MATHCOUNTS®

2013
■ Chapter Competition ■
Sprint Round
Problems 1–30

HONOR PLEDGE

I pledge to uphold the highest principles of honesty and integrity as a Mathlete®. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature ______________________________ Date __________

Printed Name ______________________________

School ______________________________

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

1. ______:______ p.m.

Marti lives in New York and wishes to call her friend Kathy who lives in Honolulu. The chart below shows the times in several cities when it is 12:00 noon in New York. If Marti calls Kathy when the time is 6:30 p.m. in New York, what time is it in Honolulu?

City	**Time when it is 12:00 noon in New York**
Chicago, IL	11:00 AM
Denver, CO	10:00 AM
Los Angeles, CA	9:00 AM
Honolulu, HI	7:00 AM

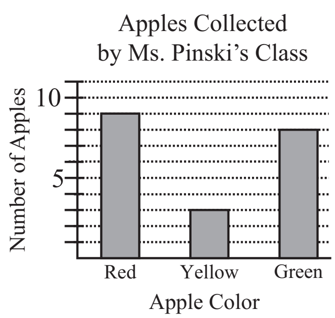
2. ______________

What is the value of $1 - 2 + 4 - 8 + 16 - 32 + 64 - 128 + 256 - 512 + 1024$?

3. ______________ minutes

At a university's graduation ceremony, 690 names are to be read at a pace of one name every 10 seconds. How many minutes will it take to read all of the names?

4. ______________ red apples



The apples collected by Ms. Pinski's class are represented in the bar graph shown. How many more red apples than yellow apples were collected?

5. ______________ cm

The perimeter of a particular rectangle is 18 cm, and the length of the rectangle is one-third its perimeter. What is the width of the rectangle?

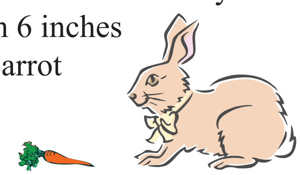
6. ______________

One-half of the sum of n and 8 is equal to 7. What is the value of n?

7. ______________ When $(37 \times 45) - 15$ is simplified, what is the units digit?

8. ______________ One witness to a crime said that the suspect was 25 years old and 69 inches tall. A second witness claimed that the suspect was 35 years old and 74 inches tall. The third witness reported that the suspect was 65 inches tall and 35 years old. Each witness correctly identified either the suspect's height or age, but not both. If a is the suspect's age in years, and b is the suspect's height in inches, what is the value of the sum $a + b$?

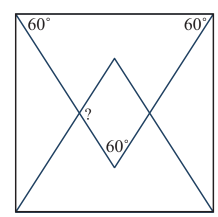
9. ______________ times Zeno's pet rabbit always jumps halfway to a carrot no matter how far away he is from the carrot. He will eat the carrot if he lands within 6 inches of it. How many times must Zeno's rabbit jump to eat a carrot that is initially 12 feet away?



10. ______________ outfits

Mellie has 6 pairs of pants and 10 shirts. She then buys 2 more pairs of pants. If an outfit consists of a pair of pants and a shirt, how many more outfits can Mellie make now compared to the number that she could make before this purchase?

11. ______________ degrees Two equilateral triangles are drawn in a square, as shown. In degrees, what is the measure of each obtuse angle in the rhombus formed by the intersection of the two triangles?



12. $ ______________ Seven pounds of Mystery Meat and four pounds of Tastes Like Chicken cost \$78.00. Tastes Like Chicken costs \$3.00 more per pound than Mystery Meat. In dollars, how much does a pound of Mystery Meat cost?

13. ____________ cm

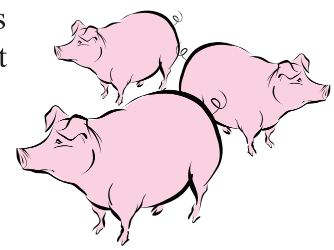
The perimeter of a rectangle is 22 cm and its area is 24 cm^2. What is the smaller of the two integer dimensions of the rectangle?

14. ____________ blocks

On Tuesdays, Mr. Cansetti walks from his home, located at one end of Main Street, to the grocery store at the other end of Main Street. On his way down Main Street, he passes the post office and the police station, in that order. Mr. Cansetti's home is 7.5 blocks from to the police station. The post office is 6 blocks from the grocery store and 3.5 blocks from the police station. How many blocks does Mr. Cansetti walk from his home to the grocery store?

15. ____________ pigs

Farmer Hank has fewer than 100 pigs on his farm. If he groups the pigs five to a pen, there are always three pigs left over. If he groups the pigs seven to a pen, there is always one pig left over. However, if he groups the pigs three to a pen, there are no pigs left over. What is the greatest number of pigs that Farmer Hank could have on his farm?

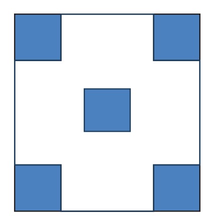


16. ____________ days

Three people can paint 5 rooms in 2 days. Working at this same rate, how many days will it take 6 people to paint 15 rooms?

17. ____________

At the school's carnival, one game featured this unique square dartboard with five smaller, shaded squares, shown here. The length of a side of the square dartboard is 4 times the length of a side of any of the five congruent, shaded squares. To win a prize, a player's dart has to land in a shaded region. If a player's dart randomly hits the dartboard, what is the probability of her winning a prize? Express your answer as a common fraction.

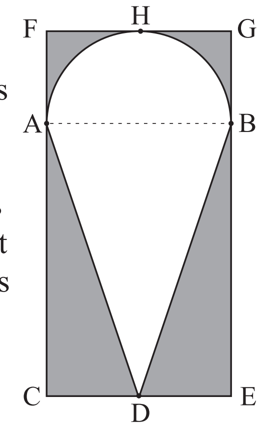


18. ____________

A line passes through the points $(-2, 8)$ and $(5, -13)$. When the equation of the line is written in the form $y = mx + b$, what is the product of m and b?

19. ____________ ft^2

Mr. Mayfeld is designing a sign for his ice cream shop. The sign will be a shape consisting of a semicircle and an isosceles triangle that he will paint to look like a cone with a scoop of ice cream. He will cut the figure out of a rectangular piece of plywood measuring 2 ft by 4 ft, as shown. The shaded regions will be cut away. If BE = 3BG and $\overline{AB}$ is parallel to $\overline{CE}$, what is the total area of the resulting figure? Express your answer as a decimal to the nearest tenth.

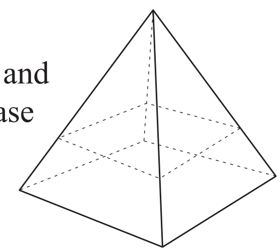


20. ____________ license plates

A state license plate contains the state logo in the center, preceded by three letters and followed by three digits. If the first two letters must both be consonants, excluding Y, how many different license plates are possible?

21. ____________ cm^3

A right square pyramid has a base with a perimeter of 36 cm and a height of 12 cm. At one-third of the distance up from the base to the apex, the pyramid is cut by a plane parallel to its base. What is the volume of the top pyramid?



22. ____________

Four integers are chosen from 1 to 10, inclusive, with repetition allowed. What is the greatest possible difference between the mean and the median of the four integers? Express your answer as a common fraction.

23. ____________ books

Jay and Mike were walking home with heavy books in their backpacks. When Mike complained about the weight in his backpack, Jay remarked, “If I take one of your books, I will be carrying twice as many books as you will be carrying, but if you take one of my books, we’ll be carrying the same number of books.” How many books is Mike carrying in his backpack?

24. ____________ cm

A right rectangular prism has a volume of 720 cm^3. Its surface area is 484 cm^2. If all edge lengths are integers, what is the length of the longest segment that can be drawn that connects two vertices? Express your answer in simplest radical form.

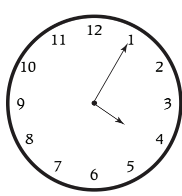
25. ______________ Avi and Hari agree to meet at their favorite restaurant between 5:00 p.m. and 6:00 p.m. They have agreed that the person who arrives first will wait for the other only 15 minutes before leaving. What is the probability that the two of them will actually meet at the restaurant, assuming that the arrival times are random within the hour? Express your answer as a common fraction.

26. ______________ When the local convenience store was bought by a new owner, the old sodas were replaced by new ones that were 20% larger. In addition, the price of the new, larger sodas was 20% less than the price of the old, smaller sodas. What is the ratio of the cost per ounce for the old soda to the cost per ounce for the new soda? Express your answer as a common fraction.

27. ___________ hours Working in pairs, Alana and Bob can complete a job in 2 hours, Bob and Cody can do the job in 3 hours, and Alana and Cody can do the same job in 4 hours. How many hours will it take for Alana, Bob and Cody working together to complete this job? Express your answer as a common fraction.

28. ______________ What fraction of the first 100 triangular numbers are evenly divisible by 7? Express your answer as a common fraction.

29. ______________



The analog clock shown has a minute hand with an arrow tip that is exactly twice as far from the clock's center as the hour hand's arrow tip. If point A is at the tip of the minute hand, and point B is at the tip of the hour hand, what is the ratio of the distance that point B travels in 3 hours to the distance that point A travels in 9 hours? Express your answer as a common fraction.

30. __________ percent What percent of the interval with endpoints −5 and 5 consists of real numbers x satisfying the inequality $x + 1 > \dfrac{8}{x-1}$?

Forms of Answers

The following list explains acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Judges will score competition answers in compliance with these rules for forms of answers.

All answers must be expressed in simplest form. A "common fraction" is to be considered a fraction in the form $\pm\frac{a}{b}$, where a and b are natural numbers and GCF(a, b) = 1. In some cases the term "common fraction" is to be considered a fraction in the form $\frac{A}{B}$, where A and B are algebraic expressions and A and B do not share a common factor. A simplified "mixed number" ("mixed numeral," "mixed fraction") is to be considered a fraction in the form $\pm N\frac{a}{b}$, where N, a and b are natural numbers, $a < b$ and GCF(a, b) = 1. Examples:

Problem: Express 8 divided by 12 as a common fraction. *Answer:* $\frac{2}{3}$ *Unacceptable:* $\frac{4}{6}$

Problem: Express 12 divided by 8 as a common fraction. *Answer:* $\frac{3}{2}$ *Unacceptable:* $\frac{12}{8}$, $1\frac{1}{2}$

Problem: Express the sum of the lengths of the radius and the circumference of a circle with a diameter of $\frac{1}{4}$ as a common fraction in terms of π. *Answer:* $\frac{1+2\pi}{8}$

Problem: Express 20 divided by 12 as a mixed number. *Answer:* $1\frac{2}{3}$ *Unacceptable:* $1\frac{8}{12}$, $\frac{5}{3}$

Ratios should be expressed as simplified common fractions unless otherwise specified. Examples:
Simplified, Acceptable Forms: $\frac{7}{2}$, $\frac{3}{\pi}$, $\frac{4-\pi}{6}$ *Unacceptable:* $3\frac{1}{2}$, $\frac{\frac{1}{4}}{3}$, 3.5, 2:1

Radicals must be simplified. A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:
Problem: Evaluate $\sqrt{15}\times\sqrt{5}$. *Answer:* $5\sqrt{3}$ *Unacceptable:* $\sqrt{75}$

Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., "How many dollars...," "How much will it cost...," "What is the amount of interest...") should be expressed in the form ($) *a.bc*, where *a* is an integer and *b* and *c* are digits. The *only* exceptions to this rule are when a is zero, in which case it may be omitted, or when b and c are both zero, in which case they may both be omitted. Examples:
Acceptable: 2.35, 0.38, .38, 5.00, 5 *Unacceptable:* 4.9, 8.0

Units of measurement are not required in answers, but they must be correct if given. When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lbs 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, $0.25 will not be accepted.

Do not make approximations for numbers (e.g., π, $\frac{2}{3}$, $5\sqrt{3}$) in the data given or in solutions unless the problem says to do so.

Do not do any intermediate rounding (other than the "rounding" a calculator performs) when calculating solutions. All rounding should be done at the end of the calculation process.

Scientific notation should be expressed in the form $a \times 10^n$ where a is a decimal, $1 \le |a| < 10$, and n is an integer. Examples:
Problem: Write 6895 in scientific notation. *Answer:* 6.895×10^3
Problem: Write 40,000 in scientific notation. *Answer:* 4×10^4 *or* 4.0×10^4

An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole number answers should be expressed in their whole number form.
Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.

The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.

MATCOUNTS®

2013
■ Chapter Competition ■
Target Round
Problems 1 and 2

Name ______________________________

School ______________________________

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of eight problems, which will be presented in pairs. Work on one pair of problems will be completed and answers will be collected before the next pair is distributed. The time limit for each pair of problems is six minutes. The first pair of problems is on the other side of this sheet. When told to do so, turn the page over and begin working. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the problem sheets. If you complete the problems before time is called, use the time remaining to check your answers.

Total Correct	Scorer's Initials

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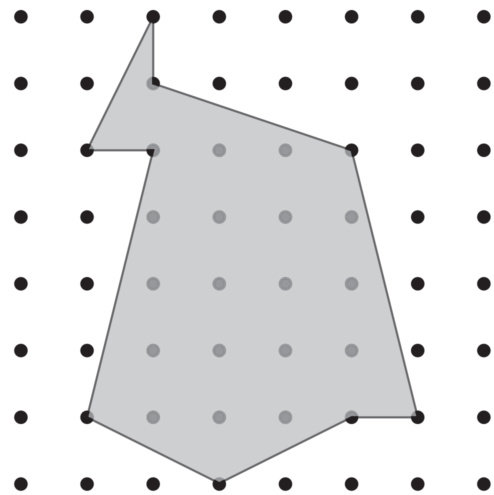
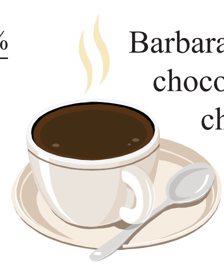
1. ____________ units^2

In the figure shown here, the distance between any two horizontally or vertically adjacent dots is one unit. What is the area of the shaded polygon? Express your answer as a decimal to the nearest tenth.



2. ____________ %



Barbara completely fills her mug with a mixture that is 15 mL of hot chocolate and 35 mL of cream. What percent of the mixture is hot chocolate?

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2013
■ Chapter Competition ■
Target Round
Problems 3 and 4

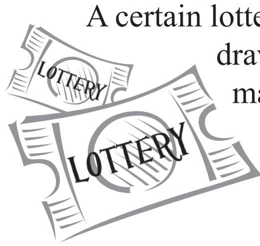
Name ______________________________

School ______________________________

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

Total Correct	Scorer's Initials

3. __________ tickets



A certain lottery has 20 million possible combinations in their weekly drawing. If you wanted to purchase 20 million tickets, how many tickets would you need to purchase each second, on average, to buy them all in one week? Express your answer to the nearest whole number.

4. __________

What is the mean of all possible positive three-digit integers in which no digit is repeated and all digits are prime? Express your answer as a decimal to the nearest hundredth.

MATCHCOUNTS®

2013

■ Chapter Competition ■

Target Round

Problems 5 and 6

Name ______________________________

School ______________________________

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

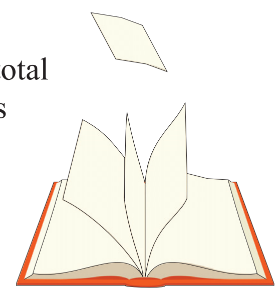
Total Correct	Scorer's Initials

5. ______________

Ray's age is half his sister's age, and his age is the square root of one-third their grandfather's age. In 5 years, Ray will be two-thirds as old as his sister will be then. What is the ratio of Ray's sister's age to their grandfather's age right now? Express your answer as a common fraction.

6. ______________

Alex added the page numbers of a book together and got a total of 888. Unfortunately, he didn't notice that one of the sheets of the book was missing with an odd page number on the front and an even page number on the back. What was the page number on the final page in the book?



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2013
■ Chapter Competition ■
Target Round
Problems 7 and 8

Name ______________________________

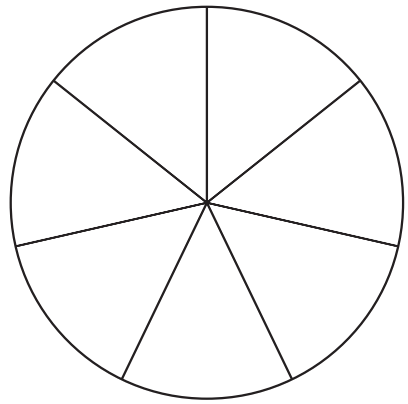
School ______________________________

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

Total Correct	Scorer's Initials

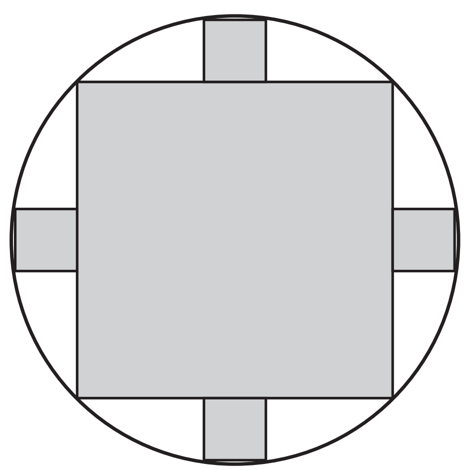
7. ____________ ways

A circular spinner has seven sections of equal size, each of which is colored either red or blue. Two colorings are considered the same if one can be rotated to yield the other. In how many ways can the spinner be colored?



8. ____________ units^2

A square is inscribed in a circle of radius 5 units. In each of the four regions bounded by a side of the square and the smaller circular arc joining the endpoints of that side, a square is drawn so that one side lies on the side of the larger square and the two opposite vertices lie on the circle, as shown. What is the total area of the five squares? Express your answer to the nearest whole number.



MATHCOUNTS®

2013
■ Chapter Competition ■
Team Round
Problems 1–10

School ______________________________

Team Members ______________________________, Captain

____

____

____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 10 problems which the team has 20 minutes to complete. Team members may work together in any way to solve the problems. Team members may talk to each other during this section of the competition. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. The team captain must record the team's official answers on his/her own competition booklet, which is the only booklet that will be scored. If the team completes the problems before time is called, use the remaining time to check your answers.

Total Correct	Scorer's Initials

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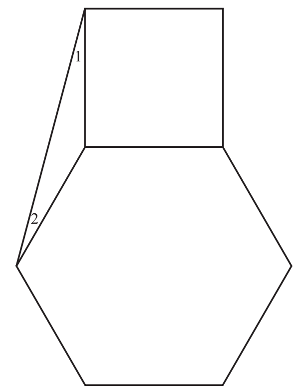
1. ______________ %

At 840,000 mi^2, Greenland is three times the size of Texas. By comparison, Iceland is only 39,800 mi^2. What percent of Texas would be covered by Iceland? Express your answer as a decimal to the nearest tenth.

2. ______________ degrees

A square and a regular hexagon are coplanar and share a common side as shown. What is the sum of the degree measures of angles 1 and 2?



3. ______________ students

A field trip costs \$9000 for the bus, plus \$125 per student. Each student pays \$375. How many students must go on the trip so that the total amount paid is equal to the total cost of the trip?

4. ______________

ΔABC has vertices at A(−3, 4), B(5, 0) and C(1, −4). What is the *x*-coordinate of the point where the median from C intersects $\overline{AB}$?

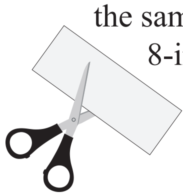
5. ______________

The sum of three primes is 125. The difference between the largest and the smallest is 50. What is the largest possible median of these three prime numbers?

6. ______________ What is the probability that a randomly selected integer from 1 to 81, inclusive, is equal to the product of two one-digit numbers? Express your answer as a common fraction.

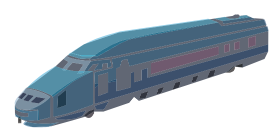
7. ______________ Shaina has one stick of length a cm and another of length b cm, where $a \neq b$. She needs a third stick with length strictly between 8 cm and 26 cm to make the third side of a triangle. What is the product ab?

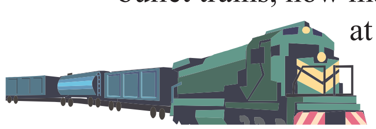
8. ___________ inches A 3-inch by 8-inch sheet of paper and a 2-inch by 12-inch sheet of paper have the same area. Using just one cut (not necessarily straight), the 3-inch by 8-inch sheet can be divided into two pieces that can be rearranged to completely cover the 2-inch by 12-inch sheet. What is the length of the cut?



9. ______________ Suppose $d = 3 + 33 + 303 + 3003 + 30{,}003 + \ldots$, where each addend after the second has one more "interior" 0 than the previous addend. If the last addend has thirty digits, what is the sum of the digits of d?

10. ___________ seconds A bullet train traveling 210 km/h is catching up to a freight train traveling 90 km/h on a parallel track. From the time the front of the bullet train catches up to the back of the freight train to the time the back of the bullet train pulls even with the front of the freight train 24 seconds elapse. If freight trains are three times as long as bullet trains, how many seconds would it take two bullet trains, each traveling at 210 km/h, to pass by each other completely when moving in opposite directions? Express your answer as a decimal to the nearest hundredth.





MATHCOUNTS

2013 Chapter Competition Solutions

Are you wondering how we could have possibly thought that a Mathlete® would be able to answer a particular Sprint Round problem without a calculator?

Are you wondering how we could have possibly thought that a Mathlete would be able to answer a particular Target Round problem in less 3 minutes?

Are you wondering how we could have possibly thought that a Mathlete would be able to answer a particular Team Round problem with less that 10 sheets of scratch paper?

The following pages provide solutions to the Sprint, Target and Team Rounds of the 2013 MATHCOUNTS® Chapter Competition. Though these solutions provide creative and concise ways of solving the problems from the competition, there are certainly numerous other solutions that also lead to the correct answer, and may even be more creative or more concise! We encourage you to find numerous solutions and representations for these MATHCOUNTS problems.

Special thanks to volunteer author Mady Bauer for sharing these solutions with us and the rest of the MATHCOUNTS community!

2013 Chapter Competition

Sprint Round

1. Marti, who lives in New York, calls Kathy, who lives in Honolulu. Marti calls at 6:30 p.m. in New York.
   The chart shows that when it is noon in New York it is 7 AM in Honolulu. That means that Honolulu is 5 hours earlier than New York.
   6:30 p.m. – 5 = 1:30 p.m. **Ans.**

2. What is the value of 1 – 2 + 4 – 8 + 16 – 32 + 64 – 128 + 256 – 512 +1024?
   1 + (– 2 + 4) + (– 8 + 16) + (– 32 + 64) + (– 128 + 256) + (– 512 + 1024) =
   1 + 2 + 8 + 32 + 128 + 512 = 683 **Ans.**

3. 690 names are read at a rate of 1 name every 10 seconds.
   This means that 60 ÷ 10 = 6 names are read every minute. Therefore, it will take 690 ÷ 6 = 115 minutes. 115 **Ans.**

4. According to the bar graph, Ms. Pinski's class collected 9 red apples, 3 yellow apples and 8 green apples.
   There are 9 – 3 = 6 more red apples than yellow apples. 6 **Ans.**

5. The perimeter of a rectangle is 18 cm. The length of the rectangle is one-third of its perimeter so what is the width?
   Let l represent the length of the rectangle and w represent its width. We have $l = (1/3) \times 18 = 6$. Using the perimeter formula, we have
   $2l + 2w = 18$
   $(2 \times 6) + 2w = 18$
   $12 + 2w = 18$
   $2w = 6$
   $w = 3$ **Ans.**

6. One-half of the sum of n and 8 is 7. Find n. Let's rewrite this as an equation and solve:
   $\frac{1}{2}(n + 8) = 7$
   $n + 8 = 14$
   $n = 6$ **Ans.**

7. When (37 × 45) – 15 is simplified, what is the units digit?
   Consider the units digits of 37 and 45. Since 7 × 5 = 35, the units digit of 37 × 45 is 5. Since 15 also has a units digit of 5, subtracting results in a units digit of 0. **Ans.**

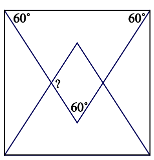
8. One witness says the suspect is 25 years old and 69 inches tall. A second witness says the suspect is 35 years old and 74 inches tall. And a third witness says the suspect is 35 years old and 65 inches tall. Each witness correctly identified the suspect's age or height but not both. Let a represent the suspect's age and b represent the suspect's height. We must find $a + b$.
   Let's assume the first witness identified the suspect's correct age, 25 years old. That means the other two witnesses were wrong about the age, so they must have correctly identified the suspect's height. But they gave two different values for the height. So, the first witness must have correctly identified the suspect's height of 69 inches, and the suspect is 35 years old. Therefore, $a = 35$, $b = 69$ and $a + b = 104$ **Ans.**

9. The rabbit jumps halfway to a carrot. If the rabbit lands within 6 inches of the carrot he will eat it. If the rabbit is originally 12 feet away, how many times must the rabbit jump in order to eat the carrot?
   After the first jump, the rabbit's distance

from the carrot is 6 feet. After the second jump, it's 3 feet. After the third jump, it's 1.5 feet, or 18 inches. After the fourth jump, it's 9 inches. After the fifth jump, it's 4.5 inches, which means the rabbit has a rather tasty carrot. 5 **Ans.**

10. Mellie has 6 pairs of pants and 10 shirts. She buys 2 more pairs of pants. How many more outfits can Mellie make now?
    Each new pair of pants can be paired with each of the 10 shirts, to make a total of $2 \times 10 = 20$ additional outfits.
    20 **Ans.**

11. Two equilateral triangles are drawn in a square. Find the measure of each obtuse angle in the rhombus formed by the intersection of the triangles.



    The rhombus formed by the intersection of the triangles has two opposite angles of 60°. The other two angles are both the same size.
    Let x represent the measure of one of the two unknown angles in the rhombus. Since the sum of the interior angles of a quadrilateral is 360°, we have
    $2x + 60 + 60 = 360$
    $2x + 120 = 240$
    $2x = 240$
    $x = 120$ **Ans.**

12. For 7 lbs of Mystery Meat and 4 lbs of Tastes Like Chicken the cost is $78. Tastes Like Chicken costs $3 more per pound than Mystery Meat. So how much does a pound of Mystery Meat cost?
    Let m represent the cost per pound of Mystery Meat, and c represent the cost per pound of Tastes Like Chicken. We have two equations: $7m + 4c = 78$ and $c = m + 3$. Substitute and solve to get
    $7m + 4(m + 3) = 78$
    $7m + 4m + 12 = 78$
    $11m = 66$
    $m = 6$ **Ans.**

13. The perimeter of a rectangle is 22 cm. The area is 24 cm^2. What is the smaller of the two integer dimensions of the rectangle?
    Let l represent the length of the rectangle and w represent its width. Then $2(l + w) = 22$ and $l + w = 11$. We also know $lw = 24$.
    The factors of 24 are:
    1×24
    2×12
    3×8
    4×6
    Of these, only 3 and 8 add to 11. The smaller of the two integer dimensions must be 3. **Ans.**

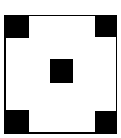
14. Mr. Cansetti's home is 7.5 blocks from the police station. The post office is 6 blocks from the grocery store and 3.5 blocks from the police station. The order in which the buildings are is Mr. Cansetti's home, the post office, the police station and then the grocery store. How far is it from his home to the store?
    The post office is 3.5 blocks from the police station. Therefore, the police station is 6 – 3.5 = 2.5 blocks from the grocery store. From Mr. Cansetti's home to the post office is 7.5 – 3.5 = 4 blocks. Now we can add it all up: 4 blocks (home to post office) + 3.5 blocks (post office to police) + 2.5 blocks (police to

grocery) is 10 blocks. 10 **Ans.**

15. Hank has less than 100 pigs. 5 pigs to a pen results in 3 pigs left over. 7 pigs to a pen results in 1 pig left over. 3 pigs to a pen results in 0 pigs left over.
Let x represent Hank's number of pigs. We know that x is divisible by 3 and ends in 3 or 8 since there is a remainder of 3 when x is divided by 5. Given that when x divided by 7 leaves a remainder of 1, we must be looking for a number ending in 2 or 7 that is divisible by 7. Those numbers less than 100 are 7, 42, and 77. Add 1 to each and choose the one that is divisible by 3. That must be 77. We have 77 + 1 = 78. Checking, we see that 78 is divisible by 3, 78 ÷ 7 = 11r1 and 78 ÷ 5 = 15r3. 78 **Ans.**

16. 3 people can paint 5 rooms in 2 days. How long does it take 6 people to paint 15 rooms.
It follows that 6 people can paint 10 rooms in 2 days, and 6 people can paint 5 rooms in 1 day. Therefore, 6 people can paint 15 rooms in 3 days. 3 **Ans.**

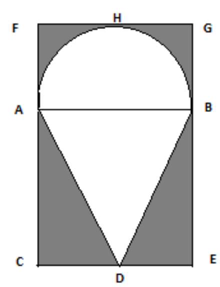
17. A square dartboard has 5 smaller shaded squares. The side of dartboard is 4 times the side of a shaded square. What's the probability that a dart lands in a shaded area?



Let x represent the length of a shaded area. Then x^2 is the area of one of the shaded squares and $5x^2$ is the entire shaded area. The length of the side of the dartboard, then, is $4x$, which means that the area of the dartboard is $16x^2$. The probability of landing in a shaded area is $5x^2/16x^2 = 5/16$ **Ans.**

18. A line passes through the points (−2, 8) and (5, −13). When the equation of the line is written in the form $y = mx + b$, what is the product of m and b.
Recall that $m = (y_2 - y_1)/(x_2 - x_1)$. So, we have $m = (8 - (-13))/(-2 - 5) = 21/(-7) = -3$. Now substituting $y = 8$, $x = -2$ and $m = -3$ into $y = mx + b$, we have
$8 = -3(-2) + b$
$8 = 6 + b$
$b = 2$
$m \times b = -3 \times 2 = -6$ **Ans.**

19. A sign has a shape consisting of a semicircle and an isosceles triangle. The rectangle measures 2 ft by 4 ft. The shaded regions will be removed. BE = 3BG and AB is parallel to CE. Find the area of the resulting region.

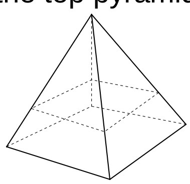


Since FG = 2 and FH = HG = 1, it follows that the semicircle with diameter AB has a radius of 1 and area π/2.
GB is the same length as the radius so it is also 1. Therefore, EB = 4 – 1 = 3, and the area of ΔABD is ½ × 2 × 3 = 3.
Thus, the total area of the isosceles triangle and the semicircle is: 3 + π/2 = 3 + 1.570795 ≈ 4.6 **Ans.**

20. A license plate has three letters followed by 3 digits. The first two letters must be consonants, excluding Y. How many different license plates are there?
There are 26 letters in the alphabet.

There are 5 vowels in the alphabet and we can't use Y.
There are 26 – 6 = 20 choices for the first two letters and 26 choices for the third letter. There are 10 choices for each of the three digits. Therefore, the number of license plates is
$20 \times 20 \times 26 \times 10 \times 10 \times 10 =$
10,400,000 **Ans.**

21. A right square pyramid has a base with a perimeter of 36 cm and a height of 12 cm. One-third of the distance from the base, the pyramid is cut by a plane parallel to its base. What is the volume of the top pyramid?



The volume of a square pyramid is (1/3)*Bh*, where *B* is the area of the base. With a perimeter of 36, each side of the base is 9. Since the plane cuts the pyramid at 1/3 of the distance from the base to the apex, it means that the height of the top pyramids is 2/3 the height of the larger pyramid, or
$(2/3) \times 12 = 8$. That also means the length of each side of the base of the top pyramid is $(2/3) \times 9 = 6$. Therefore, the volume of the top pyramid is
$(1/3) \times 6 \times 6 \times 8 = 96$ **Ans.**

22. Four integers are chosen from 1 to 10, with repetition allowed. What is the greatest possible difference between the mean and median?
To minimize the median we choose 1, 1, 1, 10. Then the median is 1, and the mean is 13/4. The difference is
$(13/4) - (4/4) = 9/4$ **Ans.**

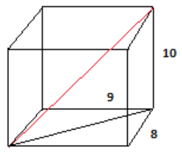
23. If Jay takes one of Mike's books, then he will be carrying twice as many books as Mike. But if Mike takes one of Jay's books, they will each be carrying the same number. We need to find out how many books Mike is carrying.
Let *m* represent the number of books Mike is carrying, *j* represent the number of books Jay is carrying. We have
$j + 1 = 2(m – 1)$
$j + 1 = 2m - 2$
$j = 2m – 3$
When Mike takes one of Jay's books:
$m + 1 = j – 1$
$m = j – 2$
$m = (2m – 3) – 2 = 2m - 5$
$m = 5$ **Ans.**

24. A rectangular prism has a volume of 720 cm^3. Its surface area is 484 cm^2 and all edge lengths are integers. We must determine what the longest segment is that can be drawn to connect two vertices.
Let *w*, *h* and *l* represent the prism's width, height and length, respectively.
We have $lwh = 720$ and
$2(lw + lh + wh) = 484$
$lw + lh + wh = 242$
Let's factor 720 so that we can determine what *l*, *w* and *h* are.
$720 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$
$720 = 8 \times 9 \times 10$
Let's check the surface area:
$(8 \times 9) + (8 \times 10) + (9 \times 10) =$
$72 + 80 + 90 = 242$
So we have a rectangular prism similar to the one shown here:

B

C

A



AB is the length of the longest segment connecting two vertices. Notice that triangle ABC is a right triangle. Segment AC is also the hypotenuse of a right triangle with sides of length 8 and 9. So $(AC)^2 = 8^2 + 9^2 = 64 + 81 = 145$. It follows that

$(AB)^2 = (AC)^2 + 10^2$

$(AB)^2 = 145 + 100$

$(AB)^2 = 245$

AB = $\sqrt{245}$ = $7\sqrt{5}$ **Ans.**

25. Avi and Hari agree to meet between 5:00 p.m. and 6:00 p.m. The first person who arrives will wait for the other for only 15 minutes. Find the probability that the two of them actually meet.
If Avi gets there between 5:00 and 5:45, Hari still has a 15 minute window to arrive. The probability that Avi gets there between 5:00 and 5:45 is 3/4. The probability that Hari arrives in the next 15 minute window is 1/4. So the probability that Avi gets there in the first 45 minutes and Hari gets there within the next 15 minutes is (3/4) × (1/4) = 3/16. Similarly, the probability that Hari gets there between 5:00 and 5:45 and Avi arrives within the next 15 minutes is 3/16. This leaves dealing with the last 15 minutes. There is a probability of (1/4) × (1/4) = 1/16 that both Avi and Hari arrive during those last 15 minutes.
(3/16) + (3/16) + (1/16) = 7/16 **Ans.**

26. Old sodas were replaced by new sodas that were 20% larger. The price of the new soda is 20% less than the price of the old soda. What is the ratio of the cost per ounce of the old soda to the new soda?
Let *x* represent the number of ounces that were in the old soda, and *y* represent the price of the old soda.
The cost per ounce of the old soda is *y*/*x*. At a volume that is 20% larger, the new sodas are (6/5)*x* ounces. The price of the new soda is (4/5)*y*. The cost per ounce for the new soda is (4/5)*y*]/[(6/5)*x*] = (4/6) × (*y*/*x*) = (2/3) × (*y*/*x*).
Therefore, the ratio of the cost per ounce of the old soda to the new soda is (*y*/*x*)/[(2/3) × (*y*/*x*)] = 1/(2/3) = 3/2 **Ans.**

27. Alana and Bob can complete a job in 2 hours. Bob and Cody can do the job in 3 hours. Alana and Cody can do the same job in 4 hours. How many hours will it take all three working together to complete the job?
Since Alana and Bob can complete a job in 2 hours, they can complete 1/2 the job in one hour. Body and Cody can do the job in 3 hours. So, they can complete 1/3 of the job in one hour.
Since Alana and Cody can do the same job in 4 hours they can complete 1/4 of the job in 1 hour.
This means that 2 sets of Alana, Bob and Cody could do (1/2) + (1/3) + (1/4) = (6/12) + (4/12) + (3/12) = 13/12 of the job in one hour. One set of Alana, Bob and Cody can do (13/12) × (1/2) = 13/24 of the job in one hour. It will take them 1/(13/24) = 24/13 hours. 24/13 **Ans.**

28. What fraction of the first 100 triangular numbers is evenly divisible by 7?
The *n*th triangular number is the sum of 1 through *n*. So let's look at the first several and see if we can see a pattern.
1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66,

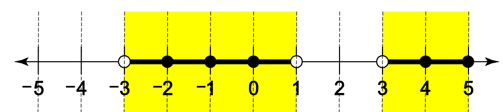
78, 91,105, 120, 136, 153, 171, 190, 210, 221
Looking at this we can see that the 6^{th} and 7^{th} triangular numbers, 21 and 28, are divisible by 7. The 13^{th} and 14^{th} triangular numbers, 105 and 120, are also divisible by 7, as are the 20^{th} and 21^{st} triangular numbers, 210 and 221. We have a pattern. There are 2 triangular numbers divisible by 7 in every 7 numbers. There are 100/7 ≈ 14 sets of 7 numbers. In each set we have 2 and they are always the last two of the 7 numbers. Therefore, we don't have to worry about the 99^{th} and 100^{th} triangular numbers. They will not be divisible by 7. That's 14 × 2 = 28 out of 100, which is 7/25 **Ans.**

29. Point A of a clock is at the tip of the minute hand. Point B is at the tip of the hour hand. Point A is twice as far from the center of the clock as point B. We must find the ratio of the distance that point B travels in 3 hours to the distance that point A travels in 9 hours.
The distance that the hour hand travels when it makes a complete revolution is $2\pi r$, where r is the distance between the tip of the hour hand and the center of the clock. The distance that the minute hand travels when it makes a complete revolution is $4\pi r$. The minute hand of a clock travels around the whole clock once per hour or one entire revolution. So point A travels $9 \times 4\pi r = 36\pi r$ in the 9 hours.
Point B travels only 1/12 of the way around the clock per hour or 360/12 = 30°. So, in 3 hours it travels 90° or 1/4 of a revolution, which is $(1/4) \times 2\pi r = (1/2)\pi r$. The ratio we must find is: $((1/2)\pi r)/(36\pi r) = 1/72$ **Ans.**

30. What percent of the interval with endpoints −5 and 5 consists of real numbers x satisfying the inequality $x + 1 > 8/(x - 1)$?
Let's create a table trying the integers from −5 to 5:

x	$x + 1$	$8/(x - 1)$
−5	−4	−8/6 = −4/3
−4	−3	−8/5
−3	−2	−8/4 = −2
−2	−1	−8/3
−1	0	−8/2 = −4
0	1	−8/1 = −8
1	2	8/0 = undefined
2	3	8/1 = 8
3	4	8/2 = 4
4	5	8/3
5	6	8/4 = 2

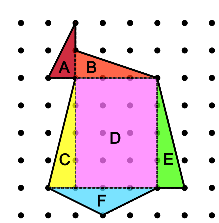
This information is represented on the number line below:



Consider the 10 intervals from −5 to 5. Only 6 of these 10 intervals consist of values that satisfy the given inequality. That's, 6/10 = 60% **Ans.**

Target Round

1. The distance between any two adjacent dots is one unit. Find the area of the shaded polygon.



The shaded polygon has been divided into 6 smaller pieces. The area of each:
A: (1/2) × 2 × 1 = 1
B: (1/2) × 3 × 1 = 1.5
C: (1/2) × 4 × 1 = 2

D: $3 \times 4 = 12$
E: $(1/2) \times 4 \times 1 = 2$
F: $(1/2) \times 4 \times 1 = 2$
Total area: $1 + 1.5 + 2 + 12 + 2 + 2 = 20.5$ **Ans.**

2. A mug is filled with a mixture that is 15 mL of hot chocolate and 35 mL of cream. What percent of the mixture is hot chocolate?
There are 15 + 35 = 50 mL in the entire mixture. So, hot chocolate accounts for 15/50 = 3/10 = 30% **Ans.**

3. A lottery has 20 million combinations. How many tickets would you need to purchase each second to buy all 20 million combinations in 1 week?
There are $60 \times 60 \times 24 \times 7 = 604{,}800$ seconds in one week. So, we have $20{,}000{,}000 \div 604{,}800 = 33.06878 \approx 33$ **Ans.**

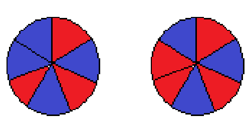
4. Find the mean of all possible positive three-digit integers in which no digit is repeated and all digits are prime.
Prime digits are 2, 3, 5 and 7.
There are 3! = 6 combinations of {2,3,5}, {2, 3, 7}, {2, 5, 7} and {3, 5, 7}.
Starting with 2 we have:
235, 253, 237, 273, 257 and 273.
Starting with 3 we have:
325, 352, 327, 372, 357 and 375.
Starting with 5 we have:
523, 532, 527, 572, 537 and 573.
Starting with 7 we have:
723, 732, 725, 752, 735 and 753.
If we were to add up all the numbers, the units column would contain 6 2s, 6 3s, 6 5s and 6 7s.
That's $(2 \times 6) + (3 \times 6) + (5 \times 6) + (7 \times 6) = 12 + 18 + 30 + 42 = 102$. That gives us a 2 in the units column, and we carry 10 over to the tens column. In the tens column you also have 6 of each number. So that's a sum of 102 plus the 10 we carried over, or 112. That means a 2 in the tens column and we carry over 11. Similarly, there are 6 of each number in the hundreds column. That's a sum of 102 plus the 11 we carried over, or 113.
The 3 is in the hundreds column so the sum is actually 11,322. The mean is 11,322/24 = 471.75 **Ans.**

5. Ray's age is half his sister's age. Ray's age is also the square root of one-third of his grandfather's age. In 5 years, Ray will be two-thirds as old as his sister will be. We need to find the ratio of Ray's sister's age to his grandfather's age.
Let r, s and g represent Ray's, Ray's sister's and Ray's grandfather's ages, respectively. Now let's create the equations. We have
$r = (1/2)s$
$r = \sqrt{(1/3)g}$
$r + 5 = (2/3)(s + 5)$
$(1/2)s + 5 = (2/3)s + 10/3$
Multiply both sides by 6 yields
$3s + 30 = 4s + 20$
$s = 10$
$r = 5$
$r^2 = 25 = (1/3)g$
$g = 25 \times 3 = 75$
$s/g = 10/75 = 2/15$ **Ans.**

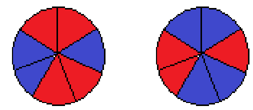
6. Alex added the page numbers of a book and got a total of 888. But there is a page missing. We must find the page number on the final page.
The sum of the pages numbers is $1 + 2 + \ldots + n - 1 + n = 888$. Since the sum of the consecutive integers from 1 to n is $(n/2) \times (n + 1)$, we can write $(n/2) \times (n + 1) = 888$. This isn't entirely accurate because 888 doesn't include the two

page numbers from the missing sheet. Multiplying by 2 to get rid of the fractions, we get $n(n + 1) = 1776$, and $n^2 + n - 1776 = 0$. We could try and solve this, but remember, the sum isn't accurate.
What this does show us is that the difference between the two roots is 1.
Notice that $\sqrt{1776} \approx 42.1426$. So, let's look at the sum of the first 42 numbers. We have $(42/2) \times 43 = 21 \times 43 = 903$, and $903 - 888 = 15$. Well, $15 = 8 + 7$ so that must be the page that is missing. Therefore, the last page must be 42.
42 **Ans.**

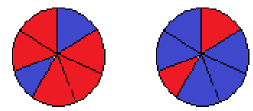
7. A circular spinner has 7 sections of equal size. Each is colored either red or blue. In how many ways can the spinner be colored?
Where you have to be careful is understanding that if you use an odd number of switches (from red to blue or vice versa) the first and last groups of sections are next to each other so the first and last groups are no longer distinct. So we're looking for even numbers of integers that sum to 7 with the exception of all 7 sections being one color.
Let's start with 6 different groups or:
1 + 2 + 1 + 1 + 1 + 1
There are 2 versions (where the red and blue sections are switched).



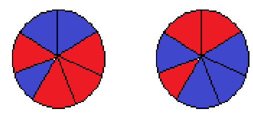
There are no other combinations of 6 groups so let's consider 4 groups..
1 + 2 + 2 + 2



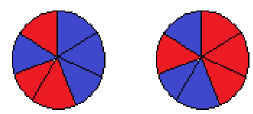
And 1 + 2 + 1 + 3



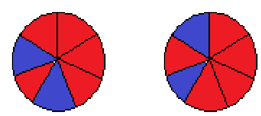
And 1 + 1 + 2 + 3



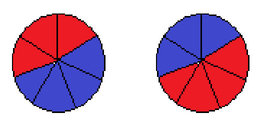
And 1 + 3 + 2 + 1



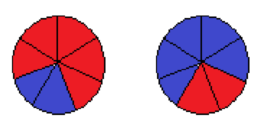
And 4 + 1 + 1 + 1



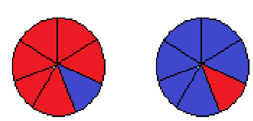
.
Now two groups: 4 + 3



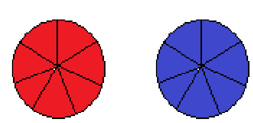
5 + 2.



6 + 1

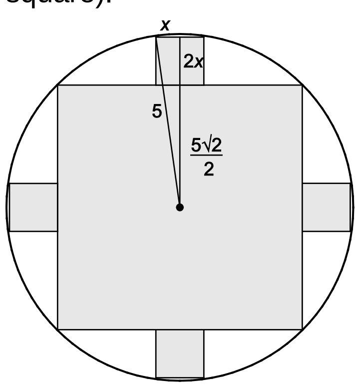


And finally all of the same color:



That's a total of 20. **Ans.**

8. A square is inscribed in a circle of radius 5 units. There are smaller squares, one in each of the regions bounded by a side of the square and the circle as shown. We must find the area of the large square and the four smaller squares.
First, let's determine the side of the large square. If we draw a diagonal in the square, that is the diameter of the circle, or 10. Let s represent the side length of the large square. Based on the properties of 45-45-90 right triangles, we know that $10 = s\sqrt{2}$; thus, $s = 5\sqrt{2}$.
Now, let's draw a right triangle whose long leg and hypotenuse originate at the center of the circle, one extending to the midpoint of the edge of one of the small squares, as shown, and one extending to a point of the same small square that is on the circle. The short leg is a segment connecting the other two (which is half of the side of the small square).



Let $2x$ represent the side length of the small square. The length of the long leg is half the length of the large square plus the length of the small square or $(5\sqrt{2})/2 + 2x$. The length of the hypotenuse is just the radius of the circle, or 5. The length of the short leg, which is half the length of a side of the small square, is x. Using the Pythagorean Theorem, we have

$x^2 + [(5\sqrt{2})/2 + 2x]^2 = 5^2$

$x^2 + 4x^2 + (10\sqrt{2})x + 25/2 = 25$

$5x^2 + (10\sqrt{2})x + 25/2 = 25$

$10x^2 + (20\sqrt{2})x + 25 = 50$

$10x^2 + (20\sqrt{2})x - 25 = 0$

$2x^2 + (4\sqrt{2})x - 5 = 0$

Let's use the quadratic equation to solve this.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $a = 2$, $b = 4\sqrt{2}$ and $c = -5$. We have

$$x = \frac{-4\sqrt{2} \pm \sqrt{(4\sqrt{2})^2 - 4(2)(-5)}}{2(2)}$$

$$x = \frac{-4\sqrt{2} \pm \sqrt{32 + 40}}{4} = \frac{-4\sqrt{2} \pm 6\sqrt{2}}{4}$$

$$x = \frac{3}{2}\sqrt{2} - \sqrt{2} = \frac{\sqrt{2}}{2}$$

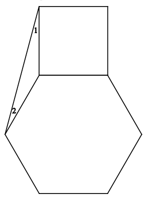
So, $2x = 2 \times \frac{\sqrt{2}}{2} = \sqrt{2}$, and the area of each small square is $(\sqrt{2})^2 = 2$. There are 4 of these squares for a total area of $4 \times 2 = 8$. The area of the large square with side length $5\sqrt{2}$ is $(5\sqrt{2})^2 = 50$. Therefore, the total area of the five squares is $50 + 8 = 58$ **Ans.**

Team Round

1. Greenland is 840,000 square miles. Iceland is only 39,800 square miles. Greenland is 3 times the size of Texas. So what percent of Texas would be covered by Iceland?
$840{,}000 \div 3 = 280{,}000$

39,800 ÷ 280,000 ≈ 14.2% Ans.

2. A square and a regular hexagon share a common side. Find the sum of the degree measures of angles 1 and 2.



Let x and y represent the measures of angles 1 and 2, respectively. Let z represent the measure of the third angle of that triangle, so $x + y = 180 – z$. In a regular hexagon, each interior angle is 120°. Adjacent to the hexagon we have an angle of 90°. That means
$z = 360 − (120 + 90) = 360 – 210 = 150$.
So, $x + y = 180 – 150 = 30$ **Ans.**

3. A trip costs \$9000 for the bus plus \$125 per student. Each student pays \$375. We must find how many students must go on the trip so that the total amount paid is equal to the total cost of the trip. Let x represent the number of students who must go on the trip. Then
$9000 + 125x = 375x$
$250x = 9000$
$x = 36$ **Ans.**

4. Triangle ABC has vertices at A(−3, 4), B(5, 0) and C(1, −4). What is the x-coordinate of the point where the median from C intersects AB?
Since [5 – (−3)]/2 = 8/2 = 4, the x-coordinate is −3 + 4 = 1 **Ans.**

5. The sum of three primes is 125. The difference between the largest and smallest prime is 50. We must find the largest possible median of these three prime numbers.
OK. Let’s list all the primes up to 125.
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31
37, 41, 43, 47, 53, 57, 61, 67, 71, 73
79, 83, 89, 97, 101, 103, 107, 109, 113
Let’s make a list of pairs of primes that are 50 apart. We have {3, 53}, {11, 61}, {17, 67}, {23, 73}, {29, 79}, {47, 97}, {53, 103}, {57, 107}. Some of these choices have a sum greater than 125. Eliminating those leaves us with {3, 53}, {11, 61}, {17, 67}, {23, 73}, (29, 79}.
Now let’s determine which of these two primes will result in a third prime to make them all sum up to 125.
3 + 53 = 56; 125 – 56 = 69; too large and not a prime
11 + 61 = 72; 125 – 72 = 53 which is a prime
17 + 67 = 84; 125 – 84 = 41 which is a prime
23 + 73 = 96; 125 – 96 = 29 which is a prime
29 + 79 = 108; 125 – 108 = 17 which is less than 29
So we have choices of {11, 53, 61}, {17, 41, 67} and {23, 29, 73}
The median of the first set is 53. The median of the second set is 41. The median of the third set is 29. The greatest is 53 **Ans.**

6. What is the probability that a randomly selected integer from 1 to 81, inclusive is equal to the product of two one-digit numbers?
Any number less than or equal to 81 is the product of two one-digit numbers except two-digit primes. Let’s see how many primes there are from 10 to 81:
11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79
That’s a total of 18 primes.
Add to this any other numbers that are multiples of these primes:
11: 22, 33, 44, 55, 66, 77 or 6 more

13: 26, 39, 52, 65, 78 or 5 more
17: 34, 51, 68 or 3 more
19: 38, 57, 76 or 3 more
23: 46, 69 or 2 more
29: 58 or 1 more
31: 62 or 1 more
37: 74 or 1 more
That's a total of 6 + 5 + 3 + 3 + 2 + 1 + 1 + 1 = 22 more. But we're not done yet: There are also a few integers that are multiples of 3 or more digits:
$50 = 2 \times 5 \times 5$
$60 = 2 \times 2 \times 3 \times 5$
$70 = 2 \times 5 \times 7$
$75 = 3 \times 5 \times 5$
$80 = 2 \times 2 \times 2 \times 2 \times 5$
That's a total of 5 more. That brings the total number of 18 + 22 + 5= 45 numbers between 1 and 81, inclusive that cannot be written as the product of two one-digit numbers. Therefore, there are 81 – 45 = 36 that are. That's a probability of 36/81 = 4/9 **Ans.**

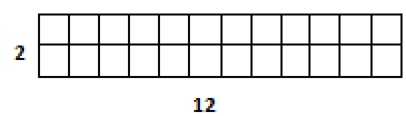
Another way to solve this problem is using the following multiplication table:

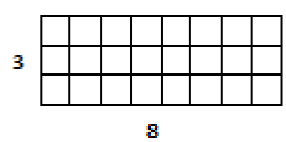
×	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

The perfect squares are located on the diagonal. Since any products located below the perfect square in each column will be duplicates, they've been grayed out. Also, 4, 6, 8, 9, 12, 18, 16, 24 and 36 are duplicates, so they've been grayed out as well. What's left are the 36 different numbers from 1 to 81, inclusive, that are the product of two one-digit numbers. Again, that's a probability of 36/81 = 4/9 **Ans.**

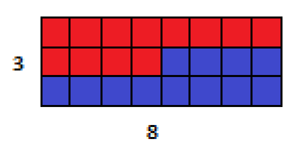
7. We have two sticks, one of length *a* cm and one of length *b* cm. With a stick whose length is strictly between 8 and 26 cm a triangle can be constructed. We must find the product *ab*.
   The length of the third side of a triangle is always less than the sum of the lengths of the two other sides. The length of the third side must also be greater than the difference of the lengths of the two sides *a* and *b*.
   Therefore, $a - b = 8$ and $a + b = 26$. Solving this system by adding the two equations, we get $2a = 24$, and $a = 17$. That means $b = 26 - 17 = 9$. Therefore, $ab = 17 \times 9 = 153$ **Ans.**

8. A 3-inch by 8-inch sheet of paper and a 2-inch by 12-inch sheet of paper have the same area. Using just one cut (not necessarily straight), the 3-inch by 8-inch sheet can be divided into two pieces that can be rearranged to completely cover the 2-inch by 12-inch sheet. Find the length of the cut.



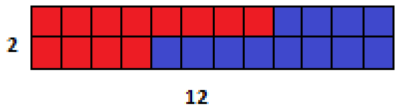


If we cut the 3-by-8 sheet into symmetric "L" shapes, it should work.



Superimposing those cuts onto the

2-by-12 sheet:



The cut is 4 + 1 + 4 = 9 inches. 9 **Ans.**

9. d = 3 + 33 + 303 + 3003 + 30,003 + … Each additional term after the second has one more "interior" 0. The last term has 30 digits. So what is the sum of the digits of d?

$$\begin{array}{r} 3 \\ 33 \\ 303 \\ 3003 \\ 30003 \\ \vdots \\ \underline{+\,300\cdots\cdots\cdots 03} \end{array}$$

Adding the 30 3s in the units column we get 3 × 30 = 90. So, d has a 0 in the units column, and we carry the 9 to the tens column. In the tens column, the sum of the 3 and 28 0s, plus the 9 that was carried is 12. So, d has a 2 in the tens column, and we carry the 1 to the hundreds column. Now in the hundreds column the sum of the 3 and 27 0s, plus the one that was carried is 4. So, d has a 4 in the hundreds column. The sum in each of the remaining 27 columns is 3. Therefore, the sum of the digits of d is (27 × 3) + 4 + 2 + 0 = 87 **Ans.**

10. A bullet train travels 210 km/h. A freight train travels 90 km/h on a parallel track. From the time the bullet train catches up to the back of the freight train to the time the back of the bullet train pulls even with the front of the freight train 24 seconds elapse. Freight trains are three times as long as bullet trains. So how many seconds would it take two bullet trains, each traveling at 210 km/h to pass by each other completely, when moving in opposite directions?
A bullet train that travels 210 km/h travels 210/(60 × 60) = 7/120 km/sec.
A freight train that travels 90 km/h travels 90/(60 × 60) = 3/120 km/sec.
So, when the bullet train starts approaching the freight train to pass, it does so at a rate of (7/120) – (3/120) = 1/30 km/sec. It takes 24 seconds for the back of the bullet train to pull even with the front of the freight train. This means that the front of the bullet train passes the entire freight train and has continued on the full length of the bullet train.
Let f and b represent the lengths of the freight train and bullet train, respectively. We have f = 3b. The number of kilometers that the bullet train travels when its rear has pulled even with the front of the freight train is 24 × (1/30) = 4/5 km. So,
$f + b = 4/5$
$3b + b = 4/5$
$4b = 4/5$
$b = 1/5$ km
Given that the two bullet trains are "nose to nose", every second the front of the one bullet train moves (7/120) × 2 = 7/60 km towards the back of the other bullet train. It will take (1/5)/(7/60) = (1/5) × (60/7) = 12/7 seconds for the front of the first bullet train to reach the back of the second bullet train. It will take another 12/7 seconds for the end of the first bullet train to reach the end of the second bullet train. The time for the two bullet trains to completely pass one another is (12/7) + (12/7) = 24/7 = 3.42857 ≈ 3.43 **Ans.**