## Eureka Remediation Tool: Grade 7 <br> Module 2, Topic A

To become mathematically proficient, students must access on-gradelevel content. This document aims to help teachers who use the Eureka curriculum to target remediation for students needing extra support before and during approaching on-grade-level work, creating opportunities for on-time remediation directly connected to the new learning.

## About this Topic

## Focus Standards:

7.NS.A.1: Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
a. Describe situations in which opposite quantities combine to make 0 .

For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.
b. Understand $p+q$ as the number located a distance $|q|$ from $p$, in the positive or negative direction depending on whether $q$ is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
c. Understand subtraction of rational numbers as adding the additive inverse, $p-q=p+(-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
d. Apply properties of operations as strategies to add and subtract rational numbers.

## Topic Overview per the Eureka Curriculum

In Topic A, students find sums and differences of signed numbers and establish rules related to the addition and subtraction of rational numbers (7.NS.A.1). Students draw upon experiences in modeling, ordering, and comparing integers and other rational numbers from Grade 6 Module 3 (6.NS.C.5, 6.NS.C.6, 6.NS.C.7). They use their previous work with adding and subtracting fractions and decimals (5.NF.A.1, 6.NS.B.3) to compute the sums and differences of rational numbers. In Lesson 1 , students play a card game called the Integer Game to understand how a number and its opposite combine to make zero. The number line is used to count up and down, serving as a visual model for finding sums. In Lessons 2 and 3 , students more formally develop their understanding of the addition of integers. They use vectors to represent integers on the number line and apply the concept of absolute value
(6.NS.C.7c) to represent the length of the vector while interpreting the sign of the integer as the vector's direction. By Lesson 4, students are efficiently adding integers using well-defined rules.
After addition rules are formalized, students begin subtracting integers in Lesson 5. They relate subtraction to removing a card from their hand in the Integer Game, realizing that subtracting a positive card has the same effect as adding or picking up a negative card. Similarly, removing (subtracting) a negative card increases students' scores the same way as adding the corresponding positive card. Therefore, students determine that subtracting a signed number is the same as adding its opposite. In Lesson 6, students deepen their understanding of subtraction using absolute value and the number line to justify that the distance between two signed numbers is the absolute value of their difference. They represent sums and differences of rational numbers using the number line in Lesson 7 and use vectors to model the sum, $p+q$, or the difference, $p-q$. As Topic A concludes, students apply the properties of operations to add and subtract rational numbers in Lessons 8 and 9 . Using the properties of operations and their fluency in adding and subtracting decimals and fractions from earlier grades, they rewrite numerical expressions in different forms to efficiently find sums and differences of signed numbers without the use of a calculator.

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## Overview

Eureka Remediation Tools include:

1. a diagnostic assessment to help teachers determine the misunderstandings or gaps in mathematical knowledge related to a specific Topic in the Eureka curriculum
2. guidance for teachers to analyze student work on the diagnostic assessment
3. suggested materials for targeted remedial instruction

> Note: The use of this guidance is not intended to delay students' engagement with on-grade-level learning. On-grade-level learning should be the focus of instructional time and be treated as an opportunity for students to "finish" learning previous skills and deepen conceptual understanding.

## Diagnostic Assessment

The diagnostic assessment is designed to be administered to targeted students prior to beginning instruction on the given Topic. When appropriate, it is broken into parts (Part A, Part B, and so on); each part addresses a different prerequisite standard and contains three problems. If a student correctly answers at least 2 out of the 3 problems, it can be assumed that he/she is ready to engage with the new content of the Topic with little to no support needed prior to engaging with the Topic. The diagnostic assessment is designed in this way so that teachers can determine the "entry point" to remedial instruction and/or opportunities for unfinished learning within the context of the new learning. The entry points and opportunities for unfinished learning will vary between students.

## Guidance for Remediation

The Remediation Guidance is designed for teacher use. It is also broken into parts (Part A, Part B, and so on) and correlates to the parts on the diagnostic assessment. Each part contains the following:

1. The focus standard: The focus standards are strategically chosen to address prerequisite skills and are purposefully arranged in the order that students typically master the skills and knowledge.
2. Why this is important for current grade level work: This section describes how the work of the prerequisite standard relates to the standard(s) addressed in the Topic of instruction.
3. Using the diagnostic assessment to identify gaps: This section identifies common errors students make on the diagnostic assessment items.
4. Remediation Resources for Targeted Instruction: The resources pinpoint specific Eureka lessons and parts of lessons for teachers to use to address gaps in mathematical knowledge. Using Eureka materials to address remediation ensures alignment to the standards, consistency in approach to learning, and similarities in strategies for solving problems.

# Diagnostic Assessment: Grade 7 Eureka Module 2, Topic A 

Part A: 5.NF.A. 1 (addition):
For problems $1-3$, perform the indicated operation and support your answer by showing your work and/or a visual fraction model.

1. Add: $\frac{2}{5}+\frac{1}{4}$
2. Add: $\frac{3}{7}+\frac{2}{3}$
3. Add: $2 \frac{5}{6}+3 \frac{3}{4}$

Part B: 5.NF.A. 1 (subtraction):
For problems 4-6, perform the indicated operation and support your answer by showing your work and/or a visual fraction model.
4. Subtract: $\frac{3}{5}-\frac{1}{2}$
5. Subtract: $\frac{5}{8}-\frac{2}{7}$
6. Subtract: $4 \frac{7}{9}-1 \frac{3}{4}$

# Diagnostic Assessment: Grade 7 Eureka Module 2, Topic A 

Part C: 6.NS.C.5:
The Island of Sumatra lies on the $5^{\text {th }}$ parallel north, meaning its latitude is $5^{\circ}$ north of the equator. The Island of New Guinea lies on the $5^{\text {th }}$ parallel south.
7. What rational number would best represent the location of the Island of New Guinea? Explain your choice.
8. The Batu Islands have a latitude $0^{\circ}$. Explain the meaning of 0 in this context.
9. The Island of Java has a latitude of $-60^{\circ}$, and the Island of Kotlin has a latitude of $60^{\circ}$. Which island is closer to the equator? Explain your choice.

Part D: 6.NS.C.6a:
10. If $a=4$ and $b=-a$, where is $b$ located on the number line? Plot and label point $b$ on the number line below.

11. If $c=-2$ and $d=-c$, where is $d$ located on the number line? Plot and label point $d$ on the number line below.

12. If $e=-e$, where is $e$ located on the number line? Plot and label point $e$ on the number line below.


# Diagnostic Assessment: Grade 7 Eureka Module 2, Topic A 

Part E: 6.NS.C.6c:
13. $g=-3.5$. Plot and label point $g$ on the number line below.

14. $h=-\frac{3}{4}$. Plot and label point $h$ on the number line below.

15. $i=-5 \frac{1}{4}$. Plot and label point $i$ on the number line below.

# Diagnostic Assessment: Grade 7 Eureka Module 2, Topic A 

Part F: 6.NS.C.7a:
16. If $-\frac{12}{7}<-0.357$, which number is further to the left on a number line oriented from left to right? Explain your choice.
17. If $-2.75>-5 \frac{3}{8}$, which number is further to the right on a number line oriented from left to right? Explain your choice.
18. If $-3 \frac{1}{8},-3.7,3.01$, and $3 \frac{8}{9}$ were all plotted on a number line oriented from left to right, which number would be furthest to the left? Furthest to the right?

Part G: 6.NS.C.7c:
Determine the value of each expression and explain your answer.
19. $\left|-\frac{23}{17}\right|=$
20. $|9.735|=$
21. $|0|=$

# Diagnostic Assessment Key: Grade 7 Eureka Module 2, Topic A 

Solutions:

1. $\frac{13}{20}$ or equivalent
2. $\frac{23}{21}$ or equivalent
3. $6 \frac{7}{12}$ or equivalent
4. $\frac{1}{10}$ or equivalent
5. $\frac{19}{56}$ or equivalent
6. $3 \frac{1}{36}$ or equivalent
7. -5
8. A latitude of $0^{\circ}$ means the Batu Islands lie on the equator. (sample)
9. Both islands are equidistant from the equator, because each are $60^{\circ}$ from the equator, just in different directions. (sample)
10. 


11.

12.

13.

14.


## Diagnostic Assessment Key: Grade 7 Eureka Module 2, Topic A

15. 


16. $-\frac{12}{7}$ is further to the left because its value is less than -0.357 . (sample)
17. -2.75 is further to the right because its value is greater than $-5 \frac{3}{8}$. (sample)
18. -3.7 would be furthest to the left, and $3 \frac{8}{9}$ would be furthest to the right.
19. $\frac{23}{17}$ because it is that far from zero on a number line. (sample)
20. 9.735 because it is that far from zero on a number line. (sample)
21. 0 because it is located at zero meaning there is no distance between it and zero. (sample)

## Remediation Guidance: Grade 7 Eureka Module 2, Topic A

Part A Focus: 5.NF.A.1: Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $2 / 3+5 / 4=$ $8 / 12+15 / 12=23 / 12$. (In general, $a / b+c / d=(a d+b c) / b d$.)

## Why this is important for current grade level work:

While the first three lessons of the target topic focus only on integers in an effort to build a solid conceptual understanding, students extend their work with adding integers to adding fractions in Lesson 4. This topic is the first time a student will be expected to perform addition with signed numbers, and, as such, each student needs to be able to devote most of their thought to handling the positive and negative values. For this to be possible, students need to be fluent in adding whole numbers, fractions, and, in limited cases towards the end of the topic, decimals. Since the target standard speaks to all rational numbers, not just integers, the bulk of the problems in Lessons 8 and 9 include fractions and/or mixed numbers. In these three problems, the denominators were chosen strategically to avoid a focus on finding the least common denominator, which is not an explicit expectation of any standard at any grade/course. Rather, the problems force students to think about how to create equivalent fractions with any common denominator. The problems scaffold in difficulty, and the severity of mistakes should determine whether this should be viewed as a true prerequisite or an opportunity for unfinished learning.

## Using the Diagnostic Assessment to identify gaps:

## Problems 1-2:

The most important look fors here are the accuracy of the answers followed by the method by which the student calculates the sums. Look for students who do not create a common denominator before adding as this shows a gap in understanding that should be filled prior to beginning Lesson 4 . Students who correctly create a common denominator larger than the product of the two given denominators should still be considered ready for the target topic.

## Problem 3:

Look for students who are unable to split the problem into an addition of wholes and an addition of parts. Students should not turn the mixed numbers into improper fractions prior to adding as this is not a requirement nor an expectation of any standard at any grade/course. If students correctly answer problems 1 and 2 but miss 3 , this should be viewed as an opportunity for unfinished learning and addressed as the target topic progresses.

## Remediation Resources for

 Targeted Instruction:5th Grade, Module 3, Topic B, Lesson(s) 3-4

Use the Concept Development portion of each Lesson and a sampling of problems from the Problem Set focused on conceptual understanding.

## Remediation Guidance: Grade 7 Eureka Module 2, Topic A

Part B Focus: 5.NF.A.1: Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $2 / 3+5 / 4=$ $8 / 12+15 / 12=23 / 12$. (In general, $a / b+c / d=(a d+b c) / b d$.)

| Why this is important for current grade level work: <br> While the first three lessons of the target topic focus only on integers in an effort to build a solid conceptual understanding, students extend their work with subtracting integers to subtracting fractions in Lesson 4. This topic is the first time a student will be expected to perform subtraction with signed numbers, and, as such, each student needs to be able to devote most of their thought to handling the positive and negative values. For this to be possible, students need to be fluent in subtraction whole numbers, fractions, and, in limited cases towards the end of the topic, decimals. Since the target standard speaks to all rational numbers, not just integers, the bulk of the problems in Lessons 8 and 9 include fractions and/or mixed numbers. In these three problems, the denominators were chosen strategically to avoid a focus on finding the least common denominator, which is not an explicit expectation of any standard at any grade/course. Rather, the problems force students to think about how to create equivalent fractions with any common denominator. The problems scaffold in difficulty, and the severity of mistakes should determine whether this should be viewed as a true prerequisite or an opportunity for unfinished learning. |  | Remediation Resources for Targeted Instruction: <br> 5th Grade, Module 3, Topic B, Lesson(s) 5-6 |
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| Using the Diagnostic Assessment to identify gaps: |  |  |
| Problems 4-5: <br> The most important look fors here are the accuracy of the answers followed by the method by which the student calculates the differences. Look for students who do not create a common denominator before subtracting as this shows a gap in understanding that should be filled prior to beginning Lesson 5. Students who correctly create a common denominator larger than the product of the two given denominators should still be considered ready for the target topic. | Problem 6: <br> Look for students who are unable to split the problem into a difference of wholes and a difference of parts. Students should not turn the mixed numbers into improper fractions prior to subtracting as this is not a requirement nor an expectation of any standard at any grade/course. If students correctly answer problems 4 and 5 but miss 6 , this should be viewed as an opportunity for unfinished learning and addressed as the target topic progresses. | Use the Concept Development portion of each Lesson and a sampling of problems from the Problem Set focused on conceptual understanding. |

## Remediation Guidance: Grade 7 <br> Eureka Module 2, Topic A

Part C Focus: 6.NS.C.5: Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

| Why this is important for current grade level work: <br> Beginning in Lesson 1, the topic uses context to build understanding of addition with signed numbers, leveraging the idea of making zero in various context. Students who have a firm understanding and ability to interpret signed numbers in a real-world context will be able to extend that understanding to sums that do not equal zero. Look for students who struggle to engage with the context, keeping them from answering the questions. Such students will struggle to engage with the target topic and build conceptual understanding through real-world context. Help the students see how numbers can be used to model real-world situations, leading them to be better equipped to solve real-world problems later in the topic. |  |  | Remediation Resources for Targeted Instruction: <br> 6th Grade, Module 3, Topic A, Lesson(s) 2-3 |
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| Using the Diagnostic Assessment to identify gaps: |  |  |  |
| Problem | Problem 8: | Problem 9: |  |
| Look for students who think 5 would be an appropriate answer as this likely shows a gap in their understanding of negative numbers and their use in realworld contexts. | Look for students who do not mention the island's location with respect to the equator as this shows a lack of interpretation within the context. Further questioning should reveal whether the student has a gap in understanding or simply did not think to reference the equator. | Look for students who think one island is closer than the other as this will hinder their pursuit to master 7.NS.A.1a. Encouraging students to use a number line to aid in their choice for this item may prove advantageous, setting the stage for repeated use of the number line in the target topic. | Use the Classwork portion of each Lesson and a sampling of problems from the Problem Set focused on conceptual understanding. |

## Remediation Guidance: Grade 7 Eureka Module 2, Topic A

Part D Focus: 6.NS.C.6a: Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3)=3$, and that 0 is its own opposite.

## Why this is important for current grade level work:

Topic A provides students with their first opportunity to add and subtract with signed numbers. Having a firm understanding of positive and negative numbers and the relationship between the two will help students access and gain the new understandings presented in this topic. It is important that students understand the language of opposite as opposed to simply negative (e.g., being able to read -3 as the opposite of three instead of just negative three). This understanding and flexibility in language will lead to students understanding additive inverse, the explicit expectation of 7.NS.A.1c.

Using the Diagnostic Assessment to identify gaps:

## Problem 10: <br> Problem 11:

Look for students who use an additive relationship between hours worked and dollars earned as opposed to a multiplicative relationship. Such a student is most likely not ready to engage with on gradelevel work.

Look for students who plot $d$ at -2 , thinking the - in the sentence is already accounted for since the value of $c$ is negative. This shows a gap in understanding that - means the opposite of and/or a gap in understanding the opposite of an opposite in the number itself. In either case the student would benefit from a look back at previous grade-level lessons prior to engaging with the target topic.

## Problem 12:

Similar to problem 11, an incorrect answer to this problem shows a gap in understanding that zero is its own opposite; however, this misunderstanding can likely be viewed as an opportunity for unfinished learning and can be filled while studying grade-level content.

## Remediation Resources for Targeted

 Instruction:6th Grade, Module 3, Topic A, Lesson(s) 1, 4-6

Use the Classwork portion of each Lesson and a sampling of problems from the Problem Set focused on conceptual understanding.

## Remediation Guidance: Grade 7 Eureka Module 2, Topic A

Part E Focus: 6.NS.C.6c: Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

## Why this is important for current grade level work:

Beyond being flexible with language in reading signed numbers, the target grade-level standard requires the use of the number line to aid in performing operations with signed numbers. That process always begins by plotting the first number in the number sentence then moving from there. If students are unable to accurately plot numbers on the number line, they will not be able to utilize the number line as an aid to adding and subtracting signed numbers (i.e., the explicit expectation of the target standard). The most important look fors here are the accuracy of the plots.

## Using the Diagnostic Assessment to identify gaps:

## Problems 13-15:

Look for students who have trouble with the fractional part of the numbers they are plotting. If students plot the numbers on the correct side of zero in each case and are close to the actual number, they should be considered ready for the target topic and will just need some additional supports to finish their learning of plotting rational numbers.

## Remediation Resources for Targeted

 Instruction:6th Grade, Module 3, Topic A, Lesson(s) 4-6

Use the Classwork portion of each Lesson and a sampling of problems from the Problem Set focused finding and positioning numbers on a number line diagram.

## Remediation Guidance: Grade 7 Eureka Module 2, Topic A

Part F Focus: 6.NS.C.7a: Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret $-3>-7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right.

## Why this is important for current grade level work:

As students utilize the number line to perform addition and subtraction of signed numbers, it is imperative that students understand which direction to move along the number line based on the sign of each number. Although the comparison of position of two numbers will not explicitly be leveraged during the target topic, this understanding should help build students' ability to determine if their sums and differences are reasonable. This will prove invaluable as students progress through their study of operations with signed numbers.

## Using the Diagnostic Assessment to identify gaps:

## Problems 16-17:

Look for students who get distracted by the values in the statement of inequality as this likely shows a gap in understanding the connection between statements of inequality and statements of relative position on a number line. Such a gap in understanding will make it difficult for students to develop the ability to assess the reasonableness of their answers. Such students could benefit from additional supports while studying the target topic.

## Problem 18:

The four numbers were chosen strategically to all appear close in value, helping teachers identify students with a gap in their understanding of signed numbers and their relative position. Although not a true prerequisite understanding/skill, this should be addressed while studying the grade-level content.

## Remediation Resources for Targeted

 Instruction:6th Grade, Module 3, Topic B,
Lesson(s) 10 and 12
Use the Classwork portion of each Lesson and a sampling of problems from the Problem Set focused on conceptual understanding.

## Remediation Guidance: Grade 7 Eureka Module 2, Topic A

Part G Focus: 6.NS.C.7c: Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of -30 dollars, write |-30| $=30$ to describe the size of the debt in dollars.

## Why this is important for current grade level work:

Both 7.NS.A.1b and 7.NS.A.1c use absolute value to help students apply the concept of distance as a strategy to help them calculate sums and differences of signed numbers. These items should determine which students have a firm understanding of the concept of absolute value. The most important look fors here are the accuracy of the answers and explanations. It is likely that students will know the absolute value of any nonzero number is positive, but that does not ensure they have a firm understanding of absolute value as distance from zero on a number line. Without the understanding of distance, it will be difficult for students to master the explicit expectations of the target grade-level standard.

## Using the Diagnostic Assessment to identify gaps:

## Problem 19:

Look for students who neglect to provide an explanation. Such students should be questioned further to ensure they understand the connection between absolute value and distance from zero on a number line.

## Problem 20:

Look for students who report a negative value as this shows the students thinks absolute value is the opposite of a number instead of the number's distance from zero.

## Problem 21:

Look for students who give an answer other than zero (or possibly think the answer is -0 ) as both show a clear misunderstanding of absolute value and/or zero as neither negative nor positive. Again, this can be viewed as an opportunity for unfinished learning and be addressed while studying the grade-level content.

Remediation Resources for Targeted Instruction:

6th Grade, Module 3, Topic B, Lesson(s) 11-13

Use the Classwork portion of each Lesson and a sampling of problems from the Problem Set focused on conceptual understanding

