Problem 1

What is the smallest sum of two 3-digit numbers that can be obtained by placing each of the six digits $4, 5, 6, 7, 8, 9$ in one of the six boxes in this addition problem?

$$\begin{array}{cccc} & \square & \square & \square \\ + & \square & \square & \square \\ \hline \end{array}$$

$\textbf{(A)}\ 947 \qquad \textbf{(B)}\ 1037 \qquad \textbf{(C)}\ 1047 \qquad \textbf{(D)}\ 1056 \qquad \textbf{(E)}\ 1245$

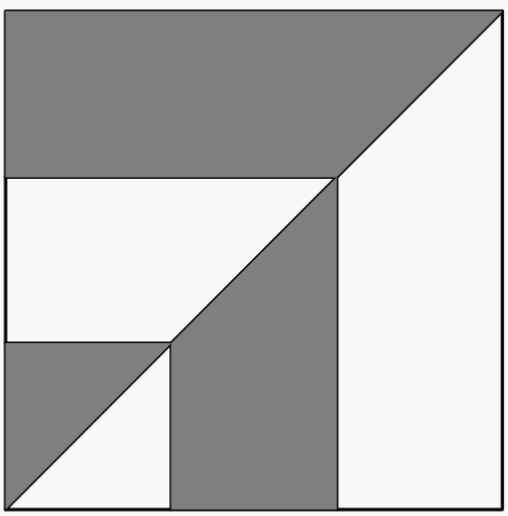
Problem 2

Which digit of $.12345$, when changed to 9, gives the largest number?

$\text{(A)}\ 1 \qquad \text{(B)}\ 2 \qquad \text{(C)}\ 3 \qquad \text{(D)}\ 4 \qquad \text{(E)}\ 5$

Problem 3

What fraction of the square is shaded?



$\text{(A)}\ \frac{1}{3} \qquad \text{(B)}\ \frac{2}{5} \qquad \text{(C)}\ \frac{5}{12} \qquad \text{(D)}\ \frac{3}{7} \qquad \text{(E)}\ \frac{1}{2}$

Problem 4

Which of the following could **not** be the unit's digit [one's digit] of the square of a whole number?

$\text{(A)}\ 1 \qquad \text{(B)}\ 4 \qquad \text{(C)}\ 5 \qquad \text{(D)}\ 6 \qquad \text{(E)}\ 8$

Problem 5

Which of the following is closest to the product $(.48017)(.48017)(.48017)$?

$\text{(A)}\ 0.011 \qquad \text{(B)}\ 0.110 \qquad \text{(C)}\ 1.10 \qquad \text{(D)}\ 11.0 \qquad \text{(E)}\ 110$

Problem 6

Which of these five numbers is the largest?

$\text{(A)}\ 13579+\frac{1}{2468} \qquad \text{(B)}\ 13579-\frac{1}{2468} \qquad \text{(C)}\ 13579\times \frac{1}{2468}$

$\text{(D)}\ 13579\div \frac{1}{2468} \qquad \text{(E)}\ 13579.2468$

Problem 7

When three different numbers from the set $\{-3, -2, -1, 4, 5\}$ are multiplied, the largest possible product is

$\text{(A)}\ 10 \qquad \text{(B)}\ 20 \qquad \text{(C)}\ 30 \qquad \text{(D)}\ 40 \qquad \text{(E)}\ 60$

Problem 8

A dress originally priced at 80 dollars was put on sale for 25% off. If 10% tax was added to the sale price, then the total selling price (in dollars) of the dress was

$\text{(A)}\ \text{45 dollars} \qquad \text{(B)}\ \text{52 dollars} \qquad \text{(C)}\ \text{54 dollars} \qquad \text{(D)}\ \text{66 dollars} \qquad \text{(E)}\ \text{68 dollars}$

Problem 9

The grading scale shown is used at Jones Junior High. The fifteen scores in Mr. Freeman's class were:

$89, \quad 72, \quad 54, \quad 97, \quad 77, \quad 92, \quad 85, \quad 74, \quad 75,$
$63, \quad 84, \quad 78, \quad 71, \quad 80, \quad 90.$

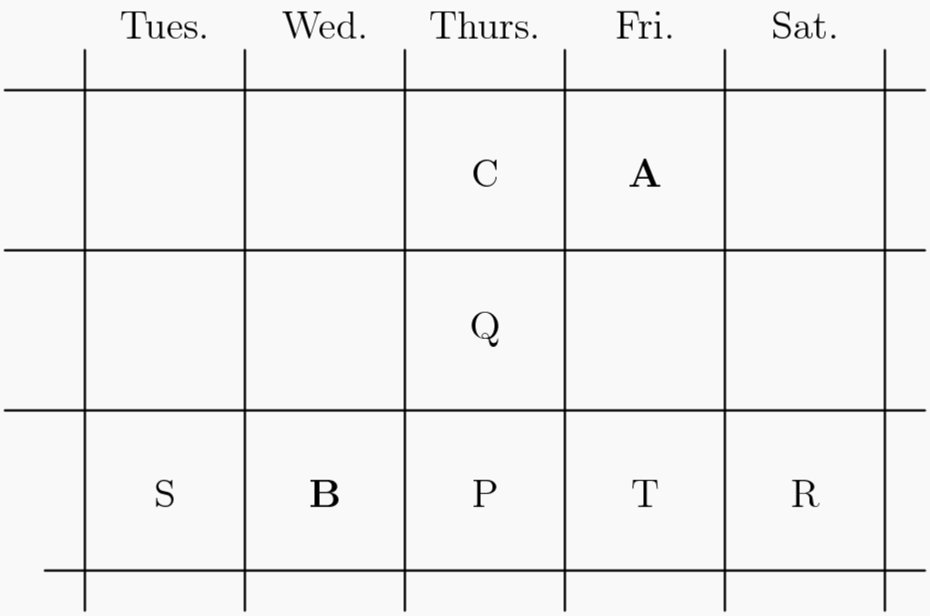
In Mr. Freeman's class, what percent of the students received a grade of C?

A:	93 - 100
B:	85 - 92
C:	75 - 84
D:	70 - 74
F:	0 - 69

$\text{(A)}\ 20\% \qquad \text{(B)}\ 25\% \qquad \text{(C)}\ 30\% \qquad \text{(D)}\ 33\frac{1}{3}\% \qquad \text{(E)}\ 40\%$

Problem 10

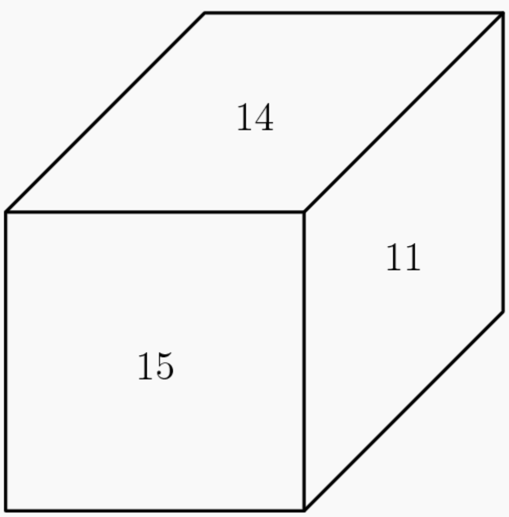
On this monthly calendar, the date behind one of the letters is added to the date behind C. If this sum equals the sum of the dates behind A and B, then the letter is



(A) P (B) Q (C) R (D) S (E) T

Problem 11

The numbers on the faces of this cube are consecutive whole numbers. The sums of the two numbers on each of the three pairs of opposite faces are equal. The sum of the six numbers on this cube is



(A) 75 (B) 76 (C) 78 (D) 80 (E) 81

Problem 12

There are twenty-four 4-digit numbers that use each of the four digits 2, 4, 5, and 7 exactly once. Listed in numerical order from smallest to largest, the number in the 17th position in the list is

(A) 4527 (B) 5724 (C) 5742 (D) 7245 (E) 7524

Problem 13

One proposal for new postage rates for a letter was 30 cents for the first ounce and 22 cents for each additional ounce (or fraction of an ounce). The postage for a letter weighing 4.5 ounces was

$\text{(A)}\ 96\text{ cents} \qquad \text{(B)}\ 1.07\text{ dollars} \qquad \text{(C)}\ 1.18\text{ dollars} \qquad \text{(D)}\ 1.20\text{ dollars} \qquad \text{(E)}\ 1.40\text{ dollars}$

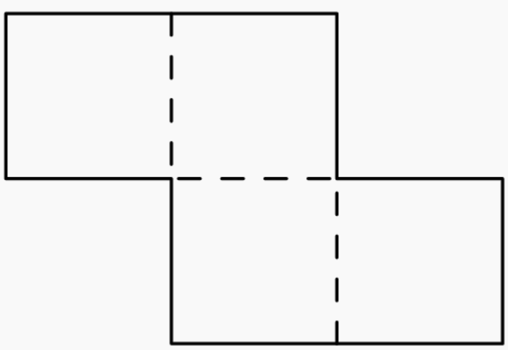
Problem 14

A bag contains only blue balls and green balls. There are 6 blue balls. If the probability of drawing a blue ball at random from this bag is $\frac{1}{4}$, then the number of green balls in the bag is

$\text{(A)}\ 12 \qquad \text{(B)}\ 18 \qquad \text{(C)}\ 24 \qquad \text{(D)}\ 30 \qquad \text{(E)}\ 36$

Problem 15

The area of this figure is 100 cm^2. Its perimeter is



[figure consists of four identical squares]

$\text{(A)}\ 20\text{ cm} \qquad \text{(B)}\ 25\text{ cm} \qquad \text{(C)}\ 30\text{ cm} \qquad \text{(D)}\ 40\text{ cm} \qquad \text{(E)}\ 50\text{ cm}$

Problem 16

$1990-1980+1970-1960+\cdots -20+10 =$

$\text{(A)}\ -990 \qquad \text{(B)}\ -10 \qquad \text{(C)}\ 990 \qquad \text{(D)}\ 1000 \qquad \text{(E)}\ 1990$

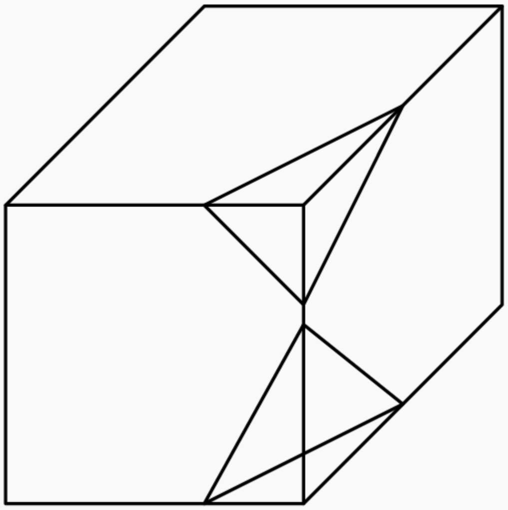
Problem 17

A straight concrete sidewalk is to be 3 feet wide, 60 feet long, and 3 **inches** thick. How many cubic yards of concrete must a contractor order for the sidewalk if concrete must be ordered in a whole number of cubic yards?

$\text{(A)}\ 2 \qquad \text{(B)}\ 5 \qquad \text{(C)}\ 12 \qquad \text{(D)}\ 20 \qquad \text{(E)}\ \text{more than 20}$

Problem 18

Each corner of a rectangular prism is cut off. Two (of the eight) cuts are shown. How many edges does the new figure have?



$\textbf{(A)}\ 24 \qquad \textbf{(B)}\ 30 \qquad \textbf{(C)}\ 36 \qquad \textbf{(D)}\ 42 \qquad \textbf{(E)}\ 48$

Assume that the planes cutting the prism do not intersect anywhere in or on the prism.

Problem 19

There are 120 seats in a row. What is the fewest number of seats that must be occupied so the next person to be seated must sit next to someone?

$\text{(A)}\ 30 \qquad \text{(B)}\ 40 \qquad \text{(C)}\ 41 \qquad \text{(D)}\ 60 \qquad \text{(E)}\ 119$

Problem 20

The annual incomes of 1,000 families range from 8,200 dollars to 98,000 dollars. In error, the largest income was entered on the computer as 980,000 dollars. The difference between the mean of the incorrect data and the mean of the actual data is

$\text{(A)}\ \text{882 dollars} \qquad \text{(B)}\ \text{980 dollars} \qquad \text{(C)}\ \text{1078 dollars} \qquad \text{(D)}\ \text{482,000 dollars} \qquad \text{(E)}\ \text{882,000 dollars}$

Problem 21

A list of 8 numbers is formed by beginning with two given numbers. Each new number in the list is the product of the two previous numbers. Find the first number if the last three are shown:

$\underline{\quad ? \quad},\ \underline{\qquad},\ \underline{\qquad},\ \underline{\qquad},\ \underline{\qquad},\ \underline{\ 16\ },\ \underline{\ 64\ },\ \underline{\ 1024\ }$

$\text{(A)}\ \frac{1}{64} \qquad \text{(B)}\ \frac{1}{4} \qquad \text{(C)}\ 1 \qquad \text{(D)}\ 2 \qquad \text{(E)}\ 4$

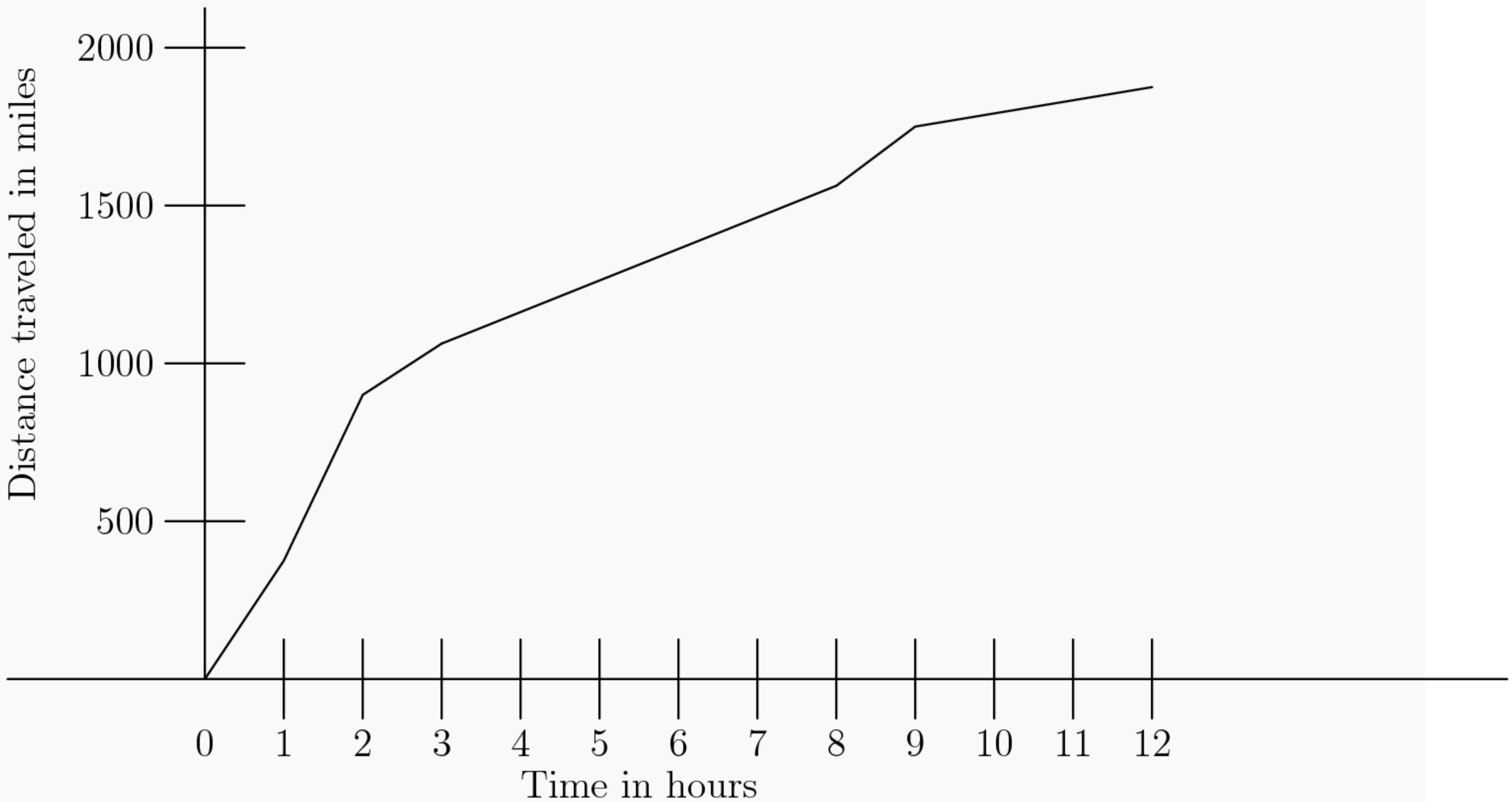
Problem 22

Several students are seated at a large circular table. They pass around a bag containing 100 pieces of candy. Each person receives the bag, takes one piece of candy and then passes the bag to the next person. If Chris takes the first and last piece of candy, then the number of students at the table could be

(A) 10 (B) 11 (C) 19 (D) 20 (E) 25

Problem 23

The graph relates the distance traveled [in miles] to the time elapsed [in hours] on a trip taken by an experimental airplane. During which hour was the average speed of this airplane the largest?



(A) first (0-1) (B) second (1-2) (C) third (2-3) (D) ninth (8-9) (E) last (11-12)

Problem 24

Three Δ's and a $\diamondsuit$ will balance nine $\bullet$'s. One Δ will balance a $\diamondsuit$ and a $\bullet$

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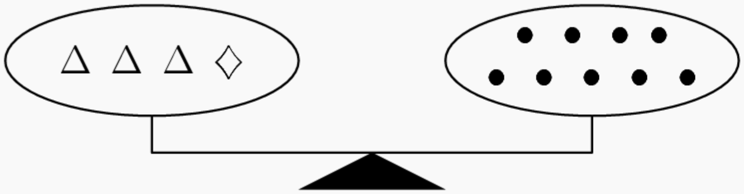
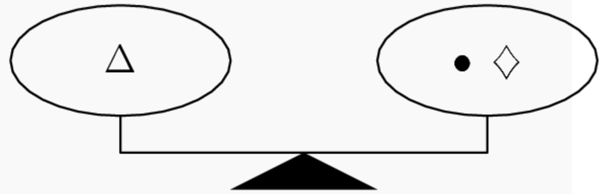
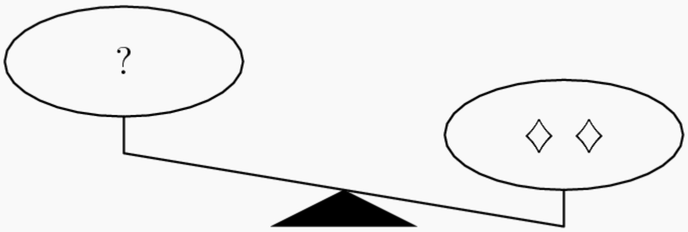




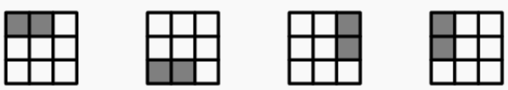
How many $\bullet$'s will balance the two $\diamondsuit$'s in this balance?



(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Problem 25

How many different patterns can be made by shading exactly two of the nine squares? Patterns that can be matched by flips and/or turns are not considered different. For example, the patterns shown below are **not** considered different.



(A) 3 (B) 6 (C) 8 (D) 12 (E) 18

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1. C In the smallest such sum, the two smallest digits are in the hundred's places, the next two digits in the ten's places and the two largest digits are in the one's places. One example is $468 + 579 = 1047$.

   Query: In how many ways can this sum of 1047 be achieved?

2. A An increase in the tenth's place gives a larger value than an increase in any of the other decimal places. Since 1 is in the tenth's place of .12345, (A) is correct.

3. E Each shaded piece above or below the diagonal is matched by an identical unshaded piece meaning $\frac{1}{2}$ of the total area is shaded.

4. E From the way we multiply whole numbers, the unit's digit of the square of a whole number is determined by the square of the unit's digit of that whole number. The possible squares of unit's digits are: $0^2 = \boxed{0}$, $1^2 = \boxed{1}$, $2^2 = \boxed{4}$, $3^2 = \boxed{9}$, $4^2 = 1\boxed{6}$, $5^2 = 2\boxed{5}$, $6^2 = 3\boxed{6}$, $7^2 = 4\boxed{9}$, $8^2 = 6\boxed{4}$, $9^2 = 8\boxed{1}$.
   Note that 2,3,7, or 8 will never occur as the unit's digit of a square.

5. B The desired product is about $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8} = 0.125$.

   OR

   The desired product is about $(.5)(.5)(.5) = 0.125$.

6. D All of the choices,except (C) and (D), are near 13,579. In (D), the result is the product (13579)(2468) while in (C) the result is much less than 13,579.

7. C For the product of three numbers to be positive, either all three of the numbers must be positive or one must be positive and two must be negative. Since there are only two positive numbers, only the latter case is possible. Thus the largest such product is $(-3)(-2)(5) = 30$.

8. D The sale price was $\frac{3}{4}(\$80) = \$60$. Thus the tax was \$6 and the total selling price was \$66.

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9. D Five of the fifteen scores [77, 75, 84, 78, 80] are in the "C range", so the desired percent is $\frac{5}{15} = \frac{1}{3} = 33\frac{1}{3}\%$

10. A Since the date behind C is one less than that behind A, the date behind the desired letter must be one more than that behind B. This date is behind P.

11. E Since 11, 12, 13, 14, and 15 are on five of the faces, the number on the remaining face must be 10 or 16. In the first case, 10 must be on the face opposite the face with 15 which is impossible since $15 + 10 = 25$ would force 14 and 11 to be on opposite faces. Thus 16 is opposite 11, 12 is opposite 15, and 14 is opposite 13. The sum on each pair of opposite faces is 27, and the desired sum, is $3 \times 27 = 81$.

12. B One-fourth of the 24 numbers in the list begin with each of the four given digits. Those in positions 1 - 6 begin with 2; those in positions 7 - 12 begin with 4; those in positions 13 - 18 begin with 5. Thus the desired number is the fifth one beginning with 5: 5247, 5274, 5427, 5472, $\boxed{5724}$, 5742.

13. C The first ounce costs \$ .30. The additional 3.5 ounces would cost 4(\$ .22) = \$ .88. Thus the postage was \$ .30 + \$ .88 = \$1.18.

14. B Since one quarter of the balls are blue and there are 6 blue balls, there must be 24 balls in the bag. Thus there are $24 - 6 = 18$ green balls.

OR

Since one quarter of the balls are blue, three quarters of them must be green. Thus there are three times as many green balls as blue balls, so there are $3 \times 6 = 18$ green balls.

15. E The total area of the four squares is 100 cm^2, so the area of each square is 25 cm^2. Thus the side of each square is 5 cm and the perimeter of the figure is 10(5 cm) = 50 cm.

16. D By grouping as shown below, there are $\frac{199+1}{2} = 100$ groups of 10 for a sum of 1000:

$$[1990 - 1980] + [1970 - 1960] + \ldots + [30 - 20] + 10$$

17. A The number of cubic feet is $3 \times 60 \times \frac{1}{4} = 45$. Since there are 27 cubic feet in 1 cubic yard, there are $\frac{45}{27} = 1\frac{2}{3}$ cubic yards of concrete required. Thus 2 cubic yards must be ordered.

OR

Since 3 feet = 1 yard, 60 feet = 20 yards. Also 3 inches = $\frac{3}{36} = \frac{1}{12}$ yard. Thus $1 \times 20 \times \frac{1}{12} = 1\frac{2}{3}$ cubic yards of concrete are needed, so 2 cubic yards must be ordered.

18. C The original prism had 12 edges. Each "cut-off" corner yields 3 additional edges, so the new figure has a total of $12 + 8 \times 3 = 36$ edges.

19. B In order for the fewest number of seats to be occupied, there must be someone in every third seat, beginning with #2 and ending with #119. There are a total of $\frac{120}{3} = 40$ occupied seats.

OR

Consider some simpler cases and make a table:

Number of seats in the row:	3	6	9	12
Number of occupied seats in the row:	1	2	3	4

In each case, the middle seat in every group of three seats must be occupied, so the desired number of occupied seats in a row of 120 seats is $\frac{120}{3} = 40$.

20. A The difference between the incorrect sum and the actual sum is \$980,000 – \$98,000 = \$882,000. Since this difference is equally shared by all 1000 families, the difference between the means is $\frac{\$882000}{1000} = \$882$.

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21. B Working backward from 1024, divide each number [e.g. 1024] by the preceding number [e.g. 64] to get the previous number [e.g. 16] in the list. Thus $64 \div 16 = 4$, $4 \div 4 = 1$, and so on:

$$\underline{\tfrac{1}{4}}\,,\ \underline{4}\,,\ \underline{1}\,,\ \underline{4}\,,\ \underline{4}\,,\ \underline{16}\,,\ \underline{64}\,,\ \underline{1024}$$

22. B Since Chris takes the last piece of candy, each person receives the same number of the other 99 pieces of candy. Thus the number of students at the table must be a factor of 99. Only (B) fulfills this condition.

23. B The rate will the largest when the graph is the "steepest". During the second hour the distance traveled is about 500 miles, so the average speed during that hour is about 500 mph. For the other hours, the speeds are less than 350 mph.

24. C The second balance shows each Δ balances ◊ •. Replace each Δ on the first balance with ◊ •. Then after removing three •'s from each side, the balance has ◊◊◊◊ on the left and •••••• on the right. Thus ◊◊ will be balanced by •••.

25. C There are 8. Be systematic. You can begin with the five cases that have one corner square. Then consider the other three cases that do not have a corner square.