AMC 8 1987

1 $.4 + .02 + .006 =$

(A) .012 (B) .066 (C) .12 (D) .24 (E).426

2 $\frac{2}{25} =$

(A) .008 (B) .08 (C) .8 (D)1.25 (E) 12.5

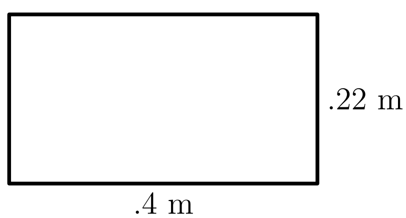
3 $2(81 + 83 + 85 + 87 + 89 + 91 + 93 + 95 + 97 + 99) =$

(A) 1600 (B) 1650 (C) 1700 (D) 1750 (E) 1800

4 Martians measure angles in clerts. There are 500 clerts in a full circle. How many clerts are there in a right angle?

(A) 90 (B) 100 (C) 125 (D) 180 (E) 250

5 The area of the rectangular region is

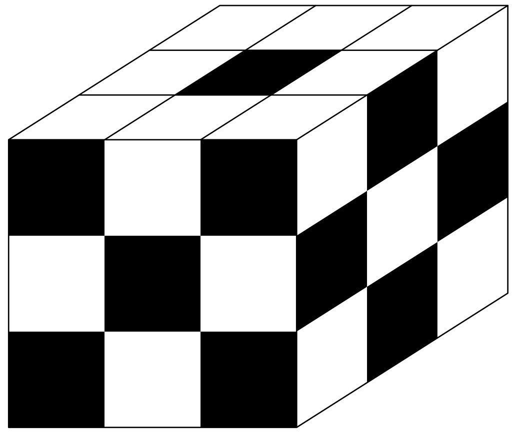


(A) .088 m^2 (B) .62 m^2 (C) .88 m^2 (D) 1.24 m^2 (E) 4.22 m^2

6 The smallest product one could obtain by multiplying two numbers in the set $\{-7, -5, -1, 1, 3\}$ is

(A) -35 (B) -21 (C) -15 (D) -1 (E) 3

7 The large cube shown is made up of 27 identical sized smaller cubes. For each face of the large cube, the opposite face is shaded the same way. The total number of smaller cubes that must have at least one face shaded is



(A) 10 (B) 16 (C) 20 (D) 22 (E) 24

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8 If A and B are nonzero digits, then the number of digits (not necessarily different) in the sum of the three whole numbers is

$$\begin{array}{cccc} 9 & 8 & 7 & 6 \\ & A & 3 & 2 \\ & & B & 1 \\ \hline \end{array}$$

(A) 4 (B) 5 (C) 6 (D) 9 (E) depends on the values of A and B

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9 When finding the sum $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}$, the least common denominator used is

(A) 120 (B) 210 (C) 420 (D) 840 (E) 5040

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10 $4(299)+3(299)+2(299)+298=$

(A) 2889 (B) 2989 (C) 2991 (D) 2999 (E) 3009

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11 The sum $2\frac{1}{7}+3\frac{1}{2}+5\frac{1}{19}$ is between

(A) 10 and $10\frac{1}{2}$ (B) $10\frac{1}{2}$ and 11 (C) 11 and $11\frac{1}{2}$ (D) $11\frac{1}{2}$ and 12 (E) 12 and $12\frac{1}{2}$

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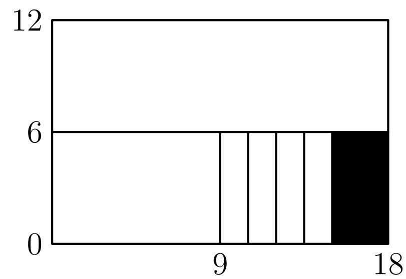
12 What fraction of the large 12 by 18 rectangular region is shaded?



(A) $\frac{1}{108}$ (B) $\frac{1}{18}$ (C) $\frac{1}{12}$ (D) $\frac{2}{9}$ (E) $\frac{1}{3}$

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13 Which of the following fractions has the largest value?

(A) $\frac{3}{7}$ (B) $\frac{4}{9}$ (C) $\frac{17}{35}$ (D) $\frac{100}{201}$ (E) $\frac{151}{301}$

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14 A computer can do $10,000$ additions per second. How many additions can it do in one hour?

(A) 6 million (B) 36 million (C) 60 million (D) 216 million (E) 360 million

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15 The sale ad read: "Buy three tires at the regular price and get the fourth tire for \$3." Sam paid \$240 for a set of four tires at the sale. What was the regular price of one tire?

(A) 59.25 dollars (B) 60 dollars (C) 70 dollars (D) 79 dollars (E) 80 dollars

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16 Joyce made 12 of her first 30 shots in the first three games of this basketball game, so her seasonal shooting average was 40%. In her next game, she took 10 shots and raised her seasonal shooting average to 50%. How many of these 10 shots did she make?

(A) 2 (B) 3 (C) 5 (D) 6 (E) 8

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17 Abby, Bret, Carl, and Dana are seated in a row of four seats numbered #1 to #4. Joe looks at them and says:

"Bret is next to Carl."
"Abby is between Bret and Carl."

However each one of Joe's statements is false. Bret is actually sitting in seat #3. Who is sitting in seat #2?

(A) Abby (B) Bret (C) Carl (D) Dana (E) There is not enough information to

18 Half the people in a room left. One third of those remaining started to dance. There were then 12 people who were not dancing. The original number of people in the room was

(A) 24 (B) 30 (C) 36 (D) 42 (E) 72

19 A calculator has a squaring key $\boxed{x^2}$ which replaces the current number displayed with its square. For example, if the display is $\boxed{000003}$ and the $\boxed{x^2}$ key is depressed, then the display becomes $\boxed{000009}$. If the display reads $\boxed{000002}$, how many times must you depress the $\boxed{x^2}$ key to produce a displayed number greater than 500?

(A) 4 (B) 5 (C) 8 (D) 9 (E) 250

20 "If a whole number n is not prime, then the whole number $n-2$ is not prime." A value of n which shows this statement to be false is

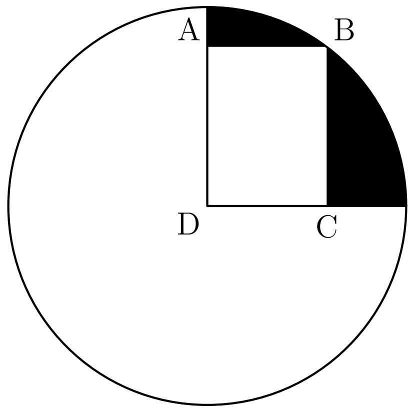
(A) 9 (B) 12 (C) 13 (D) 16 (E) 23

21 Suppose n^* means $\frac{1}{n}$, the reciprocal of n. For example, $5^* = \frac{1}{5}$. How many of the following statements are true?

i) $3^* + 6^* = 9^*$
ii) $6^* - 4^* = 2^*$
iii) $2^* \cdot 6^* = 12^*$
iv) $10^* \div 2^* = 5^*$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

22 ABCD is a rectangle, D is the center of the circle, and B is on the circle. If AD = 4 and CD = 3, then the area of the shaded region is between



(A) 4 and 5 (B) 5 and 6 (C) 6 and 7 (D) 7 and 8 (E) 8 and 9

23 Assume the adjoining chart shows the 1980 U.S. population, in millions, for each region by ethnic group. To the nearest percent, what percent of the U.S. Black population lived in the South?

	NE	MW	South	West
White	42	52	57	35
Black	5	5	15	2
Asian	1	1	1	3
Other	1	1	2	4

(A) 20% (B) 25% (C) 40% (D) 56% (E) 80%

24 A multiple choice examination consists of 20 questions. The scoring is $+5$ for each correct answer, -2 for each incorrect answer, and 0 for each unanswered question. John's score on the examination is 48. What is the maximum number of questions he could have answered correctly?

(A) 9 (B) 10 (C) 11 (D) 12 (E) 16

25 Ten balls numbered 1 to 10 are in a jar. Jack reaches into the jar and randomly removes one of the balls. Then Jill reaches into the jar and randomly removes a different ball. The probability that the sum of the two numbers on the balls removed is even is

(A) $\frac{4}{9}$ (B) $\frac{9}{19}$ (C) $\frac{1}{2}$ (D) $\frac{10}{19}$ (E) $\frac{5}{9}$

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1. E The sum is .426.

2. B $\frac{2}{25} = \frac{8}{100} = .08$.

3. E Pairing the addends as shown, we see the desired product is $(2)(5)(180) = 1800$.

$$2(\ 81 + 83 + 85 + 87 + 89 + 91 + 93 + 95 + 97 + 99\ )$$

4. C A right angle is 1/4 of a full circle, so there are $\frac{1}{4}(500) = 125$ clerts in a right angle.

5. A The area is $(.4\text{ m})(.22\text{ m}) = .088\text{ m}^2$.

6. B A negative product occurs when multiplying a positive number by a negative number. The minimum product will occur, then, when multiplying the smallest negative number by the largest positive number. In this case, that product is $(-7)(3) = -21$.

7. C The three faces shown of the larger cube must contain half the smaller shaded cubes, so there are $2(4 + 5 + 1) = 20$ shaded smaller cubes. By carefully looking at the large cube, we see that each smaller cube has at most one face (contained in a face of the large cube) shaded.

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8\. B $7 + 3 + B > 9$, so we "carry 1" from the ten's column to the hundred's column. Similarly $1 + 8 + A > 9$ since $A > 0$, so we "carry 1" from the hundred's place to the thousand's place. $1 + 9 = 10$, so the sum is a 5-digit number of the form 10CD9.

OR

Since A is a digit from 1 to 9, the sum 9876 + A32 must be between 10,000 and 100,000. Thus the sum of the three whole numbers must have 5 digits.

9\. C The least common denominator(LCD) is the least common multiple of the denominators 2,3,4,5,6,7 or $2\cdot2\cdot3\cdot5\cdot7 = 420$.

10\. B The desired sum may be written as $299(4 + 3 + 2 + 1) - 1 = 299(10) - 1 = 2990 - 1 = 2989$.

11\. B The desired sum can be rewritten as $2 + 3 + 5 + \frac{1}{7} + \frac{1}{2} + \frac{1}{19}$ which equals $10 + \frac{1}{2} + \left(\text{a number less than } \frac{1}{2}\right)$ so B is correct.

12\. C The large rectangular region can be subdivided into 24 congruent rectangular regions of which 2 are shaded.

OR

$\frac{1}{3}$ of $\frac{1}{4} = \frac{1}{12}$ of the rectangular region is shaded.

13\. E If the numerator of a fraction is less than half its denominator, then the value of the fraction is less than $\frac{1}{2}$. Consequently all the fractions other than (E) are less than $\frac{1}{2}$ while $\frac{151}{301} > \frac{1}{2}$.

14\. B There are $60\cdot60 = 3600$ seconds in an hour. Thus the computer does $3600\cdot10{,}000 = 36{,}000{,}000$ or 36 million additions in an hour.

15. D The regular price for three tires is \$240 - \$3 = \$237. Thus the regular price of one tire is $\frac{\$237}{3} = \$79$.

16. E In order to have a seasonal shooting average of 50% when having attempted 30 + 10 = 40 shots, Joyce must have made 20 of them. Thus she made 8 of her 10 shots in the next game.

17. D Since Bret is in seat #3 and statement (1) is false, Carl must be in seat #1. Then, since statement (2) is false, Abby must be in seat #4 leaving Dana in seat #2.

18. C The 12 people not dancing are $\frac{2}{3}$ of the people remaining, so 18 people remained. Thus there were $2(18) = 36$ people in the room originally.

19. A The next four numbers displayed are 4, 16, 256, and $256^2 \approx 60{,}000$. Thus 500 is exceeded on the fourth depression of the $\boxed{x^2}$ key.

20. A To show the statement is false, we must find a value of n so that n is not prime and $n - 2$ is prime. Such a value is $n = 9$.

21. C Statements i and ii are false; iii and iv are true.

i. $3* + 6* = \frac{1}{3} + \frac{1}{6} \neq \frac{1}{9}$

ii. $6* - 4* = \frac{1}{6} - \frac{1}{4} \neq \frac{1}{2}$

iii. $2*\cdot 6* = \frac{1}{2}\cdot\frac{1}{6} = \frac{1}{12} = 12*$

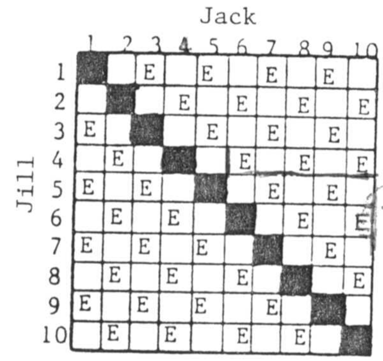
iv. $10* \div 2* = \frac{1}{10}\cdot\frac{2}{1} = \frac{1}{5} = 5*$

22. D By the Pythagorean Theorem, AC = 5. Since AC = BD, the radius of the circle is 5. The area of the shaded region, then, is $\frac{1}{4}\cdot\pi\cdot 5^2 - 3\cdot 4$ (a quarter circle with a rectangle deleted) $= \frac{25\pi}{4} - 12$. This quantity is clearly greater than $\frac{76}{4} - 12 = 7$ and less than $\frac{25(3\frac{1}{5})}{4} - 12 = 8$ since $3 < \pi < 3\frac{1}{5}$.

23. D The total Black population is the sum of the Black populations in each of the four regions or $5 + 5 + 15 + 2 = 27$ million. Thus $\frac{15}{27} = \frac{5}{9} \approx 55.56\%$ lived in the South. (The population figures were taken from the World Almanac and adjusted slightly for convenience.)

24. D If John answered 13 or more questions correctly, then his score would have been at least $13(5) - 7(2) = 51$ (13 correct, 7 incorrect). Checking the other cases, we find that John could have answered 12 correctly, 6 incorrectly, and left 2 unanswered for a score of $12(5) - 6(2) = 48$. Note that 10 correct, 1 incorrect, 9 unanswered also give a score of 48.

25\. A Since Jack and Jill cannot remove the same number, there are $10 \cdot 9 = 90$ ways they can remove the two balls from the jar as shown by the unshaded squares on the grid. Those squares representing an even sum are labeled "E". There are 40 such squares - 4 in each column (or row) since the two numbers must both be odd or both be even. The probability is $\frac{40}{90} = \frac{4}{9}$.



OR

There are 10 ways to select the first number but only 4 ways to select the second since it must have the same parity (both odd or both even) as the first. Thus the probability is $\frac{10 \cdot 4}{10 \cdot 9} = \frac{4}{9}$.