Name $\qquad$ Date $\qquad$

1. In the following two questions, lines $A B$ and $C D$ intersect at point $O$. When necessary, assume that seemingly straight lines are indeed straight lines. Determine the measures of the indicated angles.
a. Find the measure of $\angle X O C$.

b. Find the measures of $\angle A O X, \angle Y O D$, and $\angle D O B$.

2. Is it possible to draw two different triangles that both have angle measurements of $40^{\circ}$ and $50^{\circ}$ and a side length of 5 cm ? If it is possible, draw examples of these conditions, and label all vertices and angle and side measurements. If it is not possible, explain why.
3. In each of the following problems, two triangles are given. For each: (1) State if there are sufficient or insufficient conditions to show the triangles are identical, and (2) explain your reasoning.
a.


R
$T$

b.

4. In the following diagram, the length of one side of the smaller shaded square is $\frac{1}{3}$ the length of square $A B C D$. What percent of square $A B C D$ is shaded? Provide all evidence of your calculations.

5. Side $\overline{E F}$ of square $D E F G$ has a length of 2 cm and is also the radius of circle $F$. What is the area of the entire shaded region? Provide all evidence of your calculations.

6. For his latest design, a jeweler hollows out crystal cube beads (like the one in the diagram) through which the chain of a necklace is threaded. If the edge of the crystal cube is 10 mm , and the edge of the square cut is 6 mm , what is the volume of one bead? Provide all evidence of your calculations.

7. John and Joyce are sharing a piece of cake with the dimensions shown in the diagram. John is about to cut the cake at the mark indicated by the dotted lines. Joyce says this cut will make one of the pieces three times as big as the other. Is she right? Justify your response.

8. A tank measures 4 ft . in length, 3 ft . in width, and 2 ft . in height. It is filled with water to a height of 1.5 ft . A typical brick measures a length of 9 in ., a width of 4.5 in ., and a height of 3 in . How many whole bricks can be added before the tank overflows? Provide all evidence of your calculations.
9. Three vertical slices perpendicular to the base of the right rectangular pyramid are to be made at the marked locations: (1) through $\overline{A B}$, (2) through $\overline{C D}$, and (3) through vertex $E$. Based on the relative locations of the slices on the pyramid, make a reasonable sketch of each slice. Include the appropriate notation to indicate measures of equal length.

(1) Slice through $\overline{A B}$
(2) Slice through $\overline{C D}$
(3) Slice through vertex $E$

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| :--- | :--- | :--- |
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Module 6: Geometry
10. Five three-inch cubes and two triangular prisms have been glued together to form the composite threedimensional figure shown in the diagram. Find the surface area of the figure, including the base. Provide all evidence of your calculations.


A Progression Toward Mastery
$\left.\begin{array}{|l|l|l|l|l|}\hline \text { Assessment } & \begin{array}{l}\text { STEP 1 } \\ \text { Missing or } \\ \text { incorrect answer } \\ \text { and little evidence } \\ \text { of reasoning or } \\ \text { application of } \\ \text { mathematics to } \\ \text { solve the problem. }\end{array} & \begin{array}{l}\text { STEP 2 } \\ \text { Missing or incorrect } \\ \text { answer but } \\ \text { evidence of some } \\ \text { reasoning or } \\ \text { application of } \\ \text { mathematics to } \\ \text { solve the problem. }\end{array} & \begin{array}{l}\text { STEP 3 } \\ \text { A correct answer } \\ \text { with some evidence } \\ \text { of reasoning or } \\ \text { application of } \\ \text { mathematics to } \\ \text { solve the problem } \\ \text { OR an incorrect }\end{array} & \begin{array}{l}\text { STEP 4 } \\ \text { A correct answer } \\ \text { supported by } \\ \text { substantial }\end{array} \\ \text { evidence of solid } \\ \text { reasoning or } \\ \text { application of } \\ \text { mathematics to } \\ \text { solve the problem. }\end{array}\right\}$

| 3 | a $\text { 7.G.A. } 2$ | Student does not provide a response. OR <br> Student fails to provide evidence of comprehension. | Student correctly identifies triangles as identical or not identical, but no further evidence is provided. | Student correctly identifies triangles as identical or not identical but with the incorrect supporting evidence, such as giving the incorrect condition by which they are identical. | Student correctly identifies triangles as identical or not identical and supports this answer, such as giving the condition by which they are identical or the information that prevents them from being identical. |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | b $\text { 7.G.A. } 2$ | Student does not provide a response. OR <br> Student fails to provide evidence of comprehension. | Student correctly identifies triangles as identical or not identical, but no further evidence is provided. | Student correctly identifies triangles as identical or not identical but with the incorrect supporting evidence, such as giving the incorrect condition by which they are identical. | Student correctly identifies triangles as identical or not identical and supports this answer, such as giving the condition by which they are identical or the information that prevents them from being identical. |
| 4 | 7.G.B.6 | Student incorrectly calculates the percentage of the shaded area due to a combination of at least one conceptual and one calculation error or due to more than one conceptual or calculation error. | Student incorrectly calculates the percentage of the shaded area due to one conceptual error (e.g., taking the incorrect values by which to calculate percentage), but all other supporting work is correct. | Student incorrectly calculates the percentage of the shaded area due to one calculation error (e.g., not summing both shaded areas), but all other supporting work is correct. | Student correctly finds $55 \frac{5}{9} \%$ as the percentage of the shaded area, and complete evidence of calculations is shown. |
| 5 | 7.G.B.6 | Student incorrectly calculates the shaded area due to a combination of at least one conceptual and one calculation error OR due to more than one conceptual or calculation error. | Student incorrectly calculates the shaded area due to one conceptual error (e.g., taking the incorrect fraction of the area of the circle to add to the area of the square), but all other supporting work is correct. | Student incorrectly calculates the shaded area due to one calculation error (e.g., making an error in taking a fraction of $4 \pi$ ), but all other supporting work is correct. | Student correctly finds the shaded area to be either $4 \mathrm{~cm}^{2}+3 \pi \mathrm{~cm}^{2}$, or approximately $13.4 \mathrm{~cm}^{2}$, and complete evidence of calculations is shown. |
| 6 | 7.G.B.6 | Student incorrectly finds the volume due to one or more calculation errors or a combination of calculation and conceptual errors. | Student incorrectly finds the volume due to one conceptual error (e.g., calculating the volume of the hollow as a cube rather than as a rectangular prism), but all other supporting work is correct. | Student incorrectly finds the volume due to one calculation error (e.g., an arithmetic error), but all other supporting work is correct. | Student correctly finds the volume of the bead to be $640 \mathrm{~mm}^{3}$, and complete evidence of calculations is shown. |

$\left.\begin{array}{|c|c|l|l|l|l|}\hline \mathbf{7} & \text { 7.G.B.6 } & \begin{array}{l}\text { Student incorrectly } \\ \text { finds the volume due to } \\ \text { one or more calculation } \\ \text { errors or a combination } \\ \text { of calculation and } \\ \text { conceptual errors. }\end{array} & \begin{array}{l}\text { Student incorrectly finds } \\ \text { the volume due to one } \\ \text { conceptual error } \\ \text { (e.g., using the wrong } \\ \text { formula for the volume } \\ \text { of a trapezoidal prism), } \\ \text { but all other supporting } \\ \text { work is correct. }\end{array} & \begin{array}{l}\text { Student incorrectly finds } \\ \text { the volume due to one } \\ \text { calculation error (e.g., an } \\ \text { arithmetic error), but all } \\ \text { other supporting work is } \\ \text { correct. }\end{array} & \begin{array}{l}\text { Student correctly finds } \\ \text { the volume of the } \\ \text { trapezoidal prism to be } \\ 225 \mathrm{~cm}^{3} \text { and the volume } \\ \text { of the triangular prism to } \\ \text { be } 75 \mathrm{~cm}^{3} \text {, and the } \\ \text { larger piece is shown to } \\ \text { be } 3 \text { times as great as } \\ \text { the smaller piece. }\end{array} \\ \hline \mathbf{8} & \text { 7.G.B.3 } & \begin{array}{l}\text { Student incorrectly } \\ \text { finds the number of } \\ \text { bricks due to one or } \\ \text { more calculation errors } \\ \text { or a combination of } \\ \text { calculation and } \\ \text { conceptual errors. }\end{array} & \begin{array}{l}\text { Student incorrectly finds } \\ \text { the number of bricks due } \\ \text { to a calculation error } \\ \text { (e.g., using the volume of } \\ \text { water rather than the } \\ \text { volume of the unfilled } \\ \text { tank), but all other } \\ \text { supporting work is } \\ \text { correct. }\end{array} & \begin{array}{l}\text { Student incorrectly finds } \\ \text { the number of bricks due } \\ \text { to one calculation error } \\ \text { (e.g., a rounding error), } \\ \text { but all other supporting } \\ \text { work is correct. }\end{array} & \begin{array}{l}\text { Student correctly finds } \\ \text { that 85 bricks can be put } \\ \text { in the tank without the } \\ \text { tank overflowing and } \\ \text { offers complete } \\ \text { evidence of calculations. }\end{array} \\ \hline \mathbf{9} & \text { 7.G.B.6 } & \begin{array}{l}\text { Student does not } \\ \text { appropriately sketch } \\ \text { two relative trapezoids } \\ \text { according to their } \\ \text { relative positions on } \\ \text { the pyramid, and an } \\ \text { isosceles triangle is not } \\ \text { made for the slice } \\ \text { through the vertex. }\end{array} & \begin{array}{l}\text { Student sketches two } \\ \text { relative trapezoids } \\ \text { appropriately but does } \\ \text { not sketch an isosceles } \\ \text { triangle as a slice } \\ \text { through the vertex. }\end{array} & \begin{array}{l}\text { Student makes three } \\ \text { sketches but does not } \\ \text { indicate lengths of equal } \\ \text { measure. }\end{array} & \begin{array}{l}\text { Student makes three } \\ \text { sketches that indicate } \\ \text { appropriate slices at the } \\ \text { given locations on the }\end{array} \\ \text { pyramid and indicates } \\ \text { the lengths of equal }\end{array}\right\}$

Name $\qquad$ Date $\qquad$

1. In the following two questions, lines $A B$ and $C D$ intersect at point $O$. When necessary, assume that seemingly straight lines are indeed straight lines. Determine the measures of the indicated unknown angles.
a. Find the measure of $\angle X O C$.

$$
\begin{aligned}
x+10 & =25+45 \\
x+10-10 & =70-10 \\
x & =60
\end{aligned}
$$

$$
\angle X O C=60^{\circ}
$$


b. Find the measures of $\angle A O X, \angle Y O D$, and $\angle D O B$.

$$
\begin{aligned}
2 x+90+x+(60-x) & =180 \\
2 x+150-150 & =180-150 \\
2 x & =30 \\
\frac{1}{2}(2 x) & =\frac{1}{2}(30) \\
x & =15
\end{aligned}
$$



$$
\begin{aligned}
& \angle A O X=2(15)^{\circ}=30^{\circ} \\
& \angle Y O D=15^{\circ} \\
& \angle D O B=(60-15)^{\circ}=45^{\circ}
\end{aligned}
$$

2. Is it possible to draw two different triangles that both have angle measurements of $40^{\circ}$ and $50^{\circ}$ and a side length of 5 cm ? If it is possible, draw examples of these conditions, and label all vertices and angle and side measurements. If it is not possible, explain why.

One possible solution:

3. In each of the following problems, two triangles are given. For each: (1) State if there are sufficient or insufficient conditions to show the triangles are identical, and (2) explain your reasoning.

b.

The triangles are identical by the two angles and included side condition. The marked side is between the given angles. $\triangle M N O \leftrightarrow \triangle R Q P$

4. In the following diagram, the length of one side of the smaller shaded square is $\frac{1}{3}$ the length of square $A B C D$. What percent of square $A B C D$ is shaded? Provide all evidence of your calculations.

Let $x$ be the length of the side of the smaller shaded square. Then $A D=3 x$; the length of the side of the larger shaded square is $3 x-x=2 x$


Area $A_{A B C D}=(3 x)^{2}=9 x^{2}$
Area $_{\text {Large shaded }}=(2 x)^{2}=4 x^{2}$
Areasmall shaded $=(x)^{2}=x^{2}$
Areashaded $=4 x^{2}+x^{2}=5 x^{2}$
Percent Areashaded $=\frac{5 x^{2}}{9 x^{2}}(100 \%)=55 \frac{5}{9} \%$
5. Side $\overline{E F}$ of square $D E F G$ has a length of 2 cm and is also the radius of circle $F$. What is the area of the entire shaded region? Provide all evidence of your calculations.

Area $_{\text {Circle } F}=(\pi)(2 \mathrm{~cm})^{2}=4 \pi \mathrm{~cm}^{2}$
Area $_{\frac{3}{4} \text { Circle } F}=\frac{3}{4}\left(4 \pi \mathrm{~cm}^{2}\right)=3 \pi \mathrm{~cm}^{2}$
Area $a_{\text {DEFG }}=(2 \mathrm{~cm})(2 \mathrm{~cm})=4 \mathrm{~cm}^{2}$
Area ${ }_{\text {Shaded Region }}=4 \mathrm{~cm}^{2}+3 \pi \mathrm{~cm}^{2}$
Area $_{\text {shaded Region }} \approx 13.4 \mathrm{~cm}^{2}$

6. For his latest design, a jeweler hollows out crystal cube beads (like the one in the diagram) through which the chain of a necklace is threaded. If the edge of the crystal cube is 10 mm , and the edge of the square cut is 6 mm , what is the volume of one bead? Provide all evidence of your calculations.
$V_{0 l u m}$ Large $_{\text {Cube }}=(10 \mathrm{~mm})^{3}=1,000 \mathrm{~mm}^{3}$
Volume $_{\text {Hollow }}=(10 \mathrm{~mm})(6 \mathrm{~mm})(6 \mathrm{~mm})=360 \mathrm{~mm}^{3}$
Volume $_{\text {Bead }}=1,000 \mathrm{~mm}^{3}-360 \mathrm{~mm}^{3}=640 \mathrm{~mm}^{3}$

7. John and Joyce are sharing a piece of cake with the dimensions shown in the diagram. John is about to cut the cake at the mark indicated by the dotted lines. Joyce says this cut will make one of the pieces three times as big as the other. Is she right? Justify your response.

Volume Trapezoidal Prism $=\frac{1}{2}(5 \mathrm{~cm}+2.5 \mathrm{~cm})(6 \mathrm{~cm})(10 \mathrm{~cm})=225 \mathrm{~cm}^{3}$
Volume Triangular Prism $=\frac{1}{2}(2.5 \mathrm{~cm})(6 \mathrm{~cm})(10 \mathrm{~cm})=75 \mathrm{~cm}^{3}$


Joyce is right; the current cut would give $225 \mathrm{~cm}^{3}$ of cake for the trapezoidal prism piece and $75 \mathrm{~cm}^{3}$ of cake for the triangular prism piece, making the larger piece
3 times the size of the smaller piece $\left(\frac{225}{75}=3\right)$.
8. A tank measures 4 ft . in length, 3 ft . in width, and 2 ft . in height. It is filled with water to a height of 1.5 ft . A typical brick measures a length of 9 in ., a width of 4.5 in ., and a height of 3 in . How many whole bricks can be added before the tank overflows? Provide all evidence of your calculations.

Volume in tank not occupied by water:

$$
V=(4 \mathrm{ft} .)(3 \mathrm{ft} .)(0.5 \mathrm{ft} .)=6 \mathrm{ft}^{3}
$$

Volume $_{\text {Brick }}=(9 \mathrm{in}).(4.5 \mathrm{in}).(3 \mathrm{in})=.121.5 \mathrm{in}^{3}$
Conversion (in ${ }^{3}$ to $\mathrm{ft}^{3}$ ): $\left(121.5 \mathrm{in}^{3}\right)\left(\frac{1 \mathrm{ft}^{3}}{12^{3} \mathrm{in}^{3}}\right)=0.0703125 \mathrm{ft}^{3}$

Number of bricks that fit in the volume not occupied by water: $\left(\frac{6 \mathrm{ft}^{3}}{0.0703125 \mathrm{ft}^{3}}\right)=85 \frac{1}{3}$
Number of whole bricks that fit without causing overflow: 85
9. Three vertical slices perpendicular to the base of the right rectangular pyramid are to be made at the marked locations: (1) through $\overline{A B}$, (2) through $\overline{C D}$, and (3) through vertex $E$. Based on the relative locations of the slices on the pyramid, make a reasonable sketch of each slice. Include the appropriate notation to indicate measures of equal length.

Sample response:

(1) Slice through $\overline{A B}$
(2) Slice through $\overline{C D}$
(3) Slice through vertex $E$


Module 6:
10. Five three-inch cubes and two triangular prisms have been glued together to form the composite threedimensional figure. Find the surface area of the figure, including the base. Provide all evidence of your calculations.

19 square surfaces: $19(3 \mathrm{in} .)^{2}=171 \mathrm{in}^{2}$
4 triangular surfaces: $(4)\left(\frac{1}{2}\right)(3 \mathrm{in}).(4 \mathrm{in})=.24 \mathrm{in}^{2}$
$3 \times 5$ rectangular surface: $(3 \mathrm{in}).(5 \mathrm{in})=.15 \mathrm{in}^{2}$
$3 \times 4$ rectangular surface: $(3 \mathrm{in}).(4 \mathrm{in})=.12 \mathrm{in}^{2}$
$6 \times 5$ rectangular surface: $(6 \mathrm{in}).(5 \mathrm{in})=.30 \mathrm{in}^{2}$

$6 \times 4$ rectangular surface: $(6 \mathrm{in}).(4 \mathrm{in})=.24 \mathrm{in}^{2}$

Total surface area: $171 \mathrm{in}^{2}+24 \mathrm{in}^{2}+15 \mathrm{in}^{2}+12 \mathrm{in}^{2}+30 \mathrm{in}^{2}+24 \mathrm{in}^{2}=276 \mathrm{in}^{2}$

