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Ms. Hamilton's eighth-grade class wants to participate in the annual three-person-team basketball tournament.

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12. Niki usually leaves her cell phone on. If her cell phone is on but she is not actually using it, the battery will last for 24 hours. If she is using it constantly, the battery will last for only 3 hours. Since the last recharge, her phone has been on 9 hours, and during that time she has used it for 60 minutes. If she doesn't talk any more but leaves the phone on, how many more hours will the battery last?

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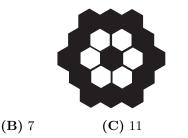
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15. Thirteen black and six white hexagonal tiles were used to create the figure below. If a new figure is created by attaching a border of white tiles with the same size and shape as the others, what will be the difference between the total number of white tiles and the total number of black tiles in the new figure?



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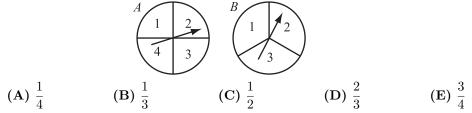
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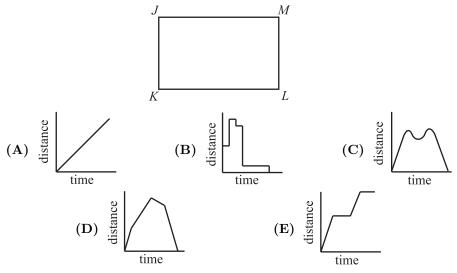
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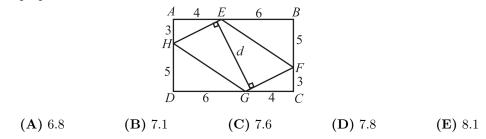
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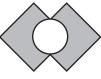
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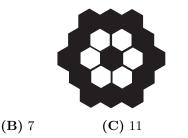
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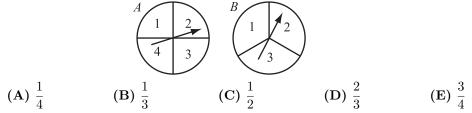
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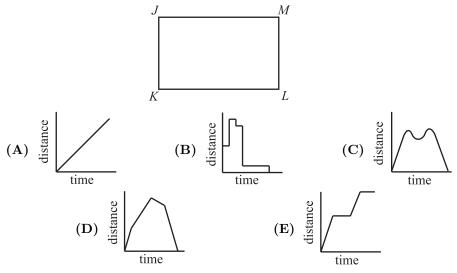
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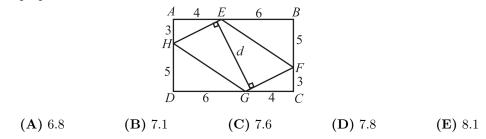
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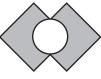
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## **SOLUTIONS**

Your School Manager will be sent at least one copy of the 2004 AMC 8 Solutions Pamphlet. It is meant to be loaned to students (but not duplicated).

## WRITE TO US

Comments about the problems and solutions for this AMC 8 should be addressed to: Ms. Bonnie Leitch, AMC 8 Chair / bleitch@earthlink.net 548 Hill Avenue, New Braunfels, TX 78130

Comments about administrative arrangements should be addressed to: MAA American Mathematics Competitions / amcinfo@unl.edu American Mathematics Competitions, University of Nebraska-Lincoln P.O. Box 880658, Lincoln, NE 68588-0658

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The AMC 10 and AMC 12 are 25-question, 75-minute contests with 5 choices of answers for each problem (A through E). Schools with high scoring students on the AMC 8 will receive an Invitation Brochure for the 2005 AMC 10. The best way to prepare for these upper level contests is to study exams from previous years. Orders for all publications listed below should be addressed to:

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ÁMC 12 (11<sup>th</sup> & 12<sup>th</sup> grade High School contests), 1990-2004, \$1.00 per copy per year.