Chapter 1.   Problems I Enjoy - George Harvey

1. Andrew arranged the numbers 1, 2, 3, ..., 11, 12 into six pairs so that the sum of any two numbers in a pair is prime and no two of these primes are equal. Find the primes and the pairs.

2. A circus performance was attended by 120 people who paid a total of $120. The men paid $5, the women $2 and the children 10 c. How many men, how many women and how many children went to the circus?

3. There are three boxes, containing respectively:

> two black marbles (BB)
> two white marbles (WW)
> one black and one white marble (BW).

The boxes are initially correctly labelled according to their contents: BB, WW and BW. The labels are then switched in such a way that every box is now incorrectly labelled. The contents of all three boxes is to be determined by drawing one marble at a time (noting its colour). What is the smallest number of drawings needed to do this?

4. "How old are your three children?" the mathematics master asks a former student. He is told their ages add to 13, and multiply to give the number on his study door (which they both can see). "I will need to know more", the master says, after a few moments reflection. "The eldest one is learning to play the violin", replies his former pupil. "Ah! In that case I can now give you their ages", the master tells him, and does so correctly.

How does he know? What are their ages? What is the number on the door?

5. A number is written on a blackboard. You may perform the following operations one after the other in any order as often as you wish.

> (i) Replace the current number on the blackboard with twice this number.
> (ii) Delete the last digit of the current number.

Can you obtain 14 from a starting number of 458?

6. Using each of the ten digits 0, 1, 2, ..., 9 just once, is it possible to form positive integers whose sum is exactly 100?

7. The only timepiece owned by a man was a wall clock which had stopped because he had forgotten to wind it. In the afternoon, he wound it while it was showing (incorrectly) 1 o'clock, then walked to a friend's place and noted that the (correct) time was 3 pm. He left his friend's place at $5 : 30$ pm that evening, taking the same route home and walking at the same average speed. On arrival, his clock showed $5 : 30$ pm. At what (correct) time did he arrive home?

8. How many differently shaped rectangles, with positive integer dimensions, have a perimeter equal to their area?

9. Let x be any number less than 1, and let y be any number greater than 1. Let S be the sum of x and y, and let P be the product of x and y. Prove that the difference between S and P must be greater than 1.

10. Prove that it is impossible to find four distinct numbers p, q, r and s which satisfy the equation

$$pq + rs = ps + qr \, .$$

Now try Problems 1 and 2 in the Noether Student Problems Book.