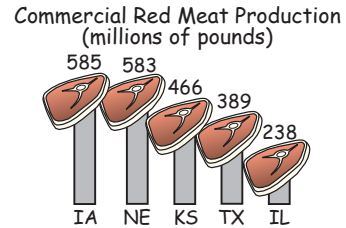
3.            ways If all of the letters of the word BEEP are used, in how many different ways can the four letters be arranged in a four-letter sequence? The two Es are indistinguishable.
4.            What is the greatest possible value of  $a$  in the system of equations  $5a + 2b = 0$  and  $ab = -10$ ?
5.            hours Office Surplus promises delivery of orders within three working hours of receipt of any order. Working hours are 8 a.m. to 5 p.m., Monday through Friday. Clyde's order is received at 3 p.m. on a Friday and takes the maximum promised number of working hours to be delivered. How many actual hours elapse from the time the order is placed until it is delivered?
6.            integers How many positive integers less than 1000 can be written using only the digits 0, 1 and 2? Two such integers to include are 101 and 12.
7.            Friday the 13<sup>th</sup>s The first day of the year 2006 was a Sunday. How many Friday the 13<sup>th</sup>s were there in 2006?
8.            What is the value of  $(4^3) \div (2^2)$ ? Express your answer in the form  $a^b$  where  $a$  and  $b$  are positive integers and  $a$  has the least possible value.
9.            Several points are plotted on a graph. For each point, the  $x$ -coordinate is the length of a side of a square while the  $y$ -coordinate is the perimeter of that same square. One such point is (2, 8) since a square with side length 2 units has a perimeter of 8 units. What is the slope of the line connecting the points? Express your answer in simplest form.
10.            sq units Triangle ABC has side lengths 5, 5 and 8 units. Triangle XYZ has side lengths 5, 5 and 6 units. What is the difference of the areas of these two triangles?



# Workout 2

1. \_\_\_\_\_ yen On a certain day, 10 U.S. dollars were worth 7.60 euros, and 1 euro was worth 155 Japanese yen. How many yen were the 10 U.S. dollars worth?
2. \_\_\_\_\_ If  $\sqrt{225+64} - \sqrt{147 - \sqrt{n}} = \sqrt{25}$ , what is the value of  $n$ ?
3. \_\_\_\_\_ watts Lighting experts recommend 150 to 200 watts of illumination for every 50 square feet of floor space. What is the minimum number of watts recommended for a room with a rectangular floor measuring 30 feet by 40 feet?

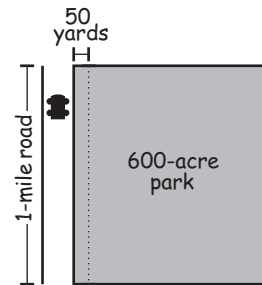
4. \_\_\_\_\_ % By what percent is the commercial red meat production for Iowa (IA) greater than that for Texas (TX), according to the data shown? Express your answer to the nearest whole number.



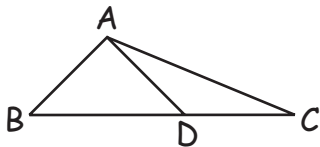
5. \_\_\_\_\_ white gumballs Anna bought 12 pieces of gum consisting of only red gumballs and white gumballs. The total cost was \$1.29. The red gumballs each cost three cents more than each white gumball, and she bought fewer red gumballs than white gumballs. How many white gumballs did she buy?

6. \_\_\_\_\_ If  $k$  is an integer and  $k > 100$ , what is the smallest possible integer value of  $\sqrt[3]{k^2}$ ?

7. \_\_\_\_\_ deer Jared wants to estimate the number of deer in a 600-acre state park. When driving along the one-mile road that is one side of the park, he can see an average distance of 50 yards into the park and counts 7 deer. An acre is 4840 square yards, and a mile is 1760 yards. If the deer are evenly distributed throughout the park, what is the best estimate of the number of deer in the park?



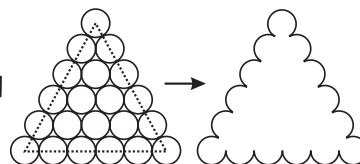
8. \_\_\_\_\_ degrees



In the figure,  $BA = AD = DC$  and point  $D$  is on segment  $BC$ . The measure of angle  $ADC$  is 135 degrees. What is the measure of angle  $ABC$ ?

9. \_\_\_\_\_ \$ In forming his budget for this year, Jamal decided to use his average cost for utilities last year to project his expenses for the future. Last year he spent an average of \$216 per month on utilities, but he anticipates a 5% increase in the annual cost of utilities. Based on this information, how much should he expect to pay for utilities each month this year?

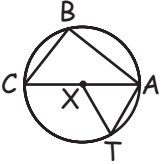
10. \_\_\_\_\_ cm Twenty-one congruent circular discs are stacked in a triangular arrangement, as shown. Connecting the centers of the three vertex discs forms an equilateral triangle. The circumference of each disc is 18 cm. What is the outside perimeter of the arrangement?




Information for problem #4 is from *USA TODAY Snapshots*®, February 8, 2006. Source: USDA.

# Warm-Up 5

- books Eighty books will be divided among six people. No two people will receive the same number of books. The person who receives the most books will receive 20; the person who receives the fewest books will receive seven. What is the maximum number of books the person with the third-largest number of books could receive?
- units In the circle with center  $X$ , the measure of angle  $AXT$  is  $60^\circ$ , and the measure of angle  $ABC$  is  $90^\circ$ . The length of segment  $AT$  is 5 units, and the length of segment  $BC$  is 6 units. What is the length of segment  $AB$ ?


- sq ft A 40-foot by 10-foot rectangular garden is enclosed by a fence. To make the garden larger, while using the same amount of fencing, its shape is changed to a square. How many square feet larger than the old garden is the new garden?
- combos Rita is selecting a sandwich at the deli. The deli has four types of meat, three types of cheese and two types of bread. A deluxe sandwich consists of exactly one meat type, two different types of cheese and one bread type. How many different deluxe sandwich combinations are possible?
- cents One zip, two zaps and three zups cost \$1.50. Two zips, three zaps and one zup cost \$1.00. Three zips, one zap and two zups cost \$1.40. What is the total cost, in cents, of one zip, one zap and one zup?
- wrappers A rectangular candy wrapper is made from a one-inch by three-inch piece of paper. What is the greatest number of wrappers that can be cut from a rectangular piece of paper measuring one foot by two feet?


- % The stocks of five companies had the following percent changes in 2006: +9.9%, +5.7%, -7.2%, +1.8% and -0.9%. What is the average percent change in 2006 for these five companies? Express your answer to the nearest hundredth.
- \_\_\_\_\_ Audrey's 300-mile trip took six hours to complete. She spent  $\frac{2}{3}$  of the total time of her trip going the first  $\frac{1}{3}$  of the distance. What is the ratio of her average speed during that first  $\frac{2}{3}$  of the time to her average speed during the remaining  $\frac{1}{3}$  of the time? Express your answer as a common fraction.
- sq ft A large game field has an area of 400 square feet. A small practice field is made with every linear dimension half as large as the game field's dimensions. What is the area of the small practice field?
- units A triangle has sides of length 5 and 6 units. The length of the third side is  $x$  units, where  $x$  is an integer. What is the largest possible perimeter of the triangle?

# Warm-Up 6

1. \_\_\_\_\_ What is the simplified value of  $(2 + 4 + 6 + \dots + 48 + 50) - (1 + 3 + 5 + \dots + 47 + 49)$ ?

2. \_\_\_\_\_ For prime numbers  $p$  and  $q$ ,  $p + q = 102$  and  $p > q$ . What is the least possible value of  $p - q$ ?

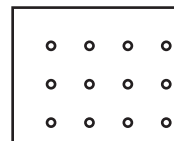
3. \_\_\_\_\_

Sally	
1. F	6. F
2. T	7. F
3. F	8. T
4. F	9. F
5. T	10. T

Sally is taking a test with 10 questions. Each answer is either True or False. She knows the correct answer for five of the questions and randomly guesses on the other five questions. What is the probability that she will have at least eight correct answers on the test? Express your answer as a common fraction.

4. \_\_\_\_\_ The sum of two numbers is 32, and the product of these two numbers is 48. What is the sum of the reciprocals of the two numbers? Express your answer as a common fraction.

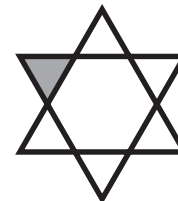
5. \_\_\_\_\_ sequins Fabric is run through a machine that fastens sequins in rows and columns such that each sequin is four inches vertically and horizontally from the next closest sequins, forming a grid as shown. There is always a four-inch space with no sequins at the beginning, end and side margins of the fabric. If a piece of fabric 60 inches by 72 inches is fed through the machine, how many sequins will be attached?



6. \_\_\_\_\_ What is the value of  $(2x^3) \div (2x)^3$  when  $x = 2007$ ? Express your answer as a common fraction.

7. \_\_\_\_\_ sq units What is the area of the quadrilateral with vertices at  $(1, 1)$ ,  $(5, 2)$ ,  $(4, 4)$  and  $(2, 3)$ ?

8. \_\_\_\_\_ A six-pointed star is formed by overlapping two congruent, equilateral triangles, placed such that each side of the original triangles is trisected. What fraction of the area of the six-pointed star is the shaded triangle? Express your answer as a common fraction.



9. \_\_\_\_\_ sq meters Alexander used exactly 20 meters of fencing around three sides of a rectangular flower bed beside his house. He did not fence the fourth side, which is an eight-meter section along the side of the house. What is the area of this flower bed?

10. \_\_\_\_\_ units Circles with centers at  $(2, 2)$  and  $(17, 10)$  are both tangent to the  $x$ -axis. What is the distance between the closest points of the two circles?



# Workout 3

- \_\_\_\_\_ <sup>feet</sup> The formula for the total surface area of a cylinder is  $SA = 2\pi r^2 + 2\pi rh$ , where  $r$  is the radius and  $h$  is the height. A particular solid right cylinder of radius 2 feet has a total surface area of  $12\pi$  square feet. What is the height of this cylinder?
- \_\_\_\_\_ A cube varies in size over time, with its largest volume being twice its smallest volume. What is its largest edge length divided by its shortest edge length? Express your answer as a decimal to the nearest hundredth.
- \_\_\_\_\_ What is the value of  $x^5 - 5x$  when  $x = 4$ ?
- \_\_\_\_\_ % Alicia recorded the statistics below at last night's basketball game. The percents were rounded to the nearest whole number, and only whole numbers of shots could be successful. What percent of the 45 attempted shots were successful?

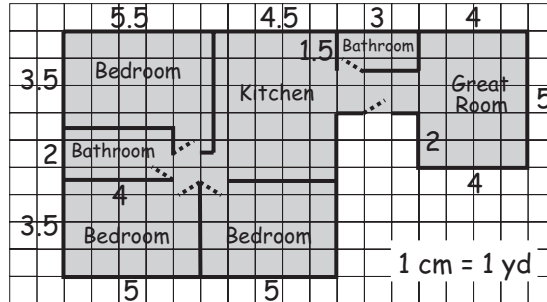
Player Name	2-Point Shots		3-Point Shots	
	Attempted	% Successful	Attempted	% Successful
Lisette	15	67	5	40
Sara	10	80	2	50
Jen	4	75	1	0
Tai	6	33	2	50

- \_\_\_\_\_ The Moisture Co. produces wipes that are sold 70 to a rectangular carton measuring 6" by 4" by 2". The carton is completely filled. The company wants to sell a carton with dimensions 6" by 4" by  $y$ " containing 100 wipes. What is the least possible value of  $y$ ? Express your answer as a decimal to the nearest tenth.
- \_\_\_\_\_ If  $40\blacklozenge$  represents a positive three-digit integer with a ones digit of  $\blacklozenge$ , and  $1\blacklozenge$  is a positive two-digit integer with a ones digit of  $\blacklozenge$ , what value of  $\blacklozenge$  makes the equation  $40\blacklozenge \div 27 = 1\blacklozenge$  true?
- \_\_\_\_\_ <sup>inches</sup> A straight, vertical 43-inch wick was lit from the top and burned downward at a constant rate of 1.5 inches per hour. Three hours later, the bottom of the wick also was lit, and it burned upward at a constant rate of 4.0 inches per hour. How many total inches had the top of the wick burned down when the burning ends met?
- \_\_\_\_\_ The sum of four consecutive integers is 2. What is the smallest of the four integers?
- \_\_\_\_\_ <sup>feet per second</sup> The distance traveled by a ball rolling down a ramp is given by the equation  $s(t) = 5t^2$ , where  $t$  is the time after the ball is released, in seconds, and  $s(t)$  is measured in feet. What is the average speed of the ball for the time  $t = 3$  through  $t = 4.5$ ? Express your answer as a decimal to the nearest tenth.
- \_\_\_\_\_ <sup>sq in</sup> In triangle ABC the length of the altitude to side AB is 6 inches, and the measures of angles A and B are 45 degrees and 60 degrees, respectively. What is the area of triangle ABC? Express your answer as a decimal to the nearest tenth.

Problem #6 is from the Team Round of the 2007 MATHCOUNTS Chapter Competition.

# Proportional Reasoning Stretch

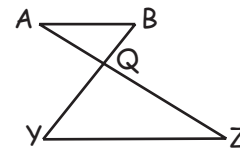
Questions #1-5 refer to the house floor plan shown below. In the figure, each pair of consecutive sides forms a right angle. The plan is drawn on graph paper with all dimensions given in centimeters.



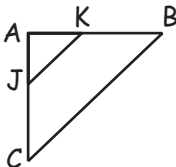
Front

- \_\_\_\_\_ feet What is the length, in feet, of the house (across the back)?
- \_\_\_\_\_ feet What is the greatest width, in feet, of the house (front to back)?
- \_\_\_\_\_ sq feet How many square feet of floor space are represented in the floor plan of the house?
- \_\_\_\_\_ sq feet How many square feet of carpet will be needed to carpet the Great Room?
- \_\_\_\_\_ tiles The two bathroom floors are to be tiled using 6-inch by 6-inch square tiles. How many tiles will be needed if allowance is not made for bathroom fixtures?

- \_\_\_\_\_ units In the figure to the right, segment AB is parallel to segment YZ. If AZ = 42 units, BQ = 12 units and QY = 24 units, what is the length of segment QZ?



- \_\_\_\_\_ units In the figure, triangle AJK is a right triangle with angle A a right angle and segment JK parallel to segment CB. If AK = 21 units, JK = 29 units and AC = 50 units, what is the length of segment CB? Express your answer as a decimal to the nearest tenth.



- \_\_\_\_\_ If  $3x = 8y$  when  $5y = 15z$ , what is the simplified value of  $\frac{x}{z}$ ?
- \_\_\_\_\_ When a square is enlarged such that its new perimeter is three times its original perimeter, what is the ratio of the square's original area to its new area? Express your answer as a common fraction.
- \_\_\_\_\_ free throws Owen made exactly 72% of his free throws during the first half of the basketball season, and he made six free throws during the second half of the season. For the entire season he made exactly 60% of his free throws. What is the fewest possible number of free throws Owen could have attempted during the second half of the season?

# Warm-Up 1

## Answers

- |        |                 |         |           |                    |              |
|--------|-----------------|---------|-----------|--------------------|--------------|
| 1. 81  | (C, F, P, S, T) | 5. 2000 | (C, F, M) | 8. 2880 or 2880.00 | (C)          |
| 2. 121 | (C)             | 6. 60   | (C, F, P) | 9. 7.5             | (C)          |
| 3. 12  | (C, F)          | 7. 5    | (C, F, P) | 10. 12             | (C, F, G, T) |
| 4. 9   | (C, E, G, T)    |         |           |                    |              |

# Warm-Up 2

## Answers

- |                |                 |                    |                    |        |                    |
|----------------|-----------------|--------------------|--------------------|--------|--------------------|
| 1. 8           | (C, F, M)       | 5. 176 or 176.00   | (C, F)             | 8. 0   | (C)                |
| 2. 55          | (C, F, M, S)    | 6. $15\frac{1}{4}$ | (C, M)             | 9. 8   | (C, E, F, G, S, T) |
| 3. 1024        | (C, M, P, S, T) | 7. 1*              | (C, G, M, P, S, T) | 10. 20 | (C, M, S)          |
| 4. 29 or 29.00 | (C, F)          |                    |                    |        |                    |

# Workout 1

## Answers

- |               |           |           |           |                   |              |
|---------------|-----------|-----------|-----------|-------------------|--------------|
| 1. 83         | (C, F, G) | 5. 38.5   | (C, F, M) | 8. 119            | (C, F, M, S) |
| 2. 25.50      | (C)       | 6. 151.58 | (C, F)    | 9. 700 or 700.00  | (C, F, M)    |
| 3. $\sqrt{5}$ | (C, E, G) | 7. 72     | (C, F, S) | 10. $\frac{4}{3}$ | (C, F)       |
| 4. 3920       | (C, F, M) |           |           |                   |              |

# Warm-Up 3

## Answers

- |            |                 |         |                    |                   |                 |
|------------|-----------------|---------|--------------------|-------------------|-----------------|
| 1. $49\pi$ | (F, M)          | 5. 34   | (C, F, M, T)       | 8. 24             | (C, F, M)       |
| 2. 5       | (C, M, P, S, T) | 6. gray | (P, S)             | 9. $\frac{7}{12}$ | (C, M, P, T)    |
| 3. 30      | (C, F, G, M, T) | 7. 10   | (C, F, G, P, S, T) | 10. 2             | (C, M, P, S, T) |
| 4. -2      | (C, P)          |         |                    |                   |                 |

\* The plural form of the units will always be provided in the answer blank even if the answer appears to require the singular form of units.

# Warm-Up 4

## Answers

- |                          |                       |                   |
|--------------------------|-----------------------|-------------------|
| 1. 25 or 25.00 (F, M, P) | 5. 66 (C, M, P, S, T) | 8. $2^4$ (C)      |
| 2. $4\pi$ (C, F, M, S)   | 6. 26 (M, P, S, T)    | 9. 4 (C, F, M, T) |
| 3. 12 (C, F, P, S, T)    | 7. 2 (C, M, P, T)     | 10. 0 (C, F, M)   |
| 4. 2 (C, F, G, S, T)     |                       |                   |

# Workout 2

## Answers

- |                   |                      |                   |
|-------------------|----------------------|-------------------|
| 1. 1178 (C, T)    | 5. 9 (C, E, F, G, T) | 8. 45 (C, F)      |
| 2. 9 (C, E, G)    | 6. 25 (C, F, G, S)   | 9. 226.80 (C)     |
| 3. 3600 (C, F, M) | 7. 231 (C, F)        | 10. 153 (C, F, P) |
| 4. 50 (C)         |                      |                   |

# Warm-Up 5

## Answers

- |                          |                         |                               |
|--------------------------|-------------------------|-------------------------------|
| 1. 17 (C, E, M, G, T)    | 5. 65 (C, F, G, M, P)   | 8. $\frac{1}{4}$ (C, F, M, S) |
| 2. 8 (C, F, M)           | 6. 96 (C, F, M, P, S)   | 9. 100 (C, F, M, P, S)        |
| 3. 225 (C, F, M)         | 7. 1.86 or +1.86 (C, F) | 10. 21 (C, E, F, G, M, P, T)  |
| 4. 24 (C, F, M, P, S, T) |                         |                               |

# Warm-Up 6

## Answers

- |                  |                 |                  |              |                   |                 |
|------------------|-----------------|------------------|--------------|-------------------|-----------------|
| 1. 25            | (C, F, P, T)    | 5. 238           | (C, M, P, S) | 8. $\frac{1}{12}$ | (C, F, G, M, P) |
| 2. 16            | (E, G, M, T)    | 6. $\frac{1}{4}$ | (C, S)       | 9. 48             | (C, F, M)       |
| 3. $\frac{1}{2}$ | (C, F, M, S, T) | 7. 6             | (C, F, M)    | 10. 5             | (C, F, M, S)    |
| 4. $\frac{2}{3}$ | (C, F, G, S)    |                  |              |                   |                 |

# Workout 3

## Answers

- |         |           |        |                 |          |                 |
|---------|-----------|--------|-----------------|----------|-----------------|
| 1. 1    | (M, F, G) | 5. 2.9 | (C, F, M)       | 8. -1    | (C, E, F, G, T) |
| 2. 1.26 | (C, F, M) | 6. 5   | (C, G)          | 9. 37.5  | (C, F, T)       |
| 3. 1004 | (C, F)    | 7. 15  | (C, F, M, P, T) | 10. 28.4 | (C, F, M)       |
| 4. 60   | (C, T)    |        |                 |          |                 |

# Proportional Reasoning Stretch

## Answers

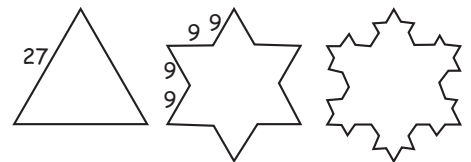
- |         |           |         |           |                  |              |
|---------|-----------|---------|-----------|------------------|--------------|
| 1. 51   | (C, F, M) | 5. 450  | (C, F, M) | 8. 8             | (C, F, G)    |
| 2. 27   | (C, F, M) | 6. 28   | (C, F, M) | 9. $\frac{1}{9}$ | (C, F, M, S) |
| 3. 1071 | (C, F, M) | 7. 72.5 | (C, F)    | 10. 15           | (C, G, M, T) |
| 4. 180  | (C, F, M) |         |           |                  |              |

# Warm-Up 7

- \_\_\_\_\_ minutes How many minutes are in 25% of one day?
- \_\_\_\_\_ points Rob's three assignment scores are 79, 80 and 84 points. What is the arithmetic mean of these three scores?
- \_\_\_\_\_ sq units What is the area of regular hexagon ABCDEF if  $AB = 2$  units? Express your answer in simplest radical form.
- \_\_\_\_\_ Anderson will roll two standard six-sided dice once. What is the probability that the two numbers rolled will be the same? Express your answer as a common fraction.
- \_\_\_\_\_ Jerry worked for one day on a project that he could have completed alone in nine days. Bill joined Jerry the next day, and they worked together for exactly three days to complete the project. How much of the job did Bill do in those three days? Express your answer as a common fraction.

- \_\_\_\_\_ What is the ones digit when  $9^{55}$  is expressed as an integer?

- \_\_\_\_\_ units In the sequence of equilateral figures shown, the middle third of each segment is replaced with two segments that are each the same length as the replaced piece. Each side of the first figure (the triangle) is 27 units. What is the perimeter of the third figure in the sequence?



- \_\_\_\_\_ pennies Sammy felt very generous. He gave Becky half of his pennies. Then an hour later, Sammy gave Janie one-fourth of his remaining pennies. Shortly after, Mary Beth borrowed half the pennies that Sammy had left. Sammy then had 12 pennies. How many pennies did Sammy give to Becky?



- \_\_\_\_\_ grams A certain material's density is  $2.5 \text{ g/cm}^3$ . Given that density = mass  $\div$  volume, what is the mass, in grams, of  $300 \text{ cm}^3$  of this material?

- \_\_\_\_\_ \$ Alex parked his car at the Leggett Airport on Monday at 7 a.m. He removed his car from the parking lot on Friday of the same week at 10 a.m. The rates at Leggett are shown on this sign. How much did Alex pay for his parking during this time period?

<p style="text-align: center;"><b>Parking</b></p> <p style="text-align: center;">\$1 per hour ..... \$7 maximum per 24-hour period</p>
--

# Warm-Up 8

1. \_\_\_\_\_ The faces of a cubical die are each labeled with a different prime number, and each of the six smallest prime numbers (2, 3, 5, 7, 11, 13) is on exactly one face of the die. The die will be rolled twice. What is the probability that the product of the two numbers rolled will be even? Express your answer as a common fraction.

2. \_\_\_\_\_ sq units What is the area of the triangle with vertices at (0, 2), (3, 2) and (3, 0) on the coordinate plane?

3. \_\_\_\_\_ Gary can select any positive two-digit integer between 23 and 98 and write it as "AB" with tens digit A and ones digit B. When he subtracts the sum  $A + B$  from his integer, the difference will be a new two-digit integer, "JK." What is the value of  $J + K$ ?

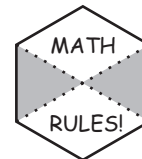
4. \_\_\_\_\_ runners During the Great Falls Marathon, 50% of the runners dropped out before reaching the first water station. By the second water station,  $\frac{1}{3}$  of the remaining runners dropped out. At the third water station, 25% of the remaining runners dropped out. By the fourth water station, 90% of the remaining runners dropped out. If the remaining six runners finished the marathon, how many runners started the race?



5. \_\_\_\_\_ If  $\sqrt[n]{96} = 2\sqrt[3]{3}$ , what is the value of  $n$ ?

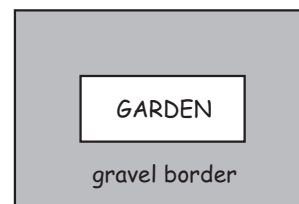
6. \_\_\_\_\_ degrees Jo's circular wheel needs to complete eight full rotations, or  $8 \times 360 = 2880^\circ$ , to roll 12 meters. How many degrees must the wheel rotate to roll 0.5 meters?

7. \_\_\_\_\_ sq inches The area of this sign in the shape of a regular hexagon is  $96\sqrt{3}$  square inches. What is the total area of the two shaded regions? Express your answer in simplest radical form.

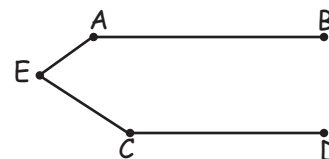


8. \_\_\_\_\_ sq units Two sides of a particular isosceles triangle are 6 and 13 units. What is the area of this triangle? Express your answer in simplest radical form.

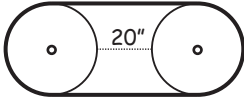
9. \_\_\_\_\_ feet A rectangular garden is 10 feet by 4 feet. A gravel border with uniform width along the sides and  $90^\circ$  corners surrounds the garden, as shown. The area of the gravel border is six times the area of the garden. What is the perimeter of the outside of the gravel border?

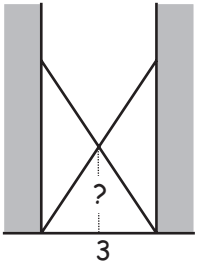


10. \_\_\_\_\_ degrees In the figure shown, segments AB and CD are parallel. What is the sum of the measures of angles BAE, AEC and ECD?



# Workout 4

- \_\_\_\_\_ mph A car that is traveling 90 feet per second is traveling how many miles per hour? There are 5280 feet in 1 mile. Express your answer to the nearest whole number.
- \_\_\_\_\_ The product  $(66)(9)(22)(39)$  has a prime factorization of the form  $(2^a)(3^b)(11^c)(13^d)$ . What is the value of  $ac - bd$ ?
- \_\_\_\_\_ inches Tony the Tortoise walked 10 inches during the first hour of his journey. He walked one and one-half times that distance during the second hour, and in general during the  $n^{\text{th}}$  hour he walked  $(1 + \frac{1}{n})$  times the distance walked during the  $(n-1)^{\text{st}}$  hour. How many inches did Tony walk during the first three hours of his journey?
- \_\_\_\_\_ \$ Big Town Auto totaled \$437,184 in revenue in August. August's revenue was 8% greater than July's revenue. July's revenue was 12% below the revenue for June. What was the revenue for June?
- \_\_\_\_\_ Let  $B(n)$  denote the sum of the digits of the binary (base 2) representation of  $n$ . Let  $T(n)$  denote the sum of the digits of the ternary (base 3) representation of  $n$ . For example,  $B(9) = B(1001_2) = 2$  and  $T(9) = T(100_3) = 1$ . What is the smallest positive integer  $n$  greater than 1 such that  $B(n) = T(n)$ ?
- \_\_\_\_\_ items Jordan buys pens for \$3 each and books for \$5 each, totaling exactly \$2008. What is the largest number of items (pens plus books) he could have bought?
- \_\_\_\_\_ blades Terrell wondered how many blades of grass were in his 60-foot by 90-foot rectangular backyard. He picked a square region three inches on a side, which contained 520 blades of grass. If the grass was uniformly distributed throughout the backyard, how many blades were in the entire backyard?
- \_\_\_\_\_ inches  Each circular pulley shown has a radius of 12 inches. The shortest distance between the pulleys is 20 inches. What is the length of the continuous belt that loops around both pulleys? Express your answer to the nearest whole number.
- \_\_\_\_\_ meters Two ladders, both 6 meters in length, are leaned up against opposite vertical walls in a 3-meter-wide corridor, as shown. How far above the ground do the two ladders cross? Express your answer as a decimal to the nearest tenth.

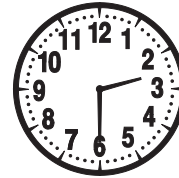

- \_\_\_\_\_ questions To pass a 30-question test, Johnny needs to answer at least 60% of the questions correctly. When Johnny received his graded test back, he saw that he needed to have answered exactly two more questions correctly to have passed the test. How many questions did he answer correctly?



# Warm-Up 9

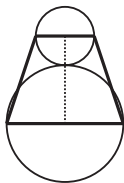
1.            combos A jar contains 100 red marbles, 100 blue marbles and 100 white marbles. All 300 marbles are the same size. How many distinct color combinations are possible when three marbles are selected from the jar? The order in which the marbles are selected does not matter.

2.            degrees How many degrees are there in the smallest angle between the two hands of a clock at 2:30?



3.            A positive 16-digit integer is such that any two consecutive digits form a multiple of either 19 or 31. If the digit 2 appears only once, what is the sum of the 16 digits?

4.            sq units



Two non-congruent circles are externally tangent to each other. Each base of an isosceles trapezoid is a diameter of one of the circles. If the distance between the centers of the circles is 9 units, what is the area of the trapezoid?

5.            gallons

To create a unique house paint color, Melton mixes together a sample that is 12 gallons of red, 2.5 gallons of yellow and 0.5 gallons of blue paint. He then mixes a main batch of paint using 30 gallons of yellow paint and enough red and blue paint so as to maintain the original ratio. How many total gallons of paint did he use when making the sample and the main batch?



6.            In a math class, each student's final grade is the average of the scores on  $n$  tests. If Alfred makes a 97 on the last test, his grade will be exactly 90. If he makes a 73 on the last test, his grade will be exactly 87. What is the value of  $n$ ?

7.            units The dimensions of two boxes are  $a$  by  $b$  by  $c$  and  $d$  by  $e$  by  $f$ , respectively. If  $a < c$ ,  $d < b$ ,  $e < a$ ,  $b < e$  and  $a < f$ , what is the diameter of the largest ball that can fit into both boxes? Express your answer in terms of  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $e$  and/or  $f$ .

8.            The original price of an item is reduced by 20%. This reduced price is then lowered by 10%, and finally this newest price is reduced by 50% to get a final selling price. What fraction of the original price is the final selling price? Express your answer as a common fraction.

9.            In an arithmetic sequence, the sum of the 8<sup>th</sup> and 9<sup>th</sup> terms is 40, and the sum of the 9<sup>th</sup> and 10<sup>th</sup> terms is 48. What is the positive difference between the 8<sup>th</sup> and 9<sup>th</sup> terms in this sequence?

10. (       ,       ) The graphs of  $y = 2x^3$  and  $y = 3x^2$  intersect at  $(0, 0)$  and at what other point? Express your answer as an ordered pair of common-fraction coordinates.

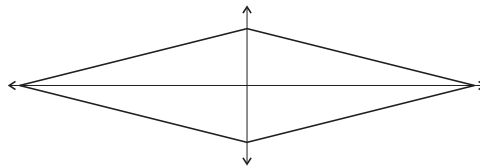
# Warm-Up 10

- \_\_\_\_\_ What is the value of  $k$  in the equation  $(21 \times 2^4) \times 5! = k!$ ?
- \_\_\_\_\_ cu in A rectangular block has faces with areas of  $48 \text{ in}^2$ ,  $72 \text{ in}^2$  and  $96 \text{ in}^2$ . What is the volume of the block?
- \_\_\_\_\_ An  $8.5''$  by  $11''$  piece of paper is cut entirely into  $0.5''$  by  $11''$  strips. What is the ratio of the total perimeter of all the strips of paper to the perimeter of the original paper? Express your answer as a common fraction.
- \_\_\_\_\_ \$ The cost at Modern Attire for a shirt and a tie is  $\$60$ . The cost for a tie and a pair of pants is  $\$66$ . The cost for a pair of pants and a sweatshirt is  $\$72$ . The cost for a sweatshirt and a jacket is  $\$100$ . The cost for a shirt and a jacket is  $\$82$ . What is the cost of one sweatshirt?
- \_\_\_\_\_ pounds A math teacher explains that he weighs 100 pounds more than half his weight. How much does he weigh?

- \_\_\_\_\_ deer For years Carl has kept a record of the number of deer he has spotted in his backyard. If the percent decrease is the same from 1998 to 2008 as it has been for the previous two 10-year spans, as shown, how many deer can he anticipate seeing in 2008?

Year	# of Deer
1978	1000
1988	900
1998	810
2008	?

- \_\_\_\_\_ sq units



What is the area enclosed on the coordinate plane by the graph of the equation  $|x| + |4y| = 20$ ?

- \_\_\_\_\_ What is the 15<sup>th</sup> term in the arithmetic sequence 7, 10, 13, ...?
- \_\_\_\_\_ Use each of the digits 3, 4, 6, 8 and 9 exactly once to create the greatest possible multiple of 6. What is that multiple of 6?
- \_\_\_\_\_ games A tournament begins with 61 people. When two people play a game, the loser is eliminated from the tournament. Eventually, only one person remains as the tournament winner. How many games have been played when the tournament winner is determined?



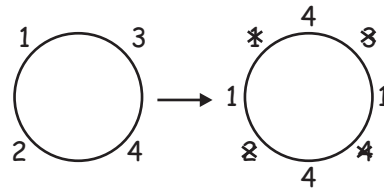
# Workout 5

1. gallons A tank contains 10,000 gallons of water at the beginning of the day on June 1. Each day, 1% of the water in the tank at the beginning of the day is lost to evaporation. How much water is left at the end of the last day of June? Express your answer to the nearest whole number.
2. sq units An acute angle of a right triangle is  $30^\circ$ , and the hypotenuse is 40 units. What is the area of the triangle? Express your answer as a decimal to the nearest tenth.
3. % Larry bought two coats at a thrift store and then sold the coats a week later for \$135 each. He made a profit of 25% on the first coat but lost 25% on the second coat. What total percent did he lose on the sale of the two coats? Express your answer to the nearest hundredth.



4. feet A Super-Duper bouncy ball is dropped straight down from a height of 80 feet. Each time the ball hits the ground it bounces straight back up  $\frac{3}{4}$  of the height from which it just fell. How many total feet had the ball traveled when it hit the ground the third time?
5.  $^\circ\text{F}$  A method for estimating the conversion between  $^\circ\text{Celsius}$  and  $^\circ\text{Fahrenheit}$  is to double the Celsius temperature and then add  $30^\circ$  to get the Fahrenheit temperature. The exact formula for the conversion is  $F = \frac{9}{5}C + 32$ . If the Celsius temperature is  $10^\circ$ , what is the difference between the estimated Fahrenheit temperature and the actual Fahrenheit temperature?
6. \_\_\_\_\_ An integer is pseudoperfect if it is the sum of two or more of its positive divisors. (A divisor may be used only once in the sum.) For instance, 20 is pseudoperfect because its divisors 1, 4, 5 and 10 have a sum of 20. What is the sum of all the pseudoperfect integers between 50 and 60?

7. \_\_\_\_\_ The numbers 1, 2, 3 and 4 are placed in any order about a circle. At each turn of a game, a new circle is formed with four new entries, each of which is the square of the difference of each pair of adjacent numbers. Each new entry is placed between the two numbers from which it was calculated, and the old numbers are erased. What is the largest possible number ever to appear when playing the game if any initial ordering of the numbers 1, 2, 3 and 4 may be used?



8. sq ft A 20-foot-high rectangular room has a floor that measures 18' by 15'. Its doorway measures 3' by 12', and its only window measures 7' by 10'. How many square feet of wall space does the room have?
9. hours A boat has a speed of 6 mph in still water. The boat can travel 30 miles with the current in the same time in which it can travel 18 miles against the current. How many hours are necessary for the boat to travel 36 miles against the current?
10. birds A bird collection has exactly four types of birds (eagles, doves, crows and sparrows). The eagles and doves make up 60% of the collection, and the doves and crows make up 20% of the collection. If the 18 crows in the collection represent 5% of the total number of birds, how many of the birds are sparrows?

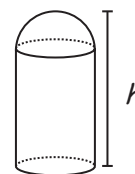
# Warm-Up 11

1. \_\_\_\_\_ sq in A square and a circle overlap such that a vertex of the square is at the center of the circle. The 4-inch radius of the circle is one-half the length of a side of the square. What is the area of the portion of the square region that is outside the circular region? Express your answer in terms of  $\pi$ .

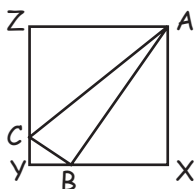
2. \_\_\_\_\_ What is the remainder when  $2008^{2007}$  is divided by 5?

3. \_\_\_\_\_ triangles How many non-congruent triangles, each with a perimeter of 15 units, can be constructed with all integral side lengths?

4. \_\_\_\_\_ meters This container is made from a right circular cylinder and a hemisphere of the same radius on top. The volume of the entire container is  $108\pi$  cubic meters. The base of the container has a radius of 3 meters. What is the height of the container from the bottom base to the top of the hemisphere?



5. \_\_\_\_\_ sq units Triangle ABC has its vertices located on square AXYZ, as shown. If  $XB = 7$ ,  $BY = 3$  and  $YC = 2$ , what is the area of triangle ABC?



6. \_\_\_\_\_ Gary will roll two standard six-sided dice once. What is the probability that the difference between the two numbers rolled will be a multiple of 2? Express your answer as a common fraction.

7. \_\_\_\_\_ The point  $(8, 10)$  is the same distance from the point  $(0, y)$  as it is from the  $x$ -axis. What is the greatest possible value of  $y$ ?

8. \_\_\_\_\_ items Twenty people each competed in a scavenger hunt. Twenty percent of the people each found 80% of the items, and 80% of the people each found 20% of the items. Those who found the most items found a dozen items each. How many items were on the list?

9. \_\_\_\_\_ A set of nine positive integers has a median of 5, a mean of 6 and a unique mode of 8. What is the largest integer that can be a member of the set?

10. \_\_\_\_\_ My piggy bank has only nickels, dimes and dollar bills. The ratio of nickels to dimes is 2:3, and the ratio of dimes to dollar bills is 10:1. What is the ratio of coins to dollar bills? Express your answer in the form  $a:b$ , where  $a$  and  $b$  are positive integers with no common factors greater than 1.

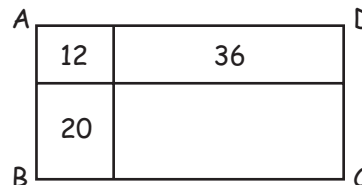


# Warm-Up 12

1. \_\_\_\_\_ % A study of 100 boys and 100 girls found that 60% of girls and 20% of boys enjoy the game Quirk. What percent of the children in the study who enjoy Quirk are girls?

2. \_\_\_\_\_ degrees The measures of the four interior angles of a convex quadrilateral are  $4x$ ,  $3x + 20$ ,  $2x + 40$  and  $x + 80$  degrees. What is the measure of the smallest interior angle of the quadrilateral?

3. \_\_\_\_\_ sq units In the figure, quadrilateral ABCD is a rectangle with integer side lengths. The areas of three smaller rectangles are given, in square units. What is the area of rectangle ABCD?



4. \_\_\_\_\_ sq units A rhombus of side length 5 units has a short diagonal of length 6 units. What is the area of the rhombus?

5. \_\_\_\_\_ cu in



A rectangular piece of paper is rolled, with no overlap, into the curved surface of a cylinder. The cylinder's volume is  $1872\pi$  cubic inches. The paper is then rolled tighter so that half the circumference of the new cylinder is overlapped paper, and the cylinder has the same height. What is the volume of the new cylinder? Express your answer in terms of  $\pi$ .

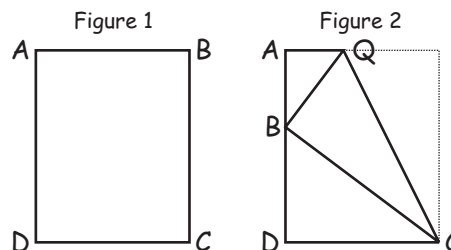
6. \_\_\_\_\_ If  $x + \frac{1}{x} = 3$ , what is the value of  $x^4 + \frac{1}{x^4}$ ?

7. \_\_\_\_\_ The function  $f(x) = 3x^2 - 6x - 11$  is graphed on a coordinate plane. What is the smallest  $y$ -coordinate of any points of the function?

8. \_\_\_\_\_ The mean of three numbers is 6 more than the least of the numbers, and it is 7 less than the greatest number. The median of the three numbers is 8. What is the sum of the three numbers?

9. \_\_\_\_\_ liters Two liters of an alcohol/water mixture is 25% alcohol. How many liters of pure water should be added to make a 10% alcohol solution?

10. \_\_\_\_\_ units In Figure 1, ABCD is a rectangular piece of paper. Point B is folded over onto its new location on side AD (see Figure 2), and the paper is creased from Q to C. In Figure 2,  $AB = 8$  units, and the area of triangle QBA is 24 square units. What is the perimeter of rectangle ABCD in Figure 1?



# Workout 6

- \_\_\_\_\_ The sides of a quadrilateral have lengths  $x$ ,  $x + 1$ ,  $x + 2$  and  $x + 3$  units. The value of the perimeter is less than 100 units and is a perfect cube. The mean of the side lengths is twice the square root of the perimeter. What is the value of  $x$ ? Express your answer as a decimal to the nearest tenth.
- \_\_\_\_\_ The sum of  $a$  and 4, the difference of  $b$  and 4, the product of  $c$  and 4 and the quotient of  $d$  and 4 are all equal to the same integer value. If the sum of  $a$ ,  $b$ ,  $c$  and  $d$  is 100, what is the ratio of  $a$  to  $d$ ? Express your answer as a common fraction.
- \_\_\_\_\_ <sup>times</sup> Kelly's heart beats 60 times per minute when she is sleeping. If she sleeps 7 hours per day for all of 2009 and 2010, how many times will her heart beat while sleeping for those two years? Express your answer in scientific notation with five significant digits.
- \_\_\_\_\_ <sup>feet</sup> A farmer's field is in the shape of regular hexagon ABCDEF. The distance from point A to point B is 420 feet. A fence post is placed at each vertex of the hexagon, and each side has 16 evenly spaced fence posts (counting the posts at the vertices). What is the distance from the center of one fence post to the center of an adjacent fence post on a side of the hexagonal field?
- \_\_\_\_\_ <sup>sq cm</sup> A circle with a radius of 2.5 cm is inscribed in a square. What is the area within the square region but outside the circular region? Express your answer as a decimal to the nearest tenth.

- \_\_\_\_\_ <sup>yards</sup> According to this data, what is the positive difference between the mean number of yards for the top five rushers and the mean number of yards for the top five passers? Express your answer as a decimal to the nearest tenth.

Playoff Leaders

Top 5 NFL Passers	Yards	Top 5 NFL Rushers	Yards
Roethlisberger PIT	680	Foster CAR	205
Delhomme CAR	655	Alexander SEA	141
Brady NE	542	Bettis PIT	137
Hasselbeck SEA	434	Parker PIT	132
Plummer DEN	420	Anderson DEN	105

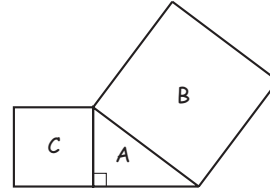
- \_\_\_\_\_ % A square is inscribed in a circle that forms a dartboard. If a dart randomly hits the dartboard, what is the probability it will hit the region outside the square but inside the circle? Express your answer as a percent to the nearest tenth.
- \_\_\_\_\_ \$ The 460 students and 20 staff at Bosney School are planning a bus trip. The bus company has large buses that can hold up to 64 passengers and small buses that can hold up to 40 passengers. The large buses cost \$960 each and the small buses cost \$680 each. What is the cost per passenger for the cheapest combination of buses?
- \_\_\_\_\_ cm What is the total perimeter of a sector of a circle with a radius of 3 cm and a central angle measuring  $90^\circ$ ? Express your answer as a decimal to the nearest tenth.
- \_\_\_\_\_ If  $\sqrt{5 + \sqrt{1 + x}} = \sqrt{2} + \sqrt{3}$ , what is the value of  $x$ ?



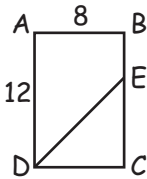
# Warm-Up 13

1.            students A teacher usually divides his class into six groups of  $n$  students each. However, on Monday, three of the students were absent, so the teacher divided the remaining students into seven groups of  $m$  students each. On Tuesday, four students were absent, so he went back to  $n$  students per group, but there was one fewer group than he usually has. How many students are in the class?

2.            sq units There are two squares placed on sides of a right triangle, as shown to the right. The area of square B is 100 square units, and the area of square C is 64 square units. What is the area of right triangle A?



3.            sq units In rectangle ABCD, point E lies on segment BC such that  $BE = \frac{1}{3}BC$ . If  $AB = 8$  units and  $DA = 12$  units, what is the area of triangle ECD?



4.            sq units What is the area of the region enclosed by the graphs of the lines  $y = -2x - 3$ ,  $y = 2x - 3$  and the  $x$ -axis? Express your answer as a decimal to the nearest tenth.

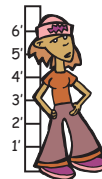
5.            A math field day's budget consists of \$432 to pay problem-writers and  $\$x$  per Mathlete for food, drinks, copying costs and trophies. A math field day for 100 Mathletes costs exactly half what a math field day for 248 Mathletes costs. What is the value of  $x$ ?

6.            units The radius of a particular circle inscribed in an equilateral triangle is 2 units. What is the perimeter of the triangle? Express your answer in simplest radical form.

7.            units Triangle ABC has a perimeter of 2007 units, and the sides have lengths that are all integers with  $AB \leq BC \leq AC$ . What is the positive difference between the largest possible length of segment AB and the smallest possible length of segment AB?

8.            What is the probability that in a group of three friends no two of them were born on the same day of the week? Express your answer as a common fraction.

9.            students Ten percent of the male students at James HS are over six feet tall. The number of female students who are taller than six feet is equal to 10% of the number of male students over six feet tall. At James HS there are 250 female students and 20% more male students than female students. How many female students are taller than six feet?



10.            values A sequence of positive integers is formed by first selecting any positive two-digit integer as the first term of the sequence. Each term after the first term is the sum of twice the tens digit and twice the ones digit of the previous term. If the second term of the sequence is 16 and the third term is 14, how many values are possible for the first term?

# Warm-Up 14

1. \_\_\_\_\_ The symbol  $*$  represents a sequence of mathematical operations. If  $12 * 6 = 35$ ,  $4 * 2 = 3$ ,  $11 * 14 = 8$ ,  $5 * 7 = 3$  and  $10 * 7 = 8$ , what is the value of  $8 * 4$ ?

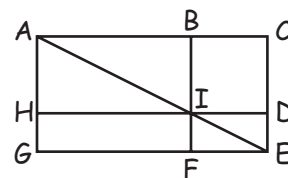
2. \_\_\_\_\_ What is the value of  $x$  in the equation  $3^{12} + 3^{12} + 3^{12} = 3^x$ ?

3. \_\_\_\_\_ If the graph of  $f(x) = 2x^2 + bx - 3$  is symmetric about the line  $x = 3$ , what is the value of  $b$ ?

4. \_\_\_\_\_ <sup>pieces</sup> Brenda gave Gail as many pieces of candy as Gail already had. Then Gail gave Brenda as many pieces of candy as Brenda needed to double her amount of candy. Now Brenda has three times as many pieces of candy as Gail. If Gail has at least one piece of candy, what is the minimum total number of pieces of candy needed for this to have occurred?

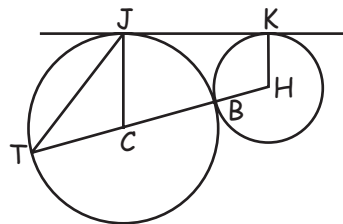
5. \_\_\_\_\_ <sup>points</sup> Point A is located at (5, 5) on the Cartesian plane. Point B is also in the plane and has integer coordinates. If  $0 < AB \leq 4$ , at how many points could B be located?

6. \_\_\_\_\_ <sup>sq units</sup> A diagonal is drawn in rectangle ACEG, as shown. Segments BF and HD are drawn parallel to sides CE and GE, respectively, with segments AE, BF and HD intersecting at point I. The area of rectangle BCDI is 36 square units. What is the area of rectangle FGHI?



7. \_\_\_\_\_ <sup>sq units</sup> What is the area of a triangle with vertices at (-5, -1), (3, 5) and (1, 9) on the coordinate plane?

8. \_\_\_\_\_ <sup>degrees</sup> In the figure, the circles with centers C and H are tangent to line JK at points J and K, respectively, and are externally tangent at point B. Points T, C, B and H are collinear. The measure of angle KHB is  $110^\circ$ . What is the measure of angle CJT?



9. \_\_\_\_\_ <sup>sets</sup> Two positive integers are relatively prime if 1 is their only common factor. How many sets of two relatively prime integers are there for which both integers in the set are greater than or equal to 2 and less than or equal to 9?

10. \_\_\_\_\_ In a study of the effects of time pressure on Mathletes, one-third of the Mathletes were given buzzers and the rest were not. All of the Mathletes answered a particular question. One-third of those with buzzers answered the question correctly, while one-fourth of those answering the question correctly had buzzers. What fraction of the students without buzzers answered the question correctly? Express your answer as a common fraction.

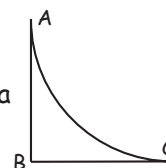


# Workout 7

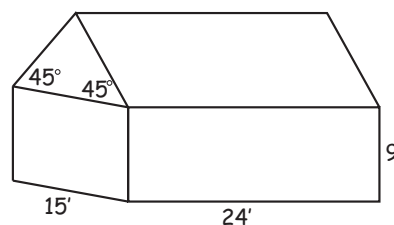
1. \_\_\_\_\_ What is the value of  $(1 + 2 + 3) + (2 + 3 + 4) + (3 + 4 + 5) + \dots + (38 + 39 + 40) + (39 + 40 + 41)$ ?
2. \_\_\_\_\_ sides If the sum of the interior angles of a particular convex polygon is  $9720^\circ$ , how many sides does the polygon have?
3. \$ \_\_\_\_\_ The 442 students and 40 staff of Beckwith School are planning a trip. Each person can go by bus or train. Using the information below, what is the least possible average cost per person for them to make this trip?

Trailmore Bus Lines	Arca Trains
Large Bus: \$1025; holds up to 64 passengers	\$20 per person for first 100 passengers
Medium Bus: \$792; holds up to 36 passengers	\$18 per person for next 300 passengers
	\$16 per person for all passengers over 400

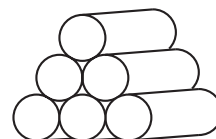
4. \_\_\_\_\_ people When a survey of 800 people was done, 55% of those surveyed said they ate candy, 65% said they ate chips and 90% said they ate hotdogs. What is the smallest possible number of people who said they ate candy, chips and hotdogs?
5. \_\_\_\_\_ cm A right circular cone has a base circumference of  $24\pi$  cm and a volume of  $1512\pi$  cm<sup>3</sup>. The cone is cut parallel to its base, and the newly formed shorter cone has a volume of  $56\pi$  cm<sup>3</sup>. What is the height of the shorter cone? Express your answer as a decimal to the nearest tenth.
6. \_\_\_\_\_ sq feet The front view of a metal sculpture is the region ABC shown, with  $AB = BC = 22$  feet and segment AB perpendicular to segment BC. Points A and C are the endpoints of a quarter-circle. What is the area of region ABC? Express your answer to the nearest whole number.



7. \_\_\_\_\_ cu ft Leonard wishes to purchase an air purifier for a small building. He needs to determine the volume of air inside the building in order to decide which model to purchase. The building is a triangular-based right prism on top of a right rectangular prism, with measurements as shown. How many cubic feet of air are in the entire building?


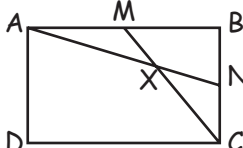
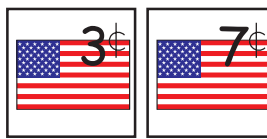


8. \_\_\_\_\_ What is the sum of the 13 smallest positive palindromes that have a tens digit of 3 and a ones digit of 7?
9. \_\_\_\_\_ feet Six pipes each having a radius of 0.5 feet are stacked in a triangular pile with three pipes on the ground tangent to each other, two in the next row and then one on top. What is the height of the pile? Express your answer in simplest radical form.



10. \_\_\_\_\_ units The side lengths of a right triangle are each an integral number of units. If one of the legs is 13 units, what is the perimeter of the triangle?

# Warm-Up 15

- \_\_\_\_\_ cm A ball is dropped from a height of 120 cm and always bounces upwards  $\frac{2}{3}$  of the height from which it falls. How high does the ball go between the third and fourth bounces? Express your answer to the nearest whole number.
- \_\_\_\_\_ Tom's graduating class has 288 students. At the graduation ceremony, the students will sit in rows with the same number of students in each row. If there must be at least 10 rows and at least 15 students in each row, then there can be  $x$  students in each row. What is the sum of all possible values of  $x$ ?
- \_\_\_\_\_ Suppose a regular hexagon has a perimeter equal to the circumference of a circle. What is the ratio of a side of the hexagon to the radius of the circle? Express your answer as a common fraction in terms of  $\pi$ .
- \_\_\_\_\_ There is a method traditionally used in some Russian villages to see which of the young women in the village are to be married the next year. Three blades of grass are folded in half and held in such a way that the six ends of the blades are visible but the rest of the blades are hidden. A young woman ties the ends together in pairs at random such that there are three knots and each end is tied to exactly one other end. If, on release, a three-blade loop is formed, the woman will be married the next year. What is the probability of getting a three-blade loop? Express your answer as a common fraction. 
- (\_\_\_\_\_, \_\_\_\_\_) The point  $A(\frac{5}{2}, 0)$  is reflected over the line  $y = \frac{1}{2}x$  to the point  $A'$ . What are the coordinates of  $A'$ ? Express any non-integer coordinate as a common fraction.
- \_\_\_\_\_ triangles How many different right triangles with integer side lengths have one leg 15 units long?
- \_\_\_\_\_ The sum of the reciprocals of three prime numbers is  $\frac{167}{285}$ . What is the sum of the three prime numbers?
- \_\_\_\_\_ Points  $M$  and  $N$  are the midpoints of sides  $AB$  and  $BC$ , respectively, of rectangle  $ABCD$ . If segments  $AN$  and  $CM$  intersect at point  $X$ , what fraction of the area of rectangle  $ABCD$  lies in the quadrilateral  $BNXM$ ? Express your answer as a common fraction. 
- \_\_\_\_\_ arr Diane has one stamp of each positive integer value 1 cent through 9 cents, inclusive. She wants to put 10 cents worth of postage in a row across the top of an envelope. If arrangements of the same stamps in a different order are considered different, how many arrangements are possible? 
- \_\_\_\_\_ integers How many positive three-digit integers have the property that the tens digit is greater than or equal to the hundreds digit and the ones digit is greater than or equal to the tens digit?

Problem #2 is from the 2007 National Competition Sprint Round.

# Warm-Up 16

1. \_\_\_\_\_ inches Four circles are stacked vertically, as shown. Each of the top three circles has a diameter that is half the diameter of the circle just below it. The total area of the four circles is  $765\pi$  square inches. What is the height of the stack?



2. \_\_\_\_\_ The fourth Farey sequence,  $F_4 = \frac{0}{1}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{1}{1}$ , is the list, written in increasing order, of all the common fractions with distinct values from 0 through 1, inclusive, that use only the integers 0, 1, 2, 3 or 4 as numerators and denominators. In the fifth Farey sequence, what is the product of the third and tenth terms? Express your answer as a common fraction.

3. \_\_\_\_\_ uses There are 64 identical-looking coins, one of which is slightly heavier than the others. A balance scale can be used to show which one of two groups of coins is heavier or that the two groups weigh the same. What is the minimum number of uses of the balance scale that is guaranteed to determine which of the coins is the heavier one?

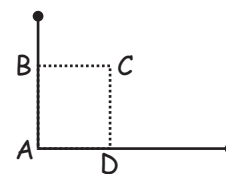


4. \_\_\_\_\_ units A square is inscribed in a right triangle with legs of 8 units and 15 units. If two of the vertices of the square lie on the hypotenuse and the other two vertices of the square lie on the legs of the triangle, what is the length of a side of the square? Express your answer as a common fraction.

5. \_\_\_\_\_ meters An apothem of a regular polygon is a line segment joining the center of the polygon to the midpoint of any side. What is the length of an apothem of a regular hexagon of side length 3 meters? Express your answer as a common fraction in simplest radical form.

6. \_\_\_\_\_ Suppose an equilateral triangle and a square have perimeters of equal length. What is the ratio of the area of the triangle to the area of the square? Express your answer as a common fraction in simplest radical form.

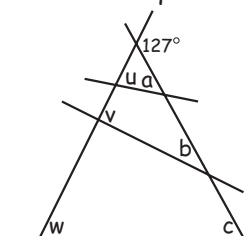
7. \_\_\_\_\_ meters Betty and Don are standing on point A of rectangle ABCD. They walk away from each other on different sides of the rectangle. Betty walks in a straight line and stops 12 meters beyond point B. Don walks in a straight line through point D and continues until point C is directly between himself and Betty. The area of rectangle ABCD is 348 square meters. How far beyond point D did Don walk?



8. \_\_\_\_\_ Suppose we replace each  $x$  in the expression  $\frac{x+1}{x-1}$  with the expression  $\frac{x+1}{x-1}$ . What is the value of the resulting expression when  $x = \frac{4}{5}$ ? Express your answer as a common fraction.

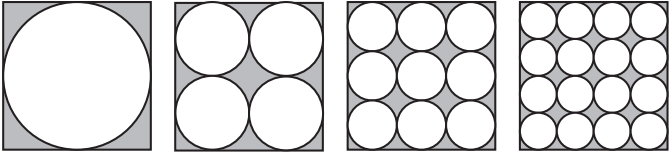
9. \_\_\_\_\_ sq units What is the area of the region in the plane bounded by the lines with the equations  $y = 0$ ,  $x = 0$ ,  $x = 3$  and  $y = 2x + 1$ ?

10. \_\_\_\_\_ degrees In the figure, if  $a + b + c = 180$  degrees, what is the value of  $u + v + w$ ?



# Workout 8

- \_\_\_\_\_ % The gravitational force on an asteroid varies inversely with the square of its distance from the sun. By what percent must the distance decrease in order that the gravitational force be multiplied by 3? Express your answer to the nearest tenth.
- \_\_\_\_\_ sq units Four unit squares contain some non-shaded circles, as shown. What is the total area of the shaded regions inside the squares? Express your answer in terms of  $\pi$ .
 


- \_\_\_\_\_ If  $\boxed{a}$  represents  $\pi a^4$ , then the volume of a sphere of radius 3 units could be represented by  $\boxed{x}$  cu units for some positive value of  $x$ . What is the value of  $x$ ? Express your answer in simplest radical form.
- \_\_\_\_\_ What is the value of  $x$  that satisfies the equation  $2^{2! \cdot 0! \cdot 0! \cdot 8!} = 2^7 \cdot (2^7)^x$ ?
- \_\_\_\_\_ Consider all of the positive five-digit integers that can be formed using each of the digits 3, 4, 5, 6 and 7 exactly once. What is the sum of these integers?
- \_\_\_\_\_ Let  $T$  be a positive integer whose only digits are 0s and 1s. If  $X = T \div 12$ , and  $X$  is an integer, what is the smallest possible value of  $X$ ?
- \_\_\_\_\_ angles What is the maximum number of acute interior angles a convex pentagon can have?
- \_\_\_\_\_ A "deletable prime" is a positive integer that (1) is prime and (2) is either a one-digit integer or, after removing one digit, results in another deletable prime. For example, 439 is deletable because 439 is prime and deleting the 9 results in another deletable prime, 43, which is deletable because removing the 4 results in the prime 3. What is the smallest deletable prime larger than 443?
- \_\_\_\_\_ What is the least positive common fraction that is an integral multiple of  $\frac{9}{28}$ ,  $\frac{12}{35}$  and  $\frac{15}{56}$ ?
- \_\_\_\_\_ % A large tank contains a 400-kg mixture of water and alcohol. The mixture is 64% alcohol by weight. At each step, 100 kg of the mixture will be drained from the tank, replaced with 100 kg of water, and then stirred. After three steps, what percent of the final solution will be alcohol?

*Problem #6 is from the 2007 National Competition Target Round.*

# Warm-Up 17


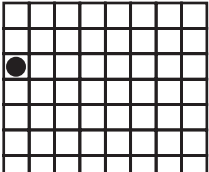
1. \_\_\_\_\_ <sup>ways</sup> Five girls (Alexandra, Betsy, Catherine, Deyola and Emily) travel with one boy (Frank) to a math contest. They have four hotel rooms, numbered 1 through 4. Each room can hold up to two people, and the boy has to have a room to himself. How many different ways are there to assign the students to the rooms, including the way shown here?
- |     |
|-----|
| 1   |
| •   |
| A,D |

2
•
C,E

3
•
F

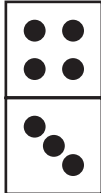
4
•
B
2. \_\_\_\_\_ <sup>sq units</sup> Two different circles are each tangent to both the  $x$ -axis and the  $y$ -axis, as shown in the figure. One of the points of intersection of the circles is  $(1, 5)$ . What is the product of the lengths of their radii?
- 
3. ( \_\_\_\_\_ , \_\_\_\_\_ ) What point on the line  $y = 2x$  is closest to the point  $(0, 5)$ ?
4. \_\_\_\_\_ What is the area of regular octagon  $ABCDEFGH$  divided by the area of quadrilateral  $ACEG$ ? Express your answer in simplest radical form.
5. \_\_\_\_\_ A fair coin is tossed four times, and at least one of the tosses results in heads. What is the probability that exactly two tosses result in heads? Express your answer as a common fraction.
6. \_\_\_\_\_ Many numbers can be made by adding two or more consecutive terms of the arithmetic sequence  $2, 5, 8, 11, \dots$ . Two such examples are  $7 = 2 + 5$  and  $24 = 5 + 8 + 11$ . What is the smallest number that can be made in at least two different ways by adding consecutive terms of this sequence?
7. \_\_\_\_\_ <sup>sq units</sup> Each of the six smaller squares in the sequence shown is formed by joining the midpoints of the sides of the next larger square. The center square has an area of 1 square unit. What is the total area of the shaded regions? Express your answer as a mixed number.
- 
8. \_\_\_\_\_ <sup>mph</sup> The cities of Smallville and Largeville are 300 miles apart. Jim left from Smallville to go to Largeville at 10 a.m. Mickey left Largeville to go to Smallville at 10:30 a.m. on the same day. Jim traveled at a constant speed that was twice Mickey's constant speed, and they both arrived at a point 90 miles from Largeville at the same time. What was Mickey's constant speed, in miles per hour?
9. \_\_\_\_\_ <sup>grids</sup> On a 5 by 5 grid of unit squares, one unit square is colored blue, one unit square is colored red, and the rest of the unit squares are white. Grids are considered different if no rotation could turn one into the other. How many different grids are there?
10. \_\_\_\_\_ Hypotenuse  $BC$  in isosceles right triangle  $ABC$  is  $x$  units, where  $x$  is a value that is a perfect square and a perfect cube greater than 1. If  $AB$  can be expressed as  $\frac{32k\sqrt{5}}{5}$  units, what is the least possible value of  $k$ ? Express your answer in simplest radical form.
-

# Warm-Up 18

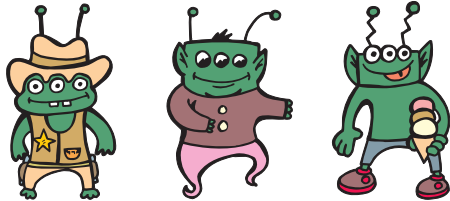
- \_\_\_\_\_ Rectangle ABCD has points E and F on sides AB and CD, respectively. If  $AE = \frac{1}{3} AB$  and  $CF = \frac{1}{4} CD$  and segments DE and BF intersect diagonal AC at G and H, respectively, what is the ratio  $AG:GH:HC$ ? Express your answer in the form  $a:b:c$ , where  $a$ ,  $b$  and  $c$  are relatively prime positive integers.
- \_\_\_\_\_ units A circle with radius 1 unit lies in the first quadrant and is tangent to both the  $x$ - and  $y$ -axes. A second larger circle lies in the first quadrant, is tangent to both axes and is externally tangent to the first circle. What is the radius of the second circle? Express your answer in simplest radical form.
- \_\_\_\_\_ terms How many terms are in the expansion of the expression  $[(3x + 2y)^2(3x - 2y)^2]^3$  after it is simplified to lowest terms?
- \_\_\_\_\_ When 97, 151 and 241 are each divided by a positive integer  $K$ , the remainder is the same. What is the largest possible value of  $K$ ?
- \_\_\_\_\_ units Equilateral triangle ABC has a side length of 6 units. Point D lies on segment BC such that  $DC = 2(BD)$ . What is the length of the altitude of triangle ADC from point C? Express your answer as a common fraction in simplest radical form.
- \_\_\_\_\_  Billy can row upstream from point A to point B in three hours. Rowing at the same rate Billy needs only one hour to row from B to A. What is the ratio of the rate of the current to the rate of Billy's rowing? Express your answer as a common fraction.
- \_\_\_\_\_ A sequence begins 1, 4, 8, 9, 16, ... and consists of all the squares and cubes of positive integers written in ascending order. Numbers that are both squares and cubes, such as 1, are written only once. What is the 50<sup>th</sup> number in this sequence?
- \_\_\_\_\_ Starting with the number 20, a list of increasing integers - not necessarily consecutive integers - has a product that is a perfect square. What is the least possible value for the last integer in the list?
- \_\_\_\_\_ pairs Kendra starts counting with  $a$  and counts by  $d$ , where  $a$  and  $d$  are both positive integers. For example, for  $a = 5$  and  $d = 3$  the sequence would be 5, 8, 11, .... The sum of two terms in Kendra's sequence is 10. How many different possible pairs  $(a, d)$  are there?
- \_\_\_\_\_ paths On an 8 by 8 grid of unit squares, a red marker starts in the unit square called (3, 1), which is the unit square in the third row and first column. Each separate move of the red marker is to a neighboring unit square horizontally, vertically or diagonally. How many paths of exactly four moves are there from (3, 1) to (4, 5)? 

Problem #3 is from the 2007 National Competition Sprint Round.

# Workout 9

- sq units In trapezoid ABCD, bases AB and CD are 13 and 39 units, respectively. Legs BC and DA are 24 and 10 units, respectively, and sides BC and DA lie on lines that are perpendicular to each other. What is the area of ABCD?
- minutes An airplane started 8 miles south and 4 miles west of a radar station. The airplane travels due northeast at a speed of 3 miles per minute. In how many minutes is it again the same distance from the radar station as it started? Express your answer as a decimal to the nearest hundredth.
- students On a 100-point test only three students had scores of 90 or above, and those scores were 97, 94 and 91. Exactly one student had a score of 59 and that was the lowest score. No score occurred more than two times. At least how many students must have had scores of 70 or above if the total of all the scores was 2431?
- \_\_\_\_\_ What is the value of the sum  $(1 \cdot 1 + 2 \cdot 3 + 3 \cdot 5 + \dots + 100 \cdot 199) + (1 \cdot 3 + 2 \cdot 5 + 3 \cdot 7 + \dots + 100 \cdot 201)$ ? The first sum is of products of the form  $(n)(2n - 1)$ , and the second sum is of products of the form  $(n)(2n + 1)$ , with  $n$  in each case going from 1 to 100.
- \_\_\_\_\_ What is the largest possible difference between the square of a three-digit integer and the square of the three-digit integer formed by reversing the digits of the original integer?
- cylinders Nicole needs to pour a cubic meter of water into cylinders that are each 1 meter tall and  $\frac{1}{3}$  meter in diameter. How many cylinders does she need?
- units Convex quadrilateral ABCD has perpendicular diagonals. If  $AB = 25$ ,  $BC = 39$  and  $CD = 60$  units, what is the length of segment DA?
- sq units What is the area of the circle that passes through  $A(-8, 0)$ ,  $B(0, 8)$  and  $C(12, 0)$ ? Express your answer in terms of  $\pi$ .
- \_\_\_\_\_ A set of dominoes consists of exactly one domino with every possible pair of integers  $(x, y)$  with  $0 \leq x \leq y \leq 6$ . What is the sum of all the integers in the set of dominoes?
- integers How many positive integers less than or equal to 50 are multiples of 3 or 4, but not 5?

# Number Theory Stretch

- \_\_\_\_\_ Using exponents when prime factors are used more than once, what is the prime factorization of 504?
- \_\_\_\_\_ factors How many positive factors does 504 have?
- \_\_\_\_\_ What is the sum of the positive factors of 504?
- \_\_\_\_\_ The base-8 number  $1342_8$  is equivalent to what base-10 integer?
- \_\_\_\_\_ What is the smallest positive integer that has a remainder of 7 when divided by 8, a remainder of 8 when divided by 9 and a remainder of 11 when divided by 12?
- \_\_\_\_\_ integers How many positive integers less than 101 are multiples of 3, 4 or 7?
- \_\_\_\_\_ What is the smallest positive integer value of  $n$  such that  $2^n + 5^n + 7^n$  is a multiple of 17?
- ( , ) What are the coordinates of the first-quadrant point that lies on the graph of  $7x + 13y = 99$  and has coordinates that are both integers? Express your answer as an ordered pair  $(x, y)$ .
- ( , , ) Bill buys wigits for \$3, gigits for \$8 and pigits for \$11. If Bill bought exactly 100 aliens for \$400, and he purchased as few gigits as possible, what is the ordered triple (wigits, gigits, pigits) that represents his purchase?  
  
wigit                  gigit                  pigit
- \_\_\_\_\_ When the sum  $2007^{2008} + 2008^{2007}$  is simplified to an integer, what is the ones digit?

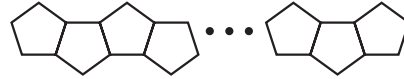


# Patterns Stretch

The following problems are from previous School Handbooks and competitions. Enjoy!

1. \_\_\_\_\_ One digit of the decimal representation of  $\frac{5}{7}$  will be chosen at random. What is the probability that the digit will be a 4? Express your answer as a common fraction. (1998 Chapter Countdown)

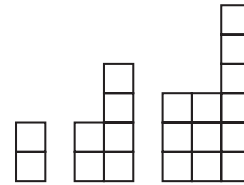
2. \_\_\_\_\_ inches A pentagon train is made by attaching regular pentagons with 1" sides so that each pentagon, except the two on the ends, is attached to exactly two other pentagons along sides, as shown. How many inches are in the perimeter of a pentagon train made from 85 pentagons? (1996 Chapter Target)



3. \_\_\_\_\_ What is the value of  $x + y$  if the sequence 2, 6, 10, ...,  $x$ ,  $y$ , 26 is an arithmetic sequence? ('04-'05 School Handbook)

4. \_\_\_\_\_ For what value of  $x$  is the equation  $x + 2x + 3x + \dots + 99x + 100x = 100$  true? Express your answer as a common fraction. ('03-'04 School Handbook)

5. \_\_\_\_\_ blocks The first three towers in a sequence are shown. The  $n^{\text{th}}$  tower is formed by stacking  $n$  blocks on top of an  $n$ -by- $n$  square of blocks. How many blocks are in the 99<sup>th</sup> tower? (1997 State Sprint)



6. \_\_\_\_\_ diagonals A diagonal of a polygon is a line containing two non-consecutive vertices. How many diagonals does a regular decagon have? ('02-'03 School Handbook)

7. \_\_\_\_\_ What is the value of the expression  $\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\dots\left(1 - \frac{1}{n+1}\right)$  when  $n = 12$ ? Express your answer as a common fraction. ('01-'02 School Handbook)

8. \_\_\_\_\_ The 25<sup>th</sup> day of the year 2003 fell on a Saturday. What day of the week did the 284<sup>th</sup> day of the year 2003 fall? (1995 State Countdown)

9. \_\_\_\_\_ If the pattern shown is continued, what is the sum of the terms in Row 12? ('00-'01 School Handbook)

Row 1 ... 2  
 Row 2 ... 2 + 4  
 Row 3 ... 2 + 4 + 6  
 Row 4 ... 2 + 4 + 6 + 8  
 Row 5 ... 2 + 4 + 6 + 8 + 10

10. \_\_\_\_\_ According to the linear function represented in this table, what is the value of  $x$  when  $y = 8$ ? ('04-'05 School Handbook)

$x$	$y$
-4	23
1	20
6	17

# PROBLEM-SOLVING STRATEGIES

NCTM's *Principles and Standards for School Mathematics* recommends that the mathematics curriculum "include numerous and varied experiences with problem solving as a method of inquiry and application." There are many problems within the MATHCOUNTS program that may be considered difficult if attacked by setting up a series of equations, but quite simple when attacked with problem-solving strategies such as looking for a pattern, drawing a diagram, making an organized list and so on.

The problem-solving method that will be used in the following discussion consists of four basic steps:

<b>FIND OUT</b>	Look at the problem. Have you seen a similar problem before? If so, how is this problem similar? How is it different? What facts do you have? What do you know that is not stated in the problem?
<b>CHOOSE A STRATEGY</b>	How have you solved similar problems in the past? What strategies do you know? Try a strategy that seems as if it will work. If it doesn't, it may lead you to one that will.
<b>SOLVE IT</b>	Use the strategy you selected and work the problem.
<b>LOOK BACK</b>	Reread the question. Did you answer the question asked? Is your answer in the correct units? Does your answer seem reasonable?

Specific strategies may vary in name. Most, however, fall into these basic categories:

- Compute or Simplify (C)
- Use a Formula (F)
- Make a Model or Diagram (M)
- Make a Table, Chart or List (T)
- Guess, Check & Revise (G)
- Consider a Simpler Case (S)
- Eliminate (E)
- Look for Patterns (P)

To assist in using these problem-solving strategies, the answers to the Warm-Ups and Workouts have been coded to indicate possible strategies. The single-letter codes above for each strategy appear in parentheses after each answer.

**NOTE:** Examples of these strategies being applied to previously published MATHCOUNTS problems are available on pages 29-39 in Volume I of the 2007-2008 MATHCOUNTS School Handbook.

# Warm-Up 7

## Answers

- |                  |           |                  |              |                 |           |
|------------------|-----------|------------------|--------------|-----------------|-----------|
| 1. 360           | (C)       | 5. $\frac{5}{9}$ | (C, F, G)    | 8. 32           | (C, F, T) |
| 2. 81            | (C, F)    | 6. 9             | (C, P, S, T) | 9. 750          | (C)       |
| 3. $6\sqrt{3}$   | (C, F, M) | 7. 144           | (C, P, S, T) | 10. 31 or 31.00 | (C)       |
| 4. $\frac{1}{6}$ | (C, S, T) |                  |              |                 |           |

# Warm-Up 8

## Answers

- |                    |                 |                 |           |                  |           |
|--------------------|-----------------|-----------------|-----------|------------------|-----------|
| 1. $\frac{11}{36}$ | (C, P, S, T)    | 5. 5            | (C, E)    | 8. $12\sqrt{10}$ | (C, F, M) |
| 2. 3               | (C, F)          | 6. 120          | (C, F, M) | 9. 68            | (C, F, M) |
| 3. 9               | (C, G, P, S, T) | 7. $32\sqrt{3}$ | (C, F, M) | 10. 360          | (F, M, S) |
| 4. 240             | (C, M, T)       |                 |           |                  |           |

# Workout 4

## Answers

- |                             |           |               |                 |        |              |
|-----------------------------|-----------|---------------|-----------------|--------|--------------|
| 1. 61                       | (C, F)    | 5. 6          | (C, G, P, T)    | 8. 163 | (C, F, M)    |
| 2. 0                        | (C, P)    | 6. 668        | (C, G, P, S, T) | 9. 2.6 | (C, F, M, P) |
| 3. 45                       | (C, P, T) | 7. 44,928,000 | (C, F)          | 10. 16 | (C)          |
| 4. 460,000 or<br>460,000.00 | (C, T)    |               |                 |        |              |

# Warm-Up 9

## Answers

- |        |              |        |                    |                                   |                    |
|--------|--------------|--------|--------------------|-----------------------------------|--------------------|
| 1. 10  | (F, M, P, T) | 5. 195 | (C, M, T)          | 8. $\frac{9}{25}$                 | (C, S)             |
| 2. 105 | (C, F, M, S) | 6. 8   | (C, F, G, T)       | 9. 4                              | (C, F, G, P, S, T) |
| 3. 72  | (C, G, P, T) | 7. $d$ | (C, E, M, P, S, T) | 10. $(\frac{3}{2}, \frac{27}{4})$ | (C, F, G, T)       |
| 4. 81  | (C, F, M)    |        |                    |                                   |                    |

# Warm-Up 10

## Answers

- |                               |                        |                     |
|-------------------------------|------------------------|---------------------|
| 1. 8 (C, F, G, P, S, T)       | 5. 200 (C, F, G)       | 8. 49 (C, F, P)     |
| 2. 576 (C, E, F, G, M, T)     | 6. 729 (C, F, P)       | 9. 98,634 (S)       |
| 3. $\frac{391}{39}$ (C, F, M) | 7. 200 (C, F, G, M, T) | 10. 60 (C, P, S, T) |
| 4. 42 or 42.00 (C, F, T)      |                        |                     |

# Workout 5

## Answers

- |                        |                            |                      |
|------------------------|----------------------------|----------------------|
| 1. 7397 (C, F, T)      | 5. 0 (C)                   | 8. 1214 (C, F, M)    |
| 2. 346.4 (C, F, M)     | 6. 110 (C, E, P, T)        | 9. 8 (C, F, G, M)    |
| 3. 6.25 (C)            | 7. 16,777,216 (C, M, P, T) | 10. 126 (C, F, M, T) |
| 4. 290 (C, F, M, P, T) |                            |                      |

# Warm-Up 11

## Answers

- |                          |                            |                    |
|--------------------------|----------------------------|--------------------|
| 1. $64 - 4\pi$ (C, F, M) | 5. 22 (C, F, M, S)         | 8. 15 (C, F, M, S) |
| 2. 2 (P, S, T)           | 6. $\frac{1}{2}$ (C, S, T) | 9. 19 (E, G, M, T) |
| 3. 7 (E, P, T)           | 7. 16 (C, F, M)            | 10. 50:3 (C, M, S) |
| 4. 13 (C, F, M)          |                            |                    |

# Warm-Up 12

## Answers

- |                        |                       |                     |
|------------------------|-----------------------|---------------------|
| 1. 75 (C, F)           | 5. $832\pi$ (C, F, M) | 8. 27 (C, G, M, T)  |
| 2. 84 (C, F, M)        | 6. 47 (C, F, P, S)    | 9. 3 (C, M, T)      |
| 3. 128 (C, F, G, M, T) | 7. -14 (C, F, G, M)   | 10. 72 (C, F, M, S) |
| 4. 24 (C, F, M)        |                       |                     |

# Workout 6

## Answers

1. 14.5 (C, E, F, G, M, T)	5. 5.4 (C, F, M)	8. 15.42 (C, G, M, T)
2. $\frac{3}{16}$ (C, E, G, M, T)	6. 402.2 (C, F)	9. 10.7 (C, F, M)
3. $1.8396 \times 10^7$ (C)	7. 36.3 (C, F, M)	10. 23 (C, G)
4. 28 (C, F, M)		

# Warm-Up 13

## Answers

1. 24 (C, G)	5. 9 or 9.00 (C)	8. $\frac{30}{49}$ (C, F)
2. 24 (C, F, M)	6. $12\sqrt{3}$ (F, M)	9. 3 (C)
3. 32 (M, S)	7. 668 (C, E, G, M, S, T)	10. 8 (C, P, T)
4. 4.5 (C, F, M)		

# Warm-Up 14

## Answers

1. 15* (G, P)	5. 48 (G, M, T)	8. 35 (C, M)
2. 13 (C, S)	6. 36 (C, F, M, P, S)	9. 19 (T)
3. -12 (C, F, G)	7. 22 (C, F, M)	10. $\frac{1}{2}$ (C, G, T)
4. 16 (C, E, T)		

\*Because the rule was not defined, it may be possible to find another rule that works for all five examples given.

# Workout 7

## Answers

1. 2457 (C, F, P, S)	5. 10.5 (C, F, M)	8. 1,472,981 (C, M, P, T)
2. 56 (F)	6. 104 (C, F, M)	9. $1+\sqrt{3}$ (C, F, M) or $\sqrt{3}+1$
3. 16.30 (E, G, P)	7. 4590 (C, F)	10. 182 (C, E, F, M, P, S)
4. 80 (C, F, M, P, T)		

\*The form  $1+\sqrt{3}$  is preferred so that there is no question as to whether the "+ 1" is under the radical.

# Warm-Up 15

## Answers

- |                    |           |                       |                    |                  |              |
|--------------------|-----------|-----------------------|--------------------|------------------|--------------|
| 1. 36              | (C, M, T) | 4. $\frac{8}{15}$     | (C, F, M, P, T)    | 7. 27            | (C, G, P, S) |
| 2. 58              | (C, G)    | 5. $(\frac{3}{2}, 2)$ | (C, F, M)          | 8. $\frac{1}{6}$ | (F, M, P, S) |
| 3. $\frac{\pi}{3}$ | (C, F, M) | 6. 4                  | (C, E, F, G, P, T) | 9. 56            | (C, E, S, T) |
|                    |           |                       |                    | 10. 165          | (C, P, S, T) |

# Warm-Up 16

## Answers

- |                       |              |                          |           |                  |              |
|-----------------------|--------------|--------------------------|-----------|------------------|--------------|
| 1. 90                 | (C, F, M)    | 5. $\frac{3\sqrt{3}}{2}$ | (F, M)    | 8. $\frac{4}{5}$ | (C, F)       |
| 2. $\frac{1}{5}$      | (C, G, P, T) | 6. $\frac{4\sqrt{3}}{9}$ | (C, F, M) | 9. 12            | (C, F, M, P) |
| 3. 4                  | (G, P, S, T) | 7. 29                    | (C, F, M) | 10. 201          | (C, F, M, P) |
| 4. $\frac{2040}{409}$ | (C, F, M)    |                          |           |                  |              |

# Workout 8

## Answers

- |               |              |              |              |                    |              |
|---------------|--------------|--------------|--------------|--------------------|--------------|
| 1. 42.3       | (C, F)       | 5. 6,666,600 | (C, P, S, T) | 8. 457             | (C, E, G, T) |
| 2. $4 - \pi$  | (C, F, M, P) | 6. 925       | (C, G, P, S) | 9. $\frac{180}{7}$ | (C, T)       |
| 3. $\sqrt{6}$ | (C, F)       | 7. 3         | (F, G, M)    | 10. 27             | (C, F, T)    |
| 4. 11,519     | (C, F, S)    |              |              |                    |              |

# Warm-Up 17

## Answers

- |               |                 |                    |                 |                 |                 |
|---------------|-----------------|--------------------|-----------------|-----------------|-----------------|
| 1. 360        | (C, F, M, S, T) | 5. $\frac{2}{5}$   | (F, M, T)       | 8. 30           | (C, G, M)       |
| 2. 26         | (C, F, M)       | 6. 55              | (E, G, M, P, T) | 9. 150          | (C, E, P, S, T) |
| 3. (2, 4)     | (C, F, M)       | 7. $16\frac{3}{4}$ | (C, F, M, P, T) | 10. $\sqrt{10}$ | (C, F, M)       |
| 4. $\sqrt{2}$ | (F, M, S)       |                    |                 |                 |                 |

# Warm-Up 18

## Answers

- |   |                                     |                    |
|---|-------------------------------------|--------------------|
| 1. 5:11:4 (C, F, M, P, S)                   | 5. $\frac{6\sqrt{21}}{7}$ (C, F, M) | 8. 30 (E, G, S, T) |
| 2. $3+2\sqrt{2}$ (F, M)<br>or $2\sqrt{2}+3$ | 6. $\frac{1}{2}$ (C, G, M, T)       | 9. 13 (M, T)       |
| 3. 7 (C, F, S)                              | 7. 1728 (P, T)                      | 10. 16 (M, T)      |
| 4. 18 (C, E, F, G, T)                       |                                     |                    |

# Workout 9

## Answers

- |                        |                   |                          |
|------------------------|-------------------|--------------------------|
| 1. 240 (F, M)          | 5. 942,480 (C, G) | 8. $104\pi$ (C, F, G, M) |
| 2. 5.66 (F, M)         | 6. 12 (C, F, M)   | 9. 168 (C, M, T)         |
| 3. 13 (E, G, T)        | 7. 52 (C, F, M)   | 10. 19 (C, M, T)         |
| 4. 1,353,400 (C, F, P) |                   |                          |

# Number Theory Stretch

## Answers

- |  |                          |                             |
|--|--------------------------|-----------------------------|
| 1. $2^3 \times 3^2 \times 7$ or $2^3 \times 3^2 \times 7^1$ (C, M) | 5. 71 (E, G, P, S, T)    | 8. (3, 6) (G, M, T)         |
| 2. 24 (C, F, P, T)   | 6. 58 (C, G, M, P, S, T) | 9. (86, 4, 10) (C, E, G, P) |
| 3. 1560 (C, T)   | 7. 3 (E, G, T)           | 10. 3 (P, S)                |
| 4. 738 (C, F)  |                          |                             |

# Patterns Stretch

## Answers

- |                              |                          |                       |
|------------------------------|--------------------------|-----------------------|
| 1. $\frac{1}{6}$ (P)         | 5. 9900 (C, F, M, P, S)  | 8. Saturday (C, G, P) |
| 2. 257 (M, P)                | 6. 35 (C, F, M, P)       | 9. 156 (C, P, S)      |
| 3. 40 (E, G, P)              | 7. $\frac{1}{13}$ (C, P) | 10. 21 (C, G, P)      |
| 4. $\frac{2}{101}$ (C, F, P) |                          |                       |

