

1. Harry and Terry are each told to calculate  $8 - (2 + 5)$ . Harry gets the correct answer. Terry ignores the parentheses and calculates  $8 - 2 + 5$ . If Harry's answer is  $H$  and Terry's answer is  $T$ , what is  $H - T$ ?

- (A)  $-10$  (B)  $-6$  (C)  $0$  (D)  $6$  (E)  $10$

2. Paul owes Paula 35 cents and has a pocket full of 5-cent coins, 10-cent coins, and 25-cent coins that he can use to pay her. What is the difference between the largest and the smallest number of coins he can use to pay her?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5



3. Isabella had a week to read a book for a school assignment. She read an average of 36 pages per day for the first three days and an average of 44 pages per day for the next three days. She then finished the book by reading 10 pages on the last day. How many pages were in the book?

- (A) 240 (B) 250 (C) 260 (D) 270 (E) 280



4. The sum of two prime numbers is 85. What is the product of these two prime numbers?

- (A) 85 (B) 91 (C) 115 (D) 133 (E) 166

5. Margie's car can go 32 miles on a gallon of gas, and gas currently costs \$4 per gallon. How many miles can Margie drive on \$20 worth of gas?

- (A) 64 (B) 128 (C) 160 (D) 320 (E) 640



6. Six rectangles each with a common base width of 2 have lengths of 1, 4, 9, 16, 25, and 36. What is the sum of the areas of the six rectangles?

- (A) 91 (B) 93 (C) 162 (D) 182 (E) 202

7. There are four more girls than boys in Ms. Raub's class of 28 students. What is the ratio of the number of girls to the number of boys in her class?

- (A) 3 : 4 (B) 4 : 3 (C) 3 : 2 (D) 7 : 4 (E) 2 : 1

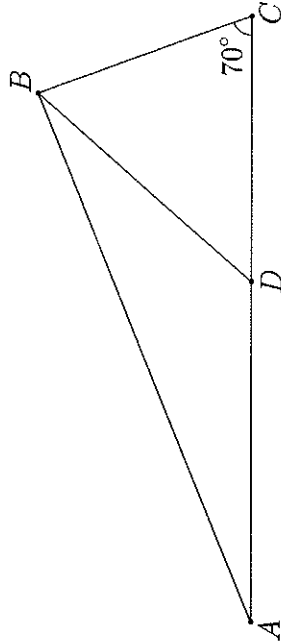
8. Eleven members of the Middle School Math Club each paid the same amount for a guest speaker to talk about problem solving at their math club meeting. They paid their guest speaker \$1A2. What is the missing digit  $A$  of this 3-digit number?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4



9. In  $\triangle ABC$ ,  $D$  is a point on side  $\overline{AC}$  such that  $BD = DC$  and  $\angle BCD$  measures  $70^\circ$ . What is the degree measure of  $\angle ADB$ ?

- (A) 100 (B) 120 (C) 135 (D) 140 (E) 150



10. The first AMC 8 was given in 1985 and it has been given annually since that time. Samantha turned 12 years old the year that she took the seventh AMC 8. In what year was Samantha born?

- (A) 1979 (B) 1980 (C) 1981 (D) 1982 (E) 1983

11. Jack wants to bike from his house to Jill's house, which is located three blocks east and two blocks north of Jack's house. After biking each block, Jack can continue either east or north, but he needs to avoid a dangerous intersection one block east and one block north of his house. In how many ways can he reach Jill's house by biking a total of five blocks?

- (A) 4 (B) 5 (C) 6 (D) 8 (E) 10



12. A magazine printed photos of three celebrities along with three photos of the celebrities as babies. The baby pictures did not identify the celebrities. Readers were asked to match each celebrity with the correct baby picture. What is the probability that a reader guessing at random will match all three correctly?

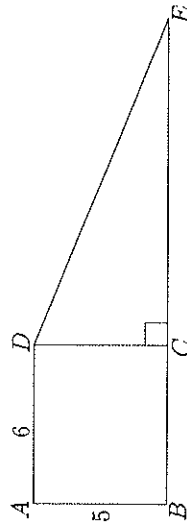
- (A)  $\frac{1}{9}$  (B)  $\frac{1}{6}$  (C)  $\frac{1}{4}$  (D)  $\frac{1}{3}$  (E)  $\frac{1}{2}$

13. If  $n$  and  $m$  are integers and  $n^2 + m^2$  is even, which of the following is impossible?

- (A)  $n$  and  $m$  are even (B)  $n$  and  $m$  are odd (C)  $n + m$  is even  
(D)  $n + m$  is odd (E) none of these is impossible

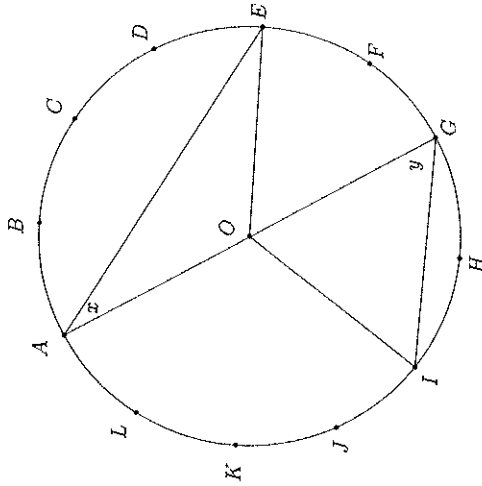
14. Rectangle  $ABCD$  and right triangle  $DCE$  have the same area. They are joined to form a trapezoid, as shown. What is  $DE$ ?

- (A) 12 (B) 13 (C) 14 (D) 15 (E) 16



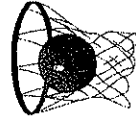
15. The circumference of the circle with center  $O$  is divided into 12 equal arcs marked the letters  $A$  through  $L$  as seen below. What is the number of degrees in the sum of angles  $x$  and  $y$ ?

- (A) 75 (B) 80 (C) 90 (D) 120 (E) 150



16. The "Middle School Eight" basketball conference has 8 teams. Every season each team plays every other conference team twice (home and away), and each team also plays 4 games against non-conference opponents. What is the total number of games in a season involving "Middle School Eight" teams?

- (A) 60 (B) 88 (C) 96 (D) 144 (E) 160



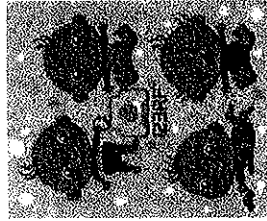
17. George walks 1 mile to school. He leaves home at the same time each day, walks at a steady speed of 3 miles per hour, and arrives just as school begins. Today he was distracted by the pleasant weather and walked the first  $\frac{1}{2}$  mile at a speed of only 2 miles per hour. At how many miles per hour must George run the last  $\frac{1}{2}$  mile in order to arrive just as school begins today?

- (A) 4 (B) 6 (C) 8 (D) 10 (E) 12



18. Four children were born at City Hospital yesterday. Assume each child is equally likely to be a boy or a girl. Which of the following outcomes is most likely?

- (A) all 4 are boys (B) all 4 are girls (C) 2 are girls and 2 are boys  
 (D) 3 are of one gender and 1 is of the other gender  
 (E) all of these outcomes are equally likely

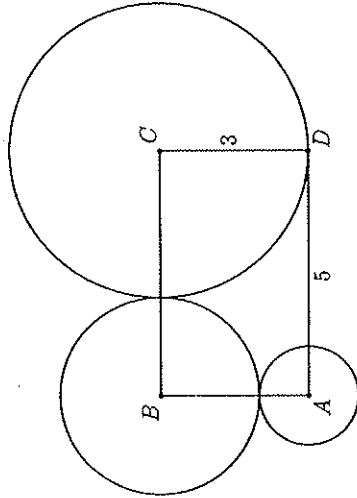


19. A cube with 3-inch edges is to be constructed from 27 smaller cubes with 1-inch edges. Twenty-one of the cubes are colored red and 6 are colored white. If the 3-inch cube is constructed to have the smallest possible white surface area showing, what fraction of the surface area is white?

- (A)  $\frac{5}{54}$  (B)  $\frac{1}{9}$  (C)  $\frac{5}{27}$  (D)  $\frac{2}{9}$  (E)  $\frac{1}{3}$

20. Rectangle  $ABCD$  has sides  $CD = 3$  and  $DA = 5$ . A circle of radius 1 is centered at  $A$ , a circle of radius 2 is centered at  $B$ , and a circle of radius 3 is centered at  $C$ . Which of the following is closest to the area of the region inside the rectangle but outside all three circles?

- (A) 3.5 (B) 4.0 (C) 4.5 (D) 5.0 (E) 5.5



21. The 7-digit numbers  $\underline{7}4\underline{A}5\underline{2}B\underline{1}$  and  $326\underline{A}B\underline{4}C$  are each multiples of 3. Which of the following could be the value of  $C$ ?

- (A) 1 (B) 2 (C) 3 (D) 5 (E) 8

22. A 2-digit number is such that the product of the digits plus the sum of the digits is equal to the number. What is the units digit of the number?

- (A) 1 (B) 3 (C) 5 (D) 7 (E) 9

23. Three members of the Euclid Middle School girls' softball team had the following conversation.

Ashley: I just realized that our uniform numbers are all 2-digit primes.

Bethany: And the sum of your two uniform numbers is the date of my birthday earlier this month.

Caitlin: That's funny. The sum of your two uniform numbers is the date of my birthday later this month.

Ashley: And the sum of your two uniform numbers is today's date.

What number does Caitlin wear?

- (A) 11 (B) 13 (C) 17 (D) 19 (E) 23

24. One day the Beverage Barn sold 252 cans of soda to 100 customers, and every customer bought at least one can of soda. What is the maximum possible median number of cans of soda bought per customer on that day?

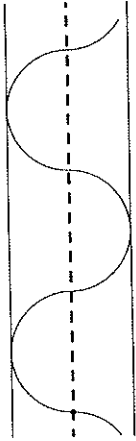
- (A) 2.5 (B) 3.0 (C) 3.5 (D) 4.0 (E) 4.5



25. A straight one-mile stretch of highway, 40 feet wide, is closed. Robert rides his bike on a path composed of semicircles as shown. If he rides at 5 miles per hour, how many hours will it take to cover the one-mile stretch?

Note: 1 mile = 5280 feet

- (A)  $\frac{\pi}{11}$     (B)  $\frac{\pi}{10}$     (C)  $\frac{\pi}{5}$     (D)  $\frac{2\pi}{5}$     (E)  $\frac{2\pi}{3}$



## SOLUTIONS

Your School Manager will be sent at least one copy of the 2014 AMC 8 Solutions Pamphlet with the report. It is meant to be loaned to students (but not duplicated).

### WRITE TO US

*Comments about the problems and solutions for this AMC 8 should be addressed to:*

Prof. Norbert Kuenzi, AMC 8 Chair  
934 Nicolet Ave  
Oshkosh, WI 54901-1634

*Comments about administrative arrangements should be addressed to:*  
MAA American Mathematics Competitions / amcinfo@maa.org

American Mathematics Competitions  
PO Box 471  
Annapolis Junction, MD 20701

### AMC 10 & AMC 12

The AMC 10 and AMC 12 are 25-question, 75-minute, multiple choice contests. All schools participating in the AMC 8 receive a brochure and registration form for the 2015 AMC 10. Schools with high scoring students on the AMC 8 should consider administering the AMC 10. The best way to prepare for these contests is to study exams from previous years. Orders for all publications listed below should be addressed to:

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1. **Answer (A):** Harry's answer is  $H = 8 - (2 + 5) = 8 - 7 = 1$ . Terry's answer is  $T = 8 - 2 + 5 = 6 + 5 = 11$ . The difference  $H - T$  is  $1 - 11 = -10$ .
2. **Answer (E):** To use the largest number of coins, Paul would use only 5-cent coins. Because  $35 = 5 \cdot 7$ , the largest number of coins Paul can use is 7. To use the smallest number of coins, Paul would use a 25-cent coin and a 10-cent coin, for a total of 2 coins. The difference between the largest and the smallest number of coins he can use is  $7 - 2 = 5$ .

3. **Answer (B):** The number of pages in the book is  
 $3 \cdot 36 + 3 \cdot 44 + 10 = 3(36 + 44) + 10 = 3 \cdot 80 + 10 = 250$ .

4. **Answer (E):** The sum of two odd primes is an even number. Since the sum 85 is odd, one of the primes must be 2, which is the only even prime. The two primes are 2 and 83, so the product is  $2 \cdot 83 = 166$ .

5. **Answer (C):** For \$20, Margie can buy  $\frac{20}{4} = 5$  gallons of gas. She can drive 32 miles on each gallon, for a total of  $32 \cdot 5 = 160$  miles.

6. **Answer (D):** The areas of the six rectangles are 2, 8, 18, 32, 50, and 72. Adding yields 182.

OR

The sum of areas is  
 $2 \cdot 1 + 2 \cdot 4 + 2 \cdot 9 + 2 \cdot 16 + 2 \cdot 25 + 2 \cdot 36 = 2(1 + 4 + 9 + 16 + 25 + 36) = 2 \cdot 91 = 182$ .

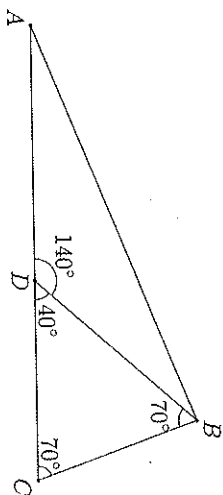
7. **Answer (B):** If there were an equal number of girls and boys, there would be 14 of each. By increasing the number of girls by 2 and decreasing the number of boys by 2, we see that there are 16 girls and 12 boys for a ratio of 16 : 12 or 4 : 3.

OR

If there were 4 fewer girls, then the class would be half boys and half girls. Remove 4 girls from the 28, and the other 24 students are evenly split into 12 boys and 12 girls. Add back the 4 girls to get 16 girls and 12 boys for a 16 : 12 ratio, which simplifies to 4 : 3.

8. **Answer (D):** The multiples of 11 between 102 and 192 are 110, 121, 132, 143, 154, 165, 176, and 187. Only 132 satisfies the condition, so  $A = 3$ .

9. **Answer (D):** Triangle  $BCD$  is isosceles, so  $\angle BCD = \angle CBD = 70^\circ$  and  $\angle BDC = 180^\circ - 2 \cdot 70^\circ = 40^\circ$ . Hence  $\angle ADB = 180^\circ - 40^\circ = 140^\circ$ .

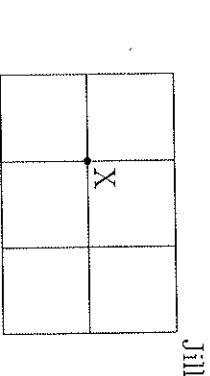


10. **Answer (A):** The seventh AMC 8 was given in 1991. So Samantha was born in  $1991 - 12 = 1979$ .

OR

Because the seventh AMC 8 was given when Samantha was 12, the first was 6 years earlier and she was 6 that first year in 1985. She was born 6 years earlier, in 1979.

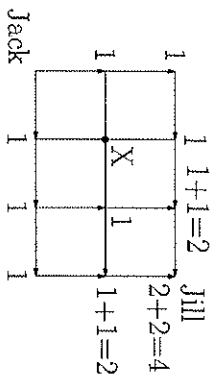
11. **Answer (A):** Let E represent traveling a block east and N represent traveling a block north. To avoid the dangerous intersection the first two blocks must be EE or NN. So there are 4 possible paths: EENN, EENEN, EENNE, and NNEEE.



OR

In the following diagram, the numbers indicate the number of ways to get to each of the intersections. In each case, the number of ways to get to any particular

intersection is the sum of the numbers of ways to get to any of the intersections leading directly to it. Thus, there are four paths to Jill's house, avoiding the dangerous intersection.



12. **Answer (B):** Call the celebrities  $L$ ,  $M$ , and  $N$ . There are six possible orderings:  $LMN$ ,  $LMN$ ,  $MLN$ ,  $MNL$ ,  $NLM$ , and  $NML$ . Only one of these identifies all three correctly. Therefore the probability is  $\frac{1}{6}$ .

13. **Answer (D):** If  $n^2 + m^2$  is even, then  $n^2$  and  $m^2$  are either both even or both odd, which means  $n$  and  $m$  are either both even or both odd. If  $n$  and  $m$  are both even, their sum is even. If  $n$  and  $m$  are both odd, their sum is even. Because  $n + m$  is never odd, (D) is the impossible choice.

14. **Answer (B):** The area of rectangle  $ABCD$  is  $5 \cdot 6 = 30$ . The area of triangle  $DCE$  is also 30, which is half of the product  $CD \cdot CE$ , so that product is 60. Because  $CD = AB = 5$ ,  $CE$  must equal  $\frac{60}{5} = 12$ , and by the Pythagorean Theorem,  $DE = \sqrt{CD^2 + CE^2} = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$ .

15. **Answer (C):** Angle  $AOE$  is  $\frac{4}{12}$  of  $360^\circ$  or  $120^\circ$  degrees, while  $\angle GOI$  is  $\frac{2}{12}$  of  $360^\circ$  or  $60^\circ$ . Both triangles are isosceles, so the equal base angles are  $\frac{60^\circ}{2}$  and  $\frac{120^\circ}{2}$  respectively. The sum of angles  $x$  and  $y$  then is  $(60^\circ + 120^\circ)/2 = 90^\circ$ .

16. **Answer (B):** Each team plays 4 non-conference games for a total of 32 games against non-conference opponents. Each team plays 7 conference games at home for a total of 56 games within the conference. The total number of games is  $32 + 56 = 88$ .

17. **Answer (B):** To walk 1 mile at 3 miles per hour requires  $\frac{1}{3}$  of an hour, or 20 minutes. This is the amount of time George allows himself to get to school. To walk  $\frac{1}{2}$  mile at 2 miles per hour requires  $\frac{1}{2} = \frac{1}{4}$  of an hour, or 15 minutes, so George has only 5 minutes to cover the remaining  $\frac{1}{2}$  mile. Because 5 minutes is  $\frac{5}{60} = \frac{1}{12}$  of an hour, George needs to run at a speed of  $\frac{1/12}{1/2} = 6$  miles per hour.

18. **Answer (D):** The 16 equally likely outcomes may be grouped as follows:

4 boys: BBBB  
 3 boys, 1 girl: BBBG, BBGB, BGBB, GBBB  
 2 boys, 2 girls: BBGG, BGBG, BGGB, GBBG, GBGB, GGBB  
 1 boy, 3 girls: BGGG, GBGG, GGBG, GGGB  
 4 girls: GGGG

There are 8 equally likely outcomes that produce 3 of one gender and 1 of the other gender, so that result is most likely.

19. **Answer (A):** The amount of white surface area is smallest when you place one white cube in the interior of the larger cube. Place each of the other 5 white cubes at the center of a face so that 1 white face and 8 red faces are visible on that face. The total surface area of the larger cube is  $6 \cdot 3^2 = 54$  square inches, so the fraction of the surface area that is white is  $\frac{5}{54}$ .

20. **Answer (B):** The areas of the quarter-circles are  $\frac{\pi}{4}$ ,  $\pi$  and  $\frac{9\pi}{4}$ . Their total area is  $\frac{7\pi}{2}$ . Using  $\frac{22}{7}$  as an approximation of  $\pi$ , this is  $\frac{7}{2} \cdot \frac{22}{7} = 11$ , leaving  $15 - 11 = 4$  for the desired area. (Using 3.14 for  $\pi$  yields 4.01.)

21. **Answer (A):** For  $\underline{74A}A\underline{2E}1$  to be a multiple of 3, the sum  $7 + 4 + A + 5 + 2 + B + 1 = 19 + A + B$  must be a multiple of 3. Therefore  $A + B$  is 1 less than a multiple of 3. The sum  $3 + 2 + 6 + A + B + 4 + C = 15 + A + B + C$  must be a multiple of 3, so  $A + B + C$  must be a multiple of 3. Since  $A + B$  is 1 less than a multiple of 3,  $C$  must be 1 more than a multiple of 3. Only choice (A) meets this requirement.

22. **Answer (E):** Test ten consecutive numbers with unit's digits 0 through 9. For 10 through 19 we find that adding the product of the digits and the sum of the digits yields the sequence 1, 3, 5, 7, 9, 11, 13, 15, 17, and 19. In this sequence only 19 meets the desired condition. It is easy to verify that 29, 39, ..., 99 also meet the desired condition.

OR

With  $a$  as the tens digit and  $b$  as the units digit, the number is  $10a + b$ . So  $a + b + ab = 10a + b$ ,  $a + ab = 10a$ ,  $ab = 9a$  and  $b = 9$ .

23. **Answer (A):** The sum of any two of the girls' uniform numbers must be no greater than 31. The only possible sums of two 2-digit primes that are no greater than 31 are

$$11 + 13 = 24, \quad 11 + 17 = 28, \quad 11 + 19 = 30, \quad \text{and} \quad 13 + 17 = 30.$$

Therefore the required dates are 24, 28, and 30. Caitlin's uniform number must appear in the two smallest sums,  $11 + 13 = 24$  and  $11 + 17 = 28$ . So Caitlin's uniform number is 11. The other two girls' uniform numbers are 13 for Ashley and 17 for Bethany.

24. **Answer (C):** Suppose the numbers of cans purchased by the 100 customers are listed in increasing order. The median is the average of the 50th and 51st numbers in the ordered list. To maximize the median, minimize the first 49 numbers by taking them all to be 1. If the 50th number is 4, then the sum of all 100 numbers would at least  $49 + 51 \cdot 4 = 253$ , which is too large. If instead the 50th number is 3 and the following numbers all equal 4, then the sum of the 100 numbers is  $49 + 3 + 50 \cdot 4 = 252$  and the median is  $(3 + 4) \div 2 = 3.5$ .

OR

To maximize the median, the largest 50 should be the same and as large as possible. The lower 49 should be as small as possible. The median of the list will be the average of the 50th and 51st numbers. If every customer has 1 can of soda, there are 152 left to distribute. Giving the upper 50 three more each gives the top 50 four cans each (200 total) and the lower 50 one each (50 total). There are 2 cans left. Giving the 50th person the extra 2 means the 50th has 3 cans, and the 51st has 4 cans for a median of  $(3 + 4) \div 2 = 3.5$ .

25. **Answer (B):** Each semicircle moves Robert 40 feet ahead, so he would have to ride  $5280 \div 40 = 132$  semicircles to cover 1 mile. Riding 132 semicircles is equal to the distance of 66 full circles. Each circle has a circumference of  $40\pi$ , so Robert rides  $66 \cdot 40\pi$  feet. Converting to miles, that is  $\frac{66 \cdot 40\pi}{5280} = \frac{\pi}{2}$  miles. Since he is riding at 5 miles per hour, it will take him  $\frac{\pi}{2} \div 5 = \frac{\pi}{10}$  hours.

OR

Each semi-circular path is  $\frac{\pi}{2}$  times as long as the straight path. Since the straight path would take  $\frac{1}{5}$  hour to ride, the curved path will take  $\frac{1}{5} \cdot \frac{\pi}{2} = \frac{\pi}{10}$  hours to ride.