Art of Problem Solving

1986 AMC 8

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1 In July 1861, 366 inches of rain fell in Cherrapunji, India. What was the average rainfall in inches per hour during that month?

(A) $\frac{366}{31 \times 24}$ **(B)** $\frac{366 \times 31}{24}$ **(C)** $\frac{366 \times 24}{31}$ **(D)** $\frac{31 \times 24}{366}$ **(E)** $366 \times 31 \times 24$

2 Which of the following numbers has the largest reciprocal?

(A) $\frac{1}{3}$ **(B)** $\frac{2}{5}$ **(C)** 1 **(D)** 5 **(E)** 1986

3 The smallest sum one could get by adding three different numbers from the set $\{7, 25, -1, 12, -3\}$ is

(A) -3 **(B)** -1 **(C)** 3 **(D)** 5 **(E)** 21

4 The product $(1.8)(40.3 + .07)$ is closest to

(A) 7 **(B)** 42 **(C)** 74 **(D)** 84 **(E)** 737

5 A contest began at noon one day and ended 1000 minutes later. At what time did the contest end?

(A) $10:00$ p.m. **(B)** midnight **(C)** $2:30$ a.m. **(D)** $4:40$ a.m. **(E)** $6:40$ a.m.

6 $\frac{2}{1-\frac{2}{3}} =$

(A) -3 **(B)** $-\frac{4}{3}$ **(C)** $\frac{2}{3}$ **(D)** 2 **(E)** 6

7 How many whole numbers are between $\sqrt{8}$ and $\sqrt{80}$?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

8 In the product $B2 \times 7B = 6396$, B is a digit. The value of B is

(A) 3 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

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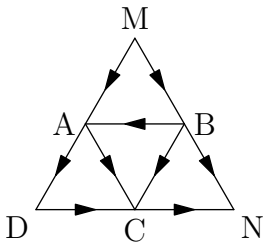

Using only the paths and the directions shown, how many different routes are there from M to N?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

10 A picture 3 feet across is hung in the center of a wall that is 19 feet wide. How many feet from the end of the wall is the nearest edge of the picture?

(A) $1\frac{1}{2}$ **(B)** 8 **(C)** $9\frac{1}{2}$ **(D)** 16 **(E)** 22

11 If $A \star B$ means $\frac{A+B}{2}$, then $(3 \star 5) \star 8$ is

(A) 6 **(B)** 8 **(C)** 12 **(D)** 16 **(E)** 30

12

TEST 1 \ TEST 2	A	B	C	D	F
A	2	2	1	0	0
B	1	4	3	0	0
C	1	3	5	2	0
D	0	0	1	1	1
F	0	0	2	1	0

The table displays the grade distribution of the 30 students in a mathematics class on the last two tests. For example, exactly one student received a "D" on Test 1 and a "C" on Test 2. What percent of the students received the same grade on both tests?

(A) 12% **(B)** 25% **(C)** $33\frac{1}{3}\%$ **(D)** 40% **(E)** 50%

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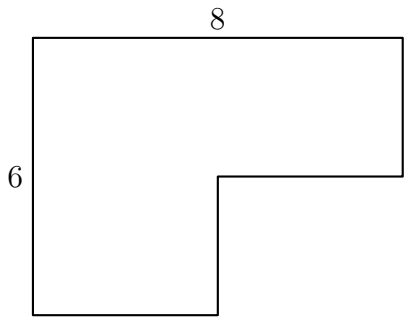


Given that all angles shown are marked, the perimeter of the polygon shown is

(A) 14 **(B)** 20 **(C)** 28 **(D)** 48 **(E)** cannot be determined from the information

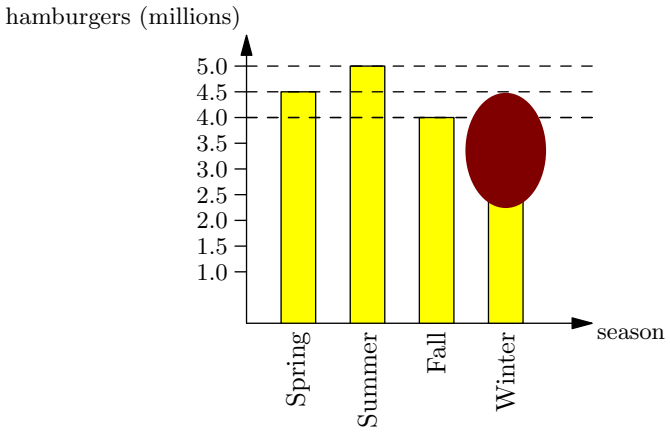
14 If $200 \le a \le 400$ and $600 \le b \le 1200$, then the largest value of the quotient $\frac{b}{a}$ is

(A) $\frac{3}{2}$ **(B)** 3 **(C)** 6 **(D)** 300 **(E)** 600

15 Sale prices at the Ajax Outlet Store are 50% below original prices. On Saturdays an additional discount of 20% off the sale price is given. What is the Saturday price of a coat whose original price is \$180?

(A) \$54 **(B)** \$72 **(C)** \$90 **(D)** \$108 **(E)** \$110

16



A bar graph shows the number of hamburgers sold by a fast food chain each season. However, the bar indicating the number sold during the winter is covered by a smudge. If exactly 25% of the chain's hamburgers are sold in the fall, how many million hamburgers are sold in the winter?

(A) 2.5 **(B)** 3 **(C)** 3.5 **(D)** 4 **(E)** 4.5

17 Let o be an odd whole number and let n be any whole number. Which of the following statements about the whole number $(o^2 + no)$ is always true?

(A) it is always odd **(B)** it is always even **(C)** it is even only if n is even **(D)** it is odd only if n

18 A rectangular grazing area is to be fenced off on three sides using part of a 100 meter rock wall as the fourth side. Fence posts are to be placed every 12 meters along the fence including the two posts where the fence meets the rock wall. What is the fewest number of posts required to fence an area 36 m by 60 m?

(A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 16

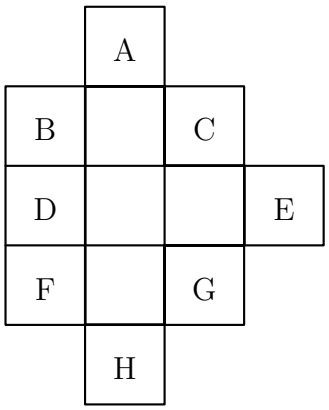
19 At the beginning of a trip, the mileage odometer read 56200 miles. The driver filled the gas tank with 6 gallons of gasoline. During the trip, the driver filled his tank again with 12 gallons of gasoline when the odometer read 56560. At the end of the trip, the driver filled the tank again with 20 gallons of gasoline. The odometer read 57060. To the nearest tenth, what was the car's average miles-per-gallon for the entire trip?

(A) 22.5 **(B)** 22.6 **(C)** 24.0 **(D)** 26.9 **(E)** 27.5

20 The value of the expression $\frac{(304)^5}{(29.7)(399)^4}$ is closest to

(A) .003 **(B)** .03 **(C)** .3 **(D)** 3 **(E)** 30

21



Suppose one of the eight lettered identical squares is included with the four squares in the T-shaped figure outlined. How many of the resulting figures can be folded into a topless cubical box?

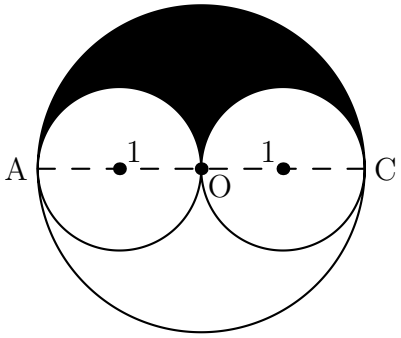
(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

22

Alan, Beth, Carlos, and Diana were discussing their possible grades in mathematics class this grading period. Alan said, "If I get an A, then Beth will get an A." Beth said, "If I get an A, then Carlos will get an A." Carlos said, "If I get an A, then Diana will get an A." All of these statements were true, but only two of the students received an A. Which two received A's?

(A) Alan, Beth **(B)**Beth, Carlos **(C)**Carlos, Diana **(D)**Alan, Diana **(E)**Beth,

23



The large circle has diameter $\overline{AC}$. The two small circles have their centers on $\overline{AC}$ and just touch at O, the center of the large circle. If each small circle has radius 1, what is the value of the ratio of the area of the shaded region to the area of one of the small circles?

(A) between $\frac{1}{2}$ and 1 **(B)** 1 **(C)** between 1 and $\frac{3}{2}$ **(D)** between $\frac{3}{2}$ and 2**(E)** canno

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24 The 600 students at King Middle School are divided into three groups of equal size for lunch. Each group has lunch at a different time. A computer randomly assigns each student to one of the three lunch groups. The probability that the three friends, Al, Bob, and Carol, will be assigned to the same lunch group is approximately:

(A) $\frac{1}{27}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{1}{8}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{1}{3}$

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25 Which of the following sets of whole numbers has the largest average?

(A) multiples of 2 between 1 and 101 **(B)** multiples of 3 between 1 and 101**(C)** multiples

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1. (A) The average rainfall per hour equals the total rainfall divided by the total number of hours. Since July has 31 days, the average is $\frac{366}{31\times 24}$ inches per hour.

2. (A) A large positive number has a small reciprocal and vice-versa. The smallest positive number $\frac{1}{3}$ has the largest reciprocal, $\frac{1}{\frac{1}{3}} = 3$.

3. (C) The three smallest numbers in the set are -3, -1, 7. Their sum is $-3 + -1 + 7 = 3$.

4. (C) The desired product is about $2(40) - .2(40) = 80 - 8 = 72$, so (C) is correct.

5. (D) Since 1000 minutes = $\frac{1000}{60}$ hours = $16\frac{2}{3}$ hours = 16 hours, 40 minutes, the contest ended 16 hours 40 minutes past noon or at 4:40 a.m.

6. (E) $\frac{2}{1-\frac{2}{3}} = \frac{2}{\frac{1}{3}} = 2\times 3 = 6$.

7. (B) Since $\sqrt{8} < \sqrt{9} = 3$, and $\sqrt{80} < \sqrt{81} = 9$, the desired whole numbers are 3,4,5,6,7,8; there are six of them.

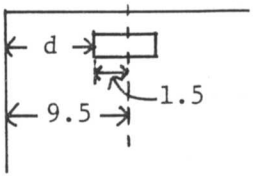
8. (E) If $B\times 2$ ends in 6, then B is 3 or 8. Since the product exceeds 6000, B must be 8.

9. (E) The possible routes are MADCN, MACN, MBADCN, MBACN, MBCN, and MBN.

Query: Can you think of a systematic method to count the routes for more complicated figures?

10. (B) Using the diagram, we see that the distance

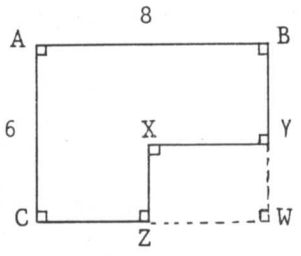
d = 9.5 feet - 1.5 feet = 8 feet.



11. (A) $(3*5)*8 = \left(\frac{3+5}{2}\right)*8 = 4*8 = \frac{4+8}{2} = 6.$

12. (D) A student received the same grade on both tests if s/he is counted on the main diagonal (from the top left to the bottom right) of the table. Thus the number of students receiving the same grade on both tests is $2 + 4 + 5 + 1 + 0 = 12$. Consequently $\frac{12}{30} = \frac{4}{10} = 40\%$ of the students received the same grade on both tests.

13. (C) Since XY = ZW and XZ = YW, the perimeter of polygon ABYXZC is equal to the perimeter of rectangle ABWC, or $2(8 + 6) = 28$. Note that the solution to the problem does not depend on the position of the point X inside the rectangle.



14. (C) The maximum value for the quotient $\frac{b}{a}$ is formed by choosing the largest possible value for b and the smallest possible value for a, or $\frac{1200}{200} = 6$.

15. (B) The sale price of the coat is 50% of \$180 or \$90. The additional discount is 20% of \$90 or \$18, so the Saturday price is \$90 - \$18 = \$72.

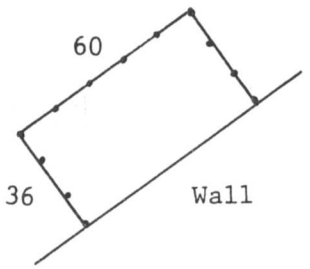
OR

The Saturday price is 80% of the sale price and the sale price is 50% of the original price, so the Saturday price is .8(.5(\$180)) = .8(\$90) = \$72.

16. (A) If the fall sales of 4 million hamburgers are 25% of the yearly sales, then the yearly sales are 16 million hamburgers. Thus the winter sales are $16 - (4.5 + 5 + 4) = 2.5$ million.

17. (E) The number o^2 is always odd. Now (no) is odd if n is odd and even if n is even. Thus the sum $(o^2 + no)$ is even if n is odd, since the sum of two odd numbers is even; the sum is odd if n is even, since the sum of an odd number and an even number is odd.

18. (B) The fewest number of posts is used if the wall serves as the longer side of the rectangular grazing area. Thus there are 6 posts on the 60 meter side (including the corners) and 3 more posts on each 36 meter side for a total of 12 posts.



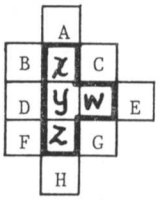
19. (D) The trip was 57,060 - 56,200 = 860 miles long and 12 + 20 = 32 gallons of gasoline were used during the trip. Thus the average number of miles per gallon was $\frac{860}{32} = 26.9$. Note that the 6 gallons needed to fill the tank at the start of the trip had no effect on the answer. No matter how many gallons were needed to fill the tank at the start, the average miles per gallon for the trip would be 26.9.

20. (D) Estimate each of the quantities to obtain the approximation:

$$\frac{(300)^5}{30(400)^4} = \frac{300}{30}\left(\frac{300}{400}\right)^4 = 10\left(\frac{3}{4}\right)^4 = 10\left(\frac{81}{256}\right)$$

$$\approx 10\left(\frac{1}{3}\right) \approx 3.$$

21. (E) Label the four squares in the T-shaped figure X,Y,Z,W as shown. First think of W as the base of the cube. Then X,Y,Z will be three of the "sides" and the fourth side could be A, E, or H. Now think of Y as the base. Then X,W,Z are three sides and B, D, or F could be the fourth side. C and G are not possible because four sides of a cube cannot come together at a point.



22. (C) If Alan received an "A", then all the others would have received an "A". If Beth received an "A", then Carlos and Diana would also have received an "A". Thus only Carlos and Diana received an "A".

23. (B) The radius of the large circle is 2 since it is a diameter of a small circle. The area of the large circle is $\pi(2)^2 = 4\pi$ and the area of each small circle is π. The shaded area is (by symmetry) half the difference of the areas of the large circle and the two small circles, i.e. $\frac{1}{2}(4\pi - 2\pi) = \pi$. Thus the desired ratio is 1.

24. (B) Al must be assigned to one of the lunch groups. The probability that Bob is assigned to the same lunch group is approximately $\frac{1}{3}$ ($\frac{199}{599}$ exactly) and the probability that Carol is assigned to that same group is also approximately $\frac{1}{3}$ ($\frac{198}{598}$). Thus the probability that all three are assigned to the same group is approximately $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.

25. (D) In a set of whole numbers which are equally spaced, the average of the numbers in the set is the average of the smallest number and the largest number. For example, the average of $\{ 1,3,5,7,9 \} = \frac{1+9}{2} = 5$, and the average of $\{ 2,5,8,11,14,17 \}$ is $\frac{2+17}{2} = 9.5$. In this problem, then, the averages are:

A: $\frac{2+100}{2} = 51$, B: $\frac{3+99}{2} = 51$,

C: $\frac{4+100}{2} = 52$, D: $\frac{5+100}{2} = 52.5$,

E: $\frac{6+96}{2} = 51$.

One could "guesstimate" that the set with the "largest" numbers should have the largest average. The numbers 5 and 100 are (overall) larger than the corresponding numbers in the other sets.