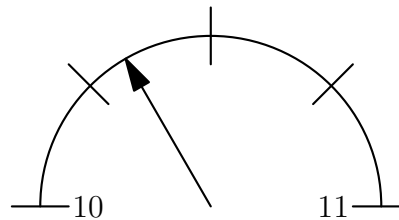# Art of Problem Solving
1988 AMC 8

AMC 8 1988

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1 The diagram shows part of a scale of a measuring device. The arrow indicates an approximate reading of

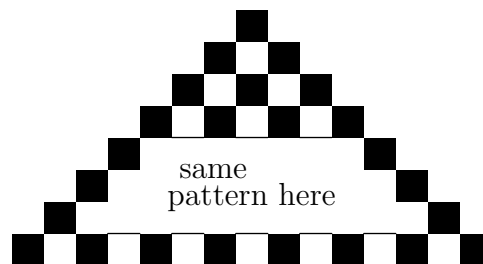(A) 10.05   (B) 10.15   (C) 10.25   (D) 10.3   (E) 10.6

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2 The product $8 \times .25 \times 2 \times .125 =$

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 1   (E) 2
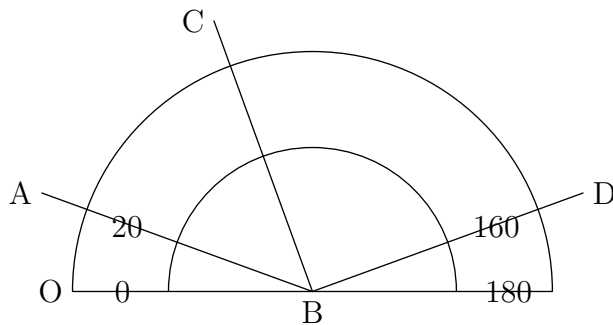
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3 $\frac{1}{10} + \frac{2}{20} + \frac{3}{30} =$

(A) .1   (B) .123   (C) .2   (D) .3   (E) .6

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4 The figure consists of alternating light and dark squares. The number of dark squares exceeds the number of light squares by (A) 7   (B) 8   (C) 9   (D) 10   (E) 11

same
pattern here

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5 If $\angle$CBD is a right angle, then this protractor indicates that the measure of $\angle$ABC is approximately
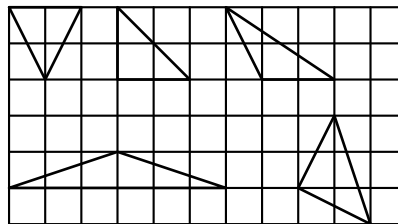

(A) 20°   (B) 40°   (C) 50°   (D) 70°   (E) 120°

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6 $\frac{(.2)^3}{(.02)^2} =$ (A) .2   (B) 2   (C) 10   (D) 15   (E) 20

---

7 $2.46 \times 8.163 \times (5.17 + 4.829)$ is closest to:

(A) 100   (B) 200   (C) 300   (D) 400   (E) 500

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8   Betty used a calculator to find the product 0.075×2.56. She forgot to enter the decimal points. The calculator showed 19200. If Betty had entered the decimal points correctly, the answer would have been

(A) .0192   (B) .192   (C) 1.92   (D) 19.2   (E) 192

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9   An isosceles triangle is a triangle with two sides of equal length. How many of the five triangles on the square grid below are isosceles?

(A) 1   (B) 2   (C) 3   (D) 4   (E) 5

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10   Chris' birthday is on a Thursday this year. What day of the week will it be 60 days after her birthday?

(A) Monday   (B) Wednesday   (C) Thursday   (D) Friday   (E) Saturday

11 $\sqrt{164}$ is

(A) 42   (B) less than 10   (C) between 10 and 11   (D) between 11 and 12   (E) betw

12   Suppose the estimated 20 billion dollar cost to send a person to the planet Mars is shared equally by the 250 million people in the U.S. Then each person's share is

(A) 40 dollars   (B) 50 dollars   (C) 80 dollars   (D) 100 dollars   (E) 125 dollars

13   If rose bushes are spaced about 1 foot apart, approximately how many bushes are needed to surround a circular patio whose radius is 12 feet?

(A) 12   (B) 38   (C) 48   (D) 75   (E) 450

14 $\Diamond$ and Δ are whole numbers and $\Diamond \times \Delta = 36$. The largest possible value of $\Diamond + \Delta$ is

(A) 12   (B) 13   (C) 15   (D) 20   (E) 37

15   The reciprocal of $\left(\frac{1}{2} + \frac{1}{3}\right)$ is

(A) $\frac{1}{6}$ (B) $\frac{2}{5}$ (C) $\frac{6}{5}$ (D) $\frac{5}{2}$ (E) 5
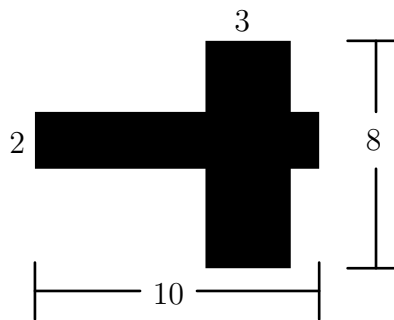
16
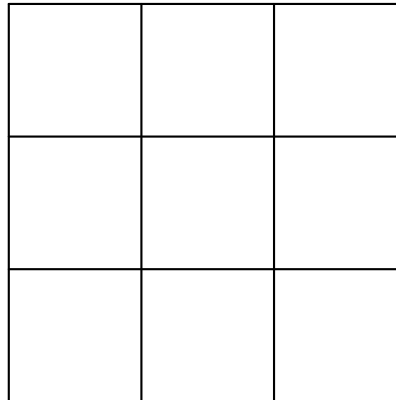
Placing no more than one x in each small square, what is the greatest number of x's that can be put on the grid shown without getting three x's in a row vertically, horizontally, or diagonally?

(A) 2   (B) 3   (C) 4   (D) 5   (E) 6

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17   The shaded region formed by the two intersecting perpendicular rectangles, in square units, is

(A) 23   (B) 38   (C) 44   (D) 46   (E) unable to be determined from the information

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18   The average weight of 6 boys is 150 pounds and the average weight of 4 girls is 120 pounds. The average weight of the 10 children is

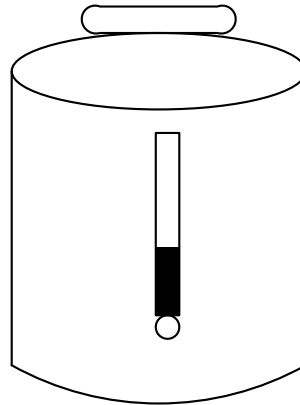(A) 135 pounds   (B) 137 pounds   (C) 138 pounds   (D) 140 pounds   (E) 141 pounds

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19   What is the $100th$ number in the arithmetic sequence: $1, 5, 9, 13, 17, 21, 25, ...$

(A) 397   (B) 399   (C) 401   (D) 403   (E) 405

---

20   The glass gauge on a cylindrical coffee maker shows that there are 45 cups left when the coffee maker is 36% full. How many cups of coffee does it hold when it is full?

(A) 80   (B) 100   (C) 125   (D) 130   (E) 262

21   A fifth number, n , is added to the set $\{3, 6, 9, 10\}$ to make the mean of the set of five numbers equal to its median. The number of possible values of n is

(A) 1   (B) 2   (C) 3   (D) 4   (E) more than 4

22   Tom's Hat Shoppe increased all original prices by 25%. Now the shoppe is having a sale where all prices are 20% off these increased prices. Which statement best describes the sale price of an item?

(A) The sale price is 5% higher than the original price. (B) The sale price is higher than the o
(C) The sale price is higher than the original price, but by more than 5%. (D) The sale price i
(E) The sale price is the same as the original price.

23   Maria buys computer disks at a price of 4 for 5 dollars and sells them at a price of 3 for 5 dollars. How many computer disks must she sell in order to make a profit of 100 dolars?

(A) 100   (B) 120   (C) 200   (D) 240   (E) 1200

24
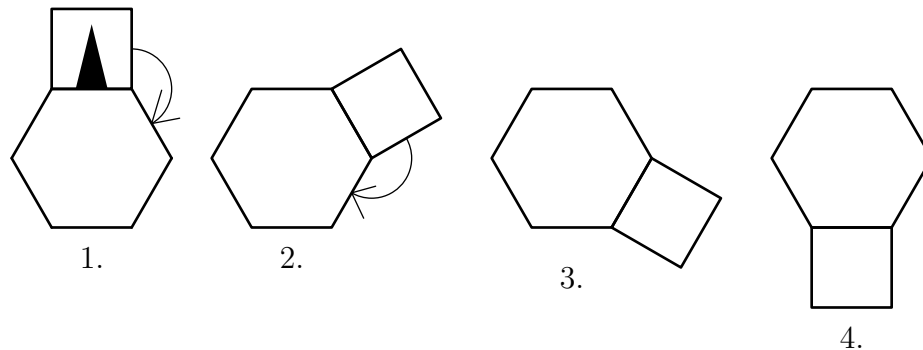
The square in the first diagram "rolls" clockwise around the fixed regular hexagon until it reaches the bottom. In which position will the solid triangle be in diagram 4?

(A)   (B)   (C)   (D)   (E)

25   A **palindrome** is a whole number that reads the same forwards and backwards. If one neglects the colon, certain times displayed on a digital watch are palindromes. Three examples are: $1:01$, $12:21$.

How many times during a 12-hour period will be palindromes?

(A) 57   (B) 60   (C) 63   (D) 90   (E) 93

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1.   D   The scale is divided into fourths and the needle is just past the $\frac{1}{4}$ mark, so the reading must be between 10.25 and 10.5.

2.   C   The product can be rewritten as $8 \times \frac{1}{4} \times 2 \times \frac{1}{8} =$

                   $8 \times \frac{1}{8} \times \frac{1}{4} \times 2 = \frac{1}{2}$

3.   D   Since $\frac{2}{20} = \frac{3}{30} = \frac{1}{10}$, the desired sum is $.1 + .1 + .1 = .3$.

4.   B   In each row, including the first, there is one more dark square than light square. Since there are 8 rows, there must be 8 more dark squares than light squares.

5.   C   Since $\angle CBD$ is a right angle, side BC must cross the protractor at 70°. Thus the measure of $\angle ABC$ is 70° - 20° = 50°.

                               OR

         $\angle ABC = \angle ABD - \angle CBD = (\angle OBD - \angle OBA) - \angle CBD =$
         $(160° - 20°) - 90° = 50°$.

6.   E $\dfrac{(.2)^3}{(.02)^2} = \dfrac{.008}{.0004} = \dfrac{80}{4} = 20$

                          OR

         $\dfrac{.2}{.02} \times \dfrac{.2}{.02} \times .2 = 10 \times 10 \times .2 = 20$.

7.   B   The product is approximately $(2.5)(8)(10) = (20)(10) = 200$.

8.   B   Although one could solve this problem by counting decimal places in the product, it is more more insightful to realize that the answer is approximately $.1(2) = .2$, so (B) is correct.

9.   D   All but the triangle in the upper right are isosceles.

10.   A   60 days is 8 weeks and 4 days, so the desired day is four days after a Thursday , that is, a Monday.

OR

60 days is 9 weeks less 3 days, so the desired day is three days before a Thursday , also a Monday.

11.   E $10^2 = 100$, $11^2 = 121$, $12^2 = 144$, $13^2 = 169$ so (E) is correct.

12.   C   The cost per person $= \dfrac{\text{total cost}}{\text{number of people}}$.

Thus $\dfrac{\$20 \text{ billion}}{250 \text{ million}} = \dfrac{2 \times 10^{10}}{2.5 \times 10^8} = .8 \times 10^2 = \$80$.

OR

Since $\dfrac{1000000000}{250000000} = 4$, the cost per person is ($4)(20) or $80.

13.   D   The circumference of the circular patio is
$2\pi(12) \approx (2)(3.14)(12) \approx 75$ feet, thus it
would take about 75 bushes to surround the patio.

14.   E   The factor pairs for 36 are 1x36, 2x18, 3x12, 4x9, and 6x6. The largest sum of such a pair is 37.
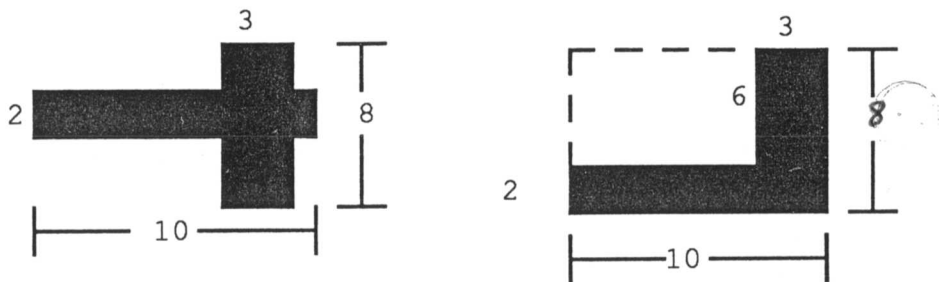
15.   C $\dfrac{1}{2} + \dfrac{1}{3} = \dfrac{5}{6}$ and its reciprocal is $\dfrac{6}{5}$.

16.   E   The arrangement pictured shows 6 X's
is possible. If there were 7 X's on a
3x3 board, then one row must
contain 3 X's.
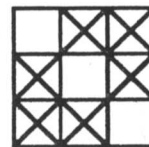
17.   B

The total shaded area is the sum of the areas of the "horizontal" rectangle and the "vertical" rectangle minus the area of the "overlapping" rectangle that is part of both of the other rectangles. Thus the desired area is $2(10) + 3(8) - 2(3) = 38$.

OR

"Slide" the rectangles as shown in the figure on the right so they form an L-shaped figure. We see that the shaded area is $(10)(2) + (6)(3) = 38$ or $(7)(2) + (8)(3) = 38$ or $(10)(8) - (7)(6) = 38$.

18.   C   The total weight of the ten children is $6(150) + 4(120) = 1380$, so the average weight is $\dfrac{1380}{10}$ or 138 pounds.

OR

The average weight of 4 boys and 4 girls is 135 pounds. The other two boys would raise this average by $\dfrac{30}{10}$ or 3 pounds.

19.   A   Adding 3 to each term in the original arithmetic sequence yields the sequence 4,8,12,16,20, ... in which the one-hundredth term is 400. Subtracting 3 from each term shows that 397 is the one-hundredth term of the original sequence.
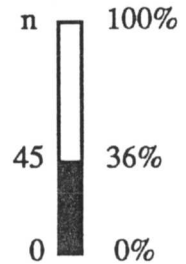
OR

We may obtain the 100th term in the sequence by adding 4 to the first term 1 a total of 99 times. Thus the 100th term is $1 + 99(4) = 397$.

20.   C   If n is the number of cups of coffee the coffeemaker will hold when it is full, then we can label the gauge as shown and write the two equivalent ratios:

$$\frac{45}{n} = \frac{36}{100} \text{ or } n = 125 \text{ cups.}$$

```
n  ┌─┐  100%
   │ │
   │ │
45 │ │  36%
   ├─┤
   │█│
 0 │█│  0%
   └─┘
```

OR

If 36% is 45 cups, then 4% is $\frac{1}{9}$(45) or 5 cups so that 100% is 25(5) or 125 cups.

21.   C   There are three possible values for the new number, n. It is either less than 6, between 6 and 9, or greater than 9. If the five elements are listed in increasing order, these three possibilities in the table below are: one of the first two, the middle one, and one of the last two. Since the median is the middle number in a set of five elements, there are three values for the median: 6, n, or 9.

					Median
n	3	6	9	10	6
3	n	6	9	10	6
3	6	n	9	10	n
3	6	9	n	10	9
3	6	9	10	n	9

Query: Can you find the values of n and which of the two alternatives occurs in the first and third cases ?

22.   E   An item whose original cost was $100, for example, will cost $25 more or $125. The sale price of a $125 item will be 80% of its current price or .8($125) = $100 - - the original cost. The same kind of comparison can be made for any original cost.

23. D   Since the computer disks are bought in groups of 4 and sold in groups of 3, it is easier to consider them in groups of 12 or dozens. Each dozen costs $15 and sells for $20 giving a profit of $5. Thus to get a profit of $100, she must sell 20 dozen or 240 computer disks.

24. A

1.   2.   3.

4.
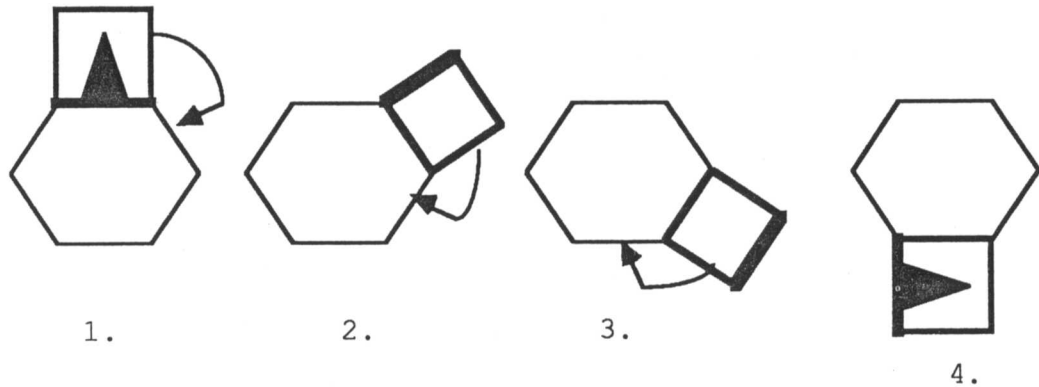
Keep track of the "bottom" side of the square in the first figure. In the fourth figure, it will appear on the left, so the solid triangle will be in the position shown in (A).

OR
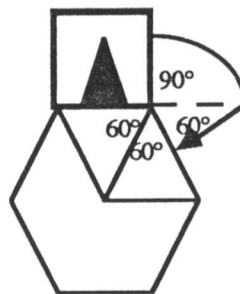
Each time the square "rolls" to the next edge of the hexagon, it turns through an angle of 150°. In going from the top to the bottom of the hexagon, the square makes three such turns for a total of 3(150) = 450°. This 450° represents one complete revolution and 1/4 of a second revolution.

25. A   In each hour from 1:00 through 9:59, there are six such times. From 3:00 to 3:59, for example, these times are 3:03, 3:13, 3:23, 3:33, 3:43, 3:53. From 10:00 to 12:59, there is one such time in each hour: 10:01, 11:11, and 12:21. Thus there are a total of 9(6) + 3 = 57 such times.